Self-Enforcing Wage Contracts

by Thomas and Worral- RES 1988

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The model

- $t = 1, 2, ...$
- $s_t \in \{1, 2, ..., S\}$, $S \geq 2$, $\Pr(s_t = s) = p(s)$ for all t.
- 1 good
- 2 types of agents:
  - Risk neutral firms: can employ 1 worker at each t and produce 1 unit of the good.
  - Risk averse workers: $u(c)$, $u' > 0$ and $u'' < 0$; 1 indivisible unit of labor
- Same discount factor $\alpha \in (0, 1)$
- No moving costs. No firing costs.

Markets

- Spot labor market: $w(s)$, $s \in S$
- Contract:
$$\delta = \{\omega_t(h_t)\}_{h_t,t}$$
where $h_t = \{s_1, s_2, ..., s^t\}$.
- No commitment.
- Punishment: spot market forever.

Firm

- Net future benefit

$$V(\delta, h_t) = w(s_t) - \omega_t(h_t) + E \sum_{\tau=t+1}^{\infty} \alpha^{\tau-t} \{w(s_\tau) - \omega_\tau(h_\tau)\}$$

- Stay in the contract if

$$V(\delta, h_t) \geq 0.$$

Worker

- Net future benefit

$$U(\delta, h_t) = u(\omega_t(h_t)) - u(w(s_t)) + E\sum_{\tau=t+1}^{\infty} \alpha^{\tau-t}\{u(\omega_\tau(h_\tau)) - u(w(s_\tau))\}$$

- Stay in the contract if

$$U(\delta, h_t) \geq 0.$$

Self enforcing

Definitions

A contract δ is self enforcing if

$$V(\delta, h_t) \geq 0$$

and

$$U(\delta, h_t) \geq 0$$

for every h_t for all t.

Efficiency

Definition

A self enforcing contract is efficient if there is no other self enforcing contract which offers both parties at least as much expected utility and one party strictly more.

Pareto Frontier

$$f_s\left(U_s^t\right) = \sup_{\delta \in \Lambda(h_{t-1},s)} \left\{ V\left(\delta; \left(h_{t-1}, s\right)\right) : U\left(\delta; \left(h_{t-1}, s\right)\right) \geq U_s^t \right\}.$$

where

$$\Lambda\left(h_{t-1}, s\right) = \left\{\delta : V\left(\delta; \left(h_{t-1}, s\right)\right) \geq 0 \text{ and } U\left(\delta; \left(h_{t-1}, s\right)\right) \geq 0\right\}$$

Pareto Frontier

$$f_s\left(U_s^t\right) = \sup_{\omega(s),\left\{U_q^{t+1}\right\}_{q=1}^{S}} w\left(s\right) - \omega\left(s\right) + \alpha E f_q\left(U_q^{t+1}\right)$$

s.t.

$$f_q\left(U_q^{t+1}\right) \geq 0\ \forall q \text{ and } U_q^{t+1} \geq 0\ \forall q \tag{1}$$

$$u\left(\omega\left(s\right)\right) - u\left(w\left(s\right)\right) + \alpha E\left(U_q^{t+1}\right) = U_s^t \tag{2}$$

Pareto Frontier

$$f_s\left(U_s^t\right) = \sup_{\omega(s),\left\{U_q^{t+1}\right\}_{q=1}^{S}} w\left(s\right) - \omega\left(s\right) + \alpha E f_q\left(U_q^{t+1}\right)$$

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- Constraint set is convex and compact.

Pareto Frontier

$$f_s\left(U_s^t\right) = \max_{\omega(s),\left\{U_q^{t+1}\right\}_{q=1}^{S}} w\left(s\right) - \omega\left(s\right) + \alpha E f_q\left(U_q^{t+1}\right)$$

s.t.

$$f_q\left(U_q^{t+1}\right) \geq 0 \ \forall q \text{ and } U_q^{t+1} \geq 0 \ \forall q \quad (1)$$

$$u\left(\omega\left(s\right)\right) - u\left(w\left(s\right)\right) + \alpha E\left(U_q^{t+1}\right) = U_s^t \quad (2)$$

Pareto Frontier

$$f_s\left(U_s^t\right) = \max_{\omega(s),\left\{U_q^{t+1}\right\}_{q=1}^S} w\left(s\right) - \omega\left(s\right) + \alpha E f_q\left(U_q^{t+1}\right)$$

s.t.

$$f_q\left(U_q^{t+1}\right) \geq 0 \ \forall q \text{ and } U_q^{t+1} \geq 0 \ \forall q \quad (1)$$

$$u\left(\omega\left(s\right)\right) - u\left(w\left(s\right)\right) + \alpha E\left(U_q^{t+1}\right) = U_s^t \quad (2)$$

- If U_q^{t+1} satisfies (1), $U_q^{t+1} \in [0, \bar{U}_q]$

Pareto Frontier

$$f_s\left(U_s^t\right) = \max_{\omega(s),\left\{U_q^{t+1}\right\}_{q=1}^S} w\left(s\right) - \omega\left(s\right) + \alpha E f_q\left(U_q^{t+1}\right)$$

s.t.

$$U_q^{t+1} \in \left[0, \bar{U}_q\right] \ \forall q \tag{1'}$$

$$u\left(\omega\left(s\right)\right) - u\left(w\left(s\right)\right) + \alpha E\left(U_q^{t+1}\right) = U_s^t \tag{2}$$

Pareto Frontier

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- FOCs

Pareto Frontier

$$f_s\left(U_s^t\right) = \max_{\omega(s),\left\{U_q^{t+1}\right\}_{q=1}^{S}} w\left(s\right) - \omega\left(s\right) + \alpha E f_q\left(U_q^{t+1}\right)$$

s.t.

$$U_q^{t+1} \in \left[0, \bar{U}_q\right] \ \forall q \tag{1'}$$

$$u\left(\omega\left(s\right)\right) - u\left(w\left(s\right)\right) + \alpha E\left(U_q^{t+1}\right) = U_s^t \tag{2}$$

- FOCs
  - $\omega\left(h_{t-1}, s\right) \in \left[\underline{\omega}_s, \bar{\omega}_s\right]$
  - $\omega\left(h_{t-1}, s\right)$ is an increasing function of U_s^t

Contract Wages

Proposition 2

For any (h_t, q), the contract wage at $t+1$ is

$$\omega(h_t, q) = \begin{cases} \bar{\omega}_q & \omega(h_t) < \bar{\omega}_q \\ \omega(h_t) & \underline{\omega}_q \leq \omega(h_t) \leq \bar{\omega}_q \\ \underline{\omega}_q & \text{if } \omega(h_t) < \underline{\omega}_q. \end{cases}$$

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- Stationary Markov Process

Wage Intervals and Spot Wages

Proposition 3

(i) For $k > q$, $\bar{\omega}_k > \bar{\omega}_q$ and $\underline{\omega}_k > \underline{\omega}_q$.

(ii) $w(s) \in [\underline{\omega}_s, \bar{\omega}_s]$ for all s, and $w(1) = \underline{\omega}_1$, $w(S) = \bar{\omega}_S$.

Comparative Statics