A Liquidity-Based Model of Security Design
by De Marzo and Duffie- ECA 1999

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The model

- Outside investors - risk neutral.
- Issuer - risk neutral
  - owns assets that generate random future cash flows $X$
  - discounts future cash flows by $\delta \in (0, 1)$
  - security design: determine payoff structure.

- Information structure:
  - Public signals: $S = (S_1, ..., S_m) \in \mathbb{R}^m$
  - Issuer’s private signal: $Z \in \mathbb{R}^n$, informative about $S$.

- Security market: Signalling game given $Z$. 
Timing

Assets Held  Security Design Chosen  Information Revealed  Security Sale  Conditional Payoffs
Issuer’s Problem

- Assume inverse demand function $P_F(q)$
- Designs and issues a security backed by assets.
  - Payoff: $F = \varphi(S) \in \mathbb{R}, \varphi(S) \leq X$.
  - Quantity: $q(Z)$
- Conditional valuation of the security after signal $E(\varphi(S) | Z)$
  - Assumption, $E(\varphi(S) | Z)$ distributed over $[f_0, f_1]$
How much to sell

- Given $F$, $Z$, and $q$, seller’s utility is

$$\delta E (X - F | Z) + \delta (1 - q) E (F | Z) + qP_F (q)$$

$$= \delta E (X | Z) + q [P_F (q) - \delta E (F | Z)]$$

- Profits of security $F$ given $f = E (F | Z)$

$$\Pi_F (f) = \sup_{q \in [0,1]} q [P_F (q) - \delta f]$$

$\implies$ Monopoly problem.
Security Design

- Choose $F$ to maximize expected profits.
- Value for the issuer

$$\sup_{F} V(F) = \sup_{F} E \left[ \Pi_F \left( E \left( F \mid Z \right) \right) \right]$$
Timing

Assets Held

Security Design Chosen

Information Revealed

Security Sale

Conditional Payoffs

$X = \text{cash flow}$

$Z = \text{issuer's information}$

$\tilde{f} = E[F|Z] = \text{conditional security payoff}$

issuer:

$\Pi_F(f)$

investors:

$\tilde{f} - P_F(q)$

$F = \text{contractual payoff}$

$q = \text{fraction sold}$

$P_F(q) = \text{market price}$
Demand for Securities

- Signalling game between investors and issuer.
- The issuer observes $Z$, computes $\bar{f} = E(F|Z)$ and chooses $Q(\bar{f})$ to sell.
- Investors only observe the fraction of the security sold in the market.
- Investors compete for the security: make 0 profits.
Equilibrium

Definition
A Bayes-Nash equilibrium is a pair \((P, Q)\) of measurable functions s.t.:

1. \(Q(\bar{f}) = \arg \max_q q[P(q) - \delta \bar{f}]\) a.s.
2. \(P(Q(\bar{f})) = E(\bar{f} | Q(\bar{f}))\) a.s.

An equilibrium \((P, Q)\) is separating if
3. \(P(Q(\bar{f})) = \bar{f}\) a.s.
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An equilibrium $(P, Q)$ is separating if

(iii) $P(Q(\bar{f})) = \bar{f}$ a.s..

- Assumption: Single-crossing property $\implies$ A separating equilibrium exists.
Separating Equilibrium

- \((Q^*, P^*)\) is a separating equilibrium, where

\[
Q^* (f) = \left( \frac{f_0}{f} \right)^{1/(1-\delta)} \quad \text{and} \quad P^* (q) = \frac{f_0}{q^{1-\delta}}
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- If \(f_0 > 0\), it is the unique separating equilibrium.
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- If \(f_0 > 0\), it is the unique separating equilibrium.
- \((Q^*, P^*)\) depends on \(F\) only through \(f_0\).
Simplified Design Problem

- Issuer’s profit:

\[ \Pi_F(f) = (1 - \delta) f_0^{1/(1-\delta)} f^{-\delta/(1-\delta)} = \Pi(f, f_0) \]

\[ \Pi_1(f, f_0) < 0 \]

\[ \Pi(f, f_0) = f_0 \Pi \left( \frac{f}{f_0}, 1 \right) \]
Design Problem

Choose $F$ such that 0 marginal benefit to the issuer of adding any incremental cash flow to the design.

- **Marginal private valuation**

$$
\gamma(F, G) = \left. \frac{d}{dk} E \left( \min (F + kG, X) \mid Z \right) \right|_{k=0^+}
$$

- **Marginal minimum private valuation**

$$
\gamma_0(F, G) = \left. \frac{d}{dk} \min_Z E \left( \min (F + kG, X) \mid Z \right) \right|_{k=0^+}
$$
\[ \nabla V (F; G) = \gamma_0 (F, G) E \left[ \Pi \left( \frac{\bar{f}}{f_0}, 1 \right) + E \left( \Pi_1 \left( \frac{\bar{f}}{f_0}, 1 \right) \left( \frac{\gamma(F, G)}{\gamma_0(F, G)} - \frac{\bar{f}}{f_0} \right) \right) \right] \]
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- Marginal information sensitivity

\[ \frac{\gamma (F, G)}{\gamma_0 (F, G)} \]

- Information sensitivity

\[ \frac{\bar{f}}{f_0} \]
Results

- If $F$ is an optimal design, then $\Pr(F = X) > 0$. 

If $Z$ has little impact on the distribution of $S$, $F = X$ is optimal. (pure equity)

A standard debt contract, $\min(X, d)$, is optimal if:

$S = X$

$F = \phi(X)$, $\phi(0) > 0$

a uniform worst case outcome of $Z$ exists.
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- A standard debt contract, $\min(X, d)$, is optimal if:
  - $S = X$
  - $F = \varphi(X), \varphi' > 0$
  - a uniform worst case outcome of $Z$ exists.
A uniform worst case exists if for all $z_0, z \in Z$ and all $I \in \mathbb{R}_+$

1. if $\mu(X \in I \mid z) > 0$, then $\mu(X \in I \mid z_0) > 0$
2. the conditional of $\mu(\cdot \mid z)$ given $X \in I$ has first-order stochastic dominance over the conditional of $\mu(\cdot \mid z_0)$ given $X \in I$