Self-Enforcing Wage Contracts
by Thomas and Worral- RES 1988

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The model

- $t = 1, 2, \ldots$
- $s_t \in \{1, 2, \ldots, S\}$, $S \geq 2$, $\Pr(s_t = s) = p(s)$ for all $t$.
- 1 good
- 2 types of agents:
  - Risk neutral firms: can employ 1 worker at each $t$ and produce 1 unit of the good.
  - Risk averse workers: $u(c), u' > 0$ and $u'' < 0$; 1 indivisible unit of labor
- Same discount factor $\alpha \in (0, 1)$
- No moving costs. No firing costs.
Markets

- Spot labor market: \( w(s), s \in S \)
- Contract:
  \[
  \delta = \{ \omega_t(h_t) \}_{h_t,t}
  \]
  where \( h_t = \{ s_1, s_2, \ldots, s^t \} \).
- No commitment.
- Punishment: spot market forever.
Firm

- Net future benefit

\[ V(\delta, h_t) = w(s_t) - \omega_t(h_t) + E \sum_{\tau=t+1}^{\infty} \alpha^{\tau-t} \{ w(s_\tau) - \omega_\tau(h_\tau) \} \]

- Stay in the contract if

\[ V(\delta, h_t) \geq 0. \]
Worker

- Net future benefit

\[ U(\delta, h_t) = u(\omega_t(h_t)) - u(w(s_t)) \]
\[ + E \sum_{\tau=t+1}^{\infty} \alpha^{\tau-t} \{ u(\omega_{\tau}(h_{\tau})) - u(w(s_{\tau})) \} \]

- Stay in the contract if

\[ U(\delta, h_t) \geq 0. \]
Definitions
A contract $\delta$ is self enforcing if

$$V(\delta, h_t) \geq 0$$

and

$$U(\delta, h_t) \geq 0$$

for every $h_t$ for all $t$. 
Definition
A self enforcing contract is efficient if there is no other self enforcing contract which offers both parties at least as much expected utility and one party strictly more.
Pareto Frontier

\[ f_s \left( U_s^t \right) = \sup_{\delta \in \Lambda \left( h_{t-1}, s \right)} \left\{ V \left( \delta; (h_{t-1}, s) \right) : U \left( \delta; (h_{t-1}, s) \right) \geq U_s^t \right\}. \]

where

\[ \Lambda \left( h_{t-1}, s \right) = \left\{ \delta : V \left( \delta; (h_{t-1}, s) \right) \geq 0 \text{ and } U \left( \delta; (h_{t-1}, s) \right) \geq 0 \right\} \]
Pareto Frontier

\[ f_s(U_s^t) = \sup_{\omega(s), \{U_{q+1}^t\}^S_{q=1}} w(s) - \omega(s) + \alpha E f_q(U_{q+1}^t) \]

s.t.

\[ f_q(U_{q+1}^t) \geq 0 \ \forall q \ \text{and} \ U_{q+1}^t \geq 0 \ \forall q \quad (1) \]
\[ u(\omega(s)) - u(w(s)) + \alpha E(U_{q+1}^t) = U_s^t \quad (2) \]
Pareto Frontier

\[
f_s \left( U_s^t \right) = \sup_{\omega(s), \{ U_q^{t+1} \}_{q=1}^S} w(s) - \omega(s) + \alpha Ef_q \left( U_q^{t+1} \right)
\]

s.t.

\[
f_q \left( U_q^{t+1} \right) \geq 0 \ \forall q \ \text{and} \ U_q^{t+1} \geq 0 \ \forall q
\]  

\[
u(\omega(s)) - u(w(s)) + \alpha E \left( U_q^{t+1} \right) = U_s^t
\]  

• Constraint set is convex and compact.
The model Contracts Pareto Frontier Wages

