A Liquidity-Based Model of Security Design

by De Marzo and Duffie- ECA 1999

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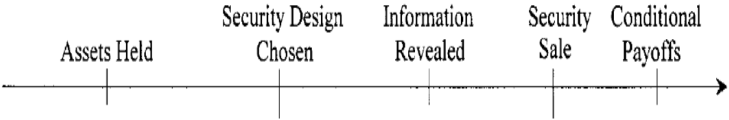
The model

- Outside investors - risk neutral.
- Issuer - risk neutral
  - owns assets that generate random future cash flows X
  - discounts future cash flows by $\delta \in (0,1)$
  - security design: determine payoff structure.
- Information structure:
  - Public signals: $S = (S_1, ..., S_m) \in \mathbb{R}^m$
  - Issuer's private signal: $Z \in \mathbb{R}^n$, informative about S.
- Security market: Signalling game given Z.

Timing


Assets Held
Security Design Chosen
Information Revealed
Security Sale
Conditional Payoffs

Issuer's Problem

- Assume inverse demand function $P_F(q)$
- Designs and issues a security backed by assets.
  - Payoff: $F = \varphi(S) \in \mathbb{R}$, $\varphi(S) \leq X$.
  - Quantity: $q(Z)$
- Conditional valuation of the security after signal $E(\varphi(S)|Z)$
  - Assumption, $E(\varphi(S)|Z)$ distributed over $[f_0, f_1]$

How much to sell

- Given F, Z, and q, seller's utility is

$$\begin{aligned} & \delta E\left(X - F \mid Z\right) + \delta\left(1 - q\right) E\left(F \mid Z\right) + q P_F\left(q\right) \\ = \ & \delta E\left(X \mid Z\right) + q\left[P_F\left(q\right) - \delta E\left(F \mid Z\right)\right] \end{aligned}$$

- Profits of security F given $f = E\left(F \mid Z\right)$

$$\Pi_F\left(f\right) = \sup_{q \in [0,1]} q\left[P_F\left(q\right) - \delta f\right]$$

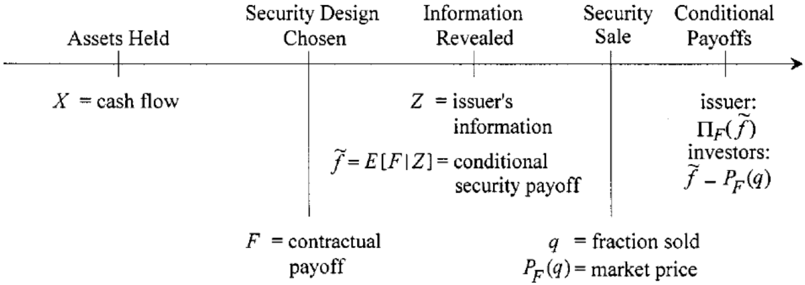
$\Longrightarrow$Monopoly problem.

Security Design

- Choose F to maximize expected profits.
- Value for the issuer

$$\sup_F V(F) = \sup_F E\left[\Pi_F\left(E\left(F\mid Z\right)\right)\right]$$

Timing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Conditional Payoffs
X = cash flow
Z = issuer's information
$\tilde{f} = E[F \mid Z]$ = conditional security payoff
issuer: $\Pi_F(\tilde{f})$
investors: $\tilde{f} - P_F(q)$
F = contractual payoff
q = fraction sold
$P_F(q)$ = market price

Demand for Securities

- Signalling game between investors and issuer.
- The issuer observes Z, computes $\bar{f} = E\left(F \mid Z\right)$ and chooses $Q\left(\bar{f}\right)$ to sell.
- Investors only observe the fraction of the security sold in the market.
- Investors compete for the security: make 0 profits.

Equilibrium

Definition

A Bayes-Nash equilibrium is a pair (P, Q) of measurable functions s.t.:
(i) $Q(\bar{f}) = \arg\max_q q[P(q) - \delta\bar{f}]$ a.s.
(ii) $P(Q(\bar{f})) = E(\bar{f} | Q(\bar{f}))$ a.s..
An equilibrium (P, Q) is separating if
(iii) $P(Q(\bar{f})) = \bar{f}$ a.s..

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- Assumption: Single-crossing property $\Longrightarrow$ A separating equilibrium exists.

Separating Equilibrium

- (Q^*, P^*) is a separating equilibrium, where

$$Q^*(f) = \left(\frac{f_0}{f}\right)^{1/(1-\delta)} \text{ and } P^*(q) = \frac{f_0}{q^{1-\delta}}$$

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  - If $f_0 > 0$, it is the unique separating equilibrium.
- (Q^*, P^*) depends on F only through f_0.

Simplified Design Problem

- Issuer's profit:

$$\Pi_F(f) = (1-\delta)\, f_0^{1/(1-\delta)} f^{-\delta/(1-\delta)} = \Pi(f, f_0)$$

$$\begin{aligned} \Pi_1(f, f_0) &< 0 \\ \Pi(f, f_0) &= f_0 \Pi\left(\frac{f}{f_0}, 1\right) \end{aligned}$$

Design Problem

Choose F such that 0 marginal benefit to the issuer of adding any incremental cash flow to the design.

- Marginal private valuation

$$\gamma\left(F,G\right) = \left.\frac{d}{dk} E\left(\min\left(F+kG,X\right) \middle| Z\right)\right|_{k=0^{+}}$$

- Marginal minimum private valuation

$$\gamma_0\left(F,G\right) = \left.\frac{d}{dk} \min_{Z} E\left(\min\left(F+kG,X\right) \middle| Z\right)\right|_{k=0^{+}}$$

FONC

$$\nabla V\left(F;G\right)=$$

$$\gamma_0\left(F,G\right)E\left[\Pi\left(\frac{\bar{f}}{f_0},1\right)+E\left(\Pi_1\left(\frac{\bar{f}}{f_0},1\right)\left(\frac{\gamma\left(F,G\right)}{\gamma_0\left(F,G\right)}-\frac{\bar{f}}{f_0}\right)\right)\right]$$

FONC

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$$\gamma_0(F,G)\, E\left[\Pi\left(\frac{\bar{f}}{f_0},1\right) + E\left(\Pi_1\left(\frac{\bar{f}}{f_0},1\right)\left(\frac{\gamma(F,G)}{\gamma_0(F,G)} - \frac{\bar{f}}{f_0}\right)\right)\right]$$

- Marginal information sensitivity

$$\frac{\gamma(F,G)}{\gamma_0(F,G)}$$

- Information sensitivity

$$\frac{\bar{f}}{f_0}$$

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- If Z has little impact on the distribution of S, $F = X$ is optimal. (pure equitiy)
- A standard debt contract, $\min(X, d)$, is optimal if:
  - $S = X$
  - $F = \varphi(X)$, $\varphi' > 0$
  - a uniform worst case outcome of Z exists.

Uniform Worst Case

A uniform worst case exists if for all $z_0, z \in Z$ and all $I \in \mathbb{R}_+$

1. if $\mu(X \in I | z) > 0$, then $\mu(X \in I | z_0) > 0$
2. the conditional of $\mu(\cdot | z)$ given $X \in I$ has first-order stochastic dominance ove rthe conditional of $\mu(\cdot | z_0)$ given $X \in I$