Pareto Frontier

\[ f_s \left( U_s^t \right) = \max_{\omega(s), \left\{ U_q^{t+1} \right\}_{q=1}^S} w(s) - \omega(s) + \alpha E_f q \left( U_q^{t+1} \right) \]

s.t.

\[ f_q \left( U_q^{t+1} \right) \geq 0 \ \forall q \ \text{and} \ U_q^{t+1} \geq 0 \ \forall q \quad (1) \]

\[ u(\omega(s)) - u(w(s)) + \alpha E \left( U_q^{t+1} \right) = U_s^t \quad (2) \]
Pareto Frontier

\[ f_s (U_s^t) = \max_{\omega(s), \{U_{q+1}^t\}_{q=1}^S} \omega (s) - \omega (s) + \alpha Ef_q (U_{q+1}^t) \]

s.t.

\[ f_q (U_{q+1}^t) \geq 0 \ \forall q \text{ and } U_{q+1}^t \geq 0 \ \forall q \]  

\[ u (\omega (s)) - u (\omega (s)) + \alpha E (U_{q+1}^t) = U_s^t \]  

- If \( U_{q+1}^t \) satisfies (1), \( U_{q+1}^t \in [0, \bar{U}_q] \)
Pareto Frontier

\[ f_s (U^t_s) = \max_{\omega(s), \{U^{t+1}_q\}_{q=1}^S} w(s) - \omega(s) + \alpha Ef_q (U^{t+1}_q) \]

s.t.

\[ U^{t+1}_q \in [0, \bar{U}_q] \quad \forall q \quad (1') \]

\[ u(\omega(s)) - u(w(s)) + \alpha E (U^{t+1}_q) = U^t_s \quad (2) \]
Pareto Frontier

\[ f_s (U_s^t) = \max_{\omega(s), \{U_q^{t+1}\}_{q=1}^S} \omega(s) - \omega(s) + \alpha Ef_q (U_q^{t+1}) \]

s.t.

\[ U_q^{t+1} \in [0, \bar{U}_q] \quad \forall q \quad (1') \]

\[ u(\omega(s)) - u(w(s)) + \alpha E (U_q^{t+1}) = U_s^t \quad (2) \]

- FOCs
Pareto Frontier

\[ f_s (U_s^t) = \max_{\omega(s), \{U_{q}^{t+1}\}_{q=1}^S} w(s) - \omega(s) + \alpha E f_q (U_{q}^{t+1}) \]

s.t.

\[ U_{q}^{t+1} \in [0, \bar{U}_q] \ \forall q \] (1')

\[ u(\omega(s)) - u(w(s)) + \alpha E (U_{q}^{t+1}) = U_s^t \] (2)

- FOCs
  - \( \omega(h_{t-1}, s) \in [\omega_s, \bar{\omega}_s] \)
  - \( \omega(h_{t-1}, s) \) is an increasing function of \( U_s^t \)
Proposition 2
For any \((h_t, q)\), the contract wage at \(t + 1\) is

\[
\omega(h_t, q) = \begin{cases} 
\bar{\omega}_q & \omega(h_t) < \bar{\omega}_q \\
\omega(h_t) & \underline{\omega}_q \leq \omega(h_t) \leq \bar{\omega}_q \\
\underline{\omega}_q & \text{if } \omega(h_t) < \underline{\omega}_q.
\end{cases}
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\end{cases}
\]

- Stationary Markov Process
Wage Intervals and Spot Wages

**Proposition 3**

(i) For $k > q$, $\bar{\omega}_k > \bar{\omega}_q$ and $\underline{\omega}_k > \underline{\omega}_q$.

(ii) $w(s) \in [\underline{\omega}_s, \bar{\omega}_s]$ for all $s$, and $w(1) = \underline{\omega}_1$, $w(S) = \bar{\omega}_S$. 
Comparative Statics

![Graph showing comparative statics for wages and discount factor. The graph includes a Pareto Frontier and comparative statics for contracts.](image-url)