Optimal Financial Crises

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Overview of the results

Three environments

- Baseline Model: deposit contracts with bank runs allow optimal risk sharing
- Costly liquidation: nominal deposit contracts with runs and accomodative monetary policy achieve first best
- Endogenous liquidation: same policy as above

Baseline Model

- $t = 0, 1, 2$
- 1 good
- 2 Assets: safe one period storage technology L, and risky long term asset X
- X cannot be liquidated at time 1 and each unit invested at 0 yields a stochastic payoff of R at time 2, $E(R) > 1$
- The realization of R becomes publicly known at time 1
- One bank and a continuum of mass 2 of ex-ante identical agents endowed with E units of the good at time 0, facing idiosyncratic unobservable shocks to preferences:
$$U(c_1, c_2) = \frac{1}{2}u(c_1) + \frac{1}{2}u(c_2)$$
- No sequential service constraint

Bank

If the bank can write contracts contingent on R, then the Bank's problem is:

$$max_{L,X,c_1(),c_2()} E[u(c_1(R)) + u(c_2(R))]$$

s.t.

$$L + X \leq E$$

$$c_1(R) \leq L$$

$$c_2(R) \leq L - c_1(R) + RX$$

$$c_1(R) \leq c_2(R)$$

Taking FOC's w.r.t. $c_1(R)$ and $C_2(R)$ in the relaxed problem in which I.C. is not imposed shows that the solution achieves first best:

$$u'(c_1(R)) \geq u'(c_2(R))$$

Solution

Theorem

The solution $(L, X, c_1(), c_2())$ to the optimal incentive compatible risk sharing problem is uniquely characterized by:

$$c_1(R) = min\{L, \frac{L + RX}{2}\}$$

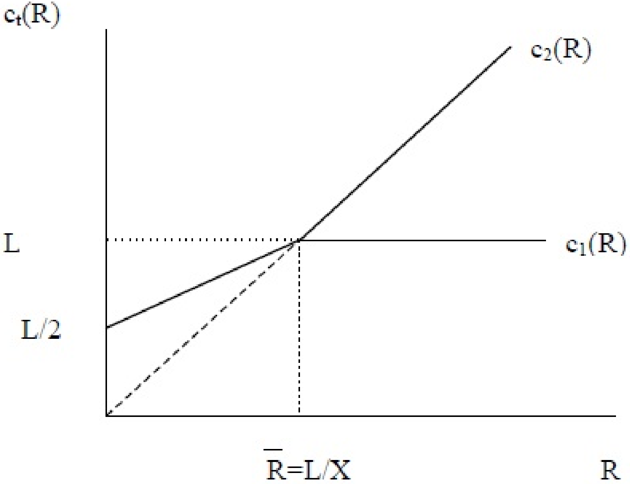
$$c_2(R) = max\{RX, \frac{L + RX}{2}\}$$

$$L + X = E$$

$$E[u'(c_1(R))] = E[u'(c_2(R))R]$$

The allocation is first best efficient.

Solution cont'd



Deposits and Runs

Assume now that the bank can only offer a "noncontingent" contract which pays $\overline{c}$ if possible at time 1 and whatever it's left at time 2.

- $\alpha(R)$ is the proportion of late consumers withdrawing early.
- $c_{21}(R), c_{22}(R)$ are equilibrium consumption of late consumers that run and wait respectively

The problem of the bank is now:

$$max_{L,X,\overline{c},\alpha(R)} E[u(c_1(R)) + u(c_2(R))]$$

s.t.

$$L + X \leq E$$

$$c_1(R) + \alpha(R)c_{21}(R) \leq L$$

$$(1 - \alpha(R))c_{22}(R) = L - c_1(R) - \alpha(R)c_{21} + RX$$

$$c_1(R) \leq \overline{c}$$

$$c_1(R) + \alpha(R)c_{21}(R) = L \text{ if } c_1(R) < \overline{c}$$

$$c_{21}(R) = c_{22}(R) = c_2(R)$$

$$c_1(R) = c_2(R) \text{ if } \alpha(R) > 0$$

$$c_1(R) \leq c_2(R)$$

From the constraints one gets:

$$c_1(R) = min\{\overline{c}, \frac{L}{1+\alpha(R)}\}$$

$$c_2(R) = max\{RX + L - \overline{c}, RX + \alpha(r)\frac{L}{1+\alpha(R)}\}$$

Solving for $\alpha(R)$

$$c_1(R) = min\{\overline{c}, \frac{L+RX}{2}\}$$

$$c_2(R) = max\{RX + L - \overline{c}, \frac{L+RX}{2}\}$$

Comparing this with the first best:

$$c_1^*(R) = min\{L^*, \frac{L^*+RX^*}{2}\}$$

$$c_2^*(R) = max\{RX^*, \frac{L^*+RX^*}{2}\}$$

Theorem

A banking system subject to runs can achieve first-best efficiency using standard deposit contracts

Theorem

An equilibrium in which runs are prevented is worse than the first best and strictly worse whenever the support of R contains zero (provided $u'(0) > E[u'(ER)R]$)

Costly runs

Banks can now invest at time 1 in a riskless asset with return $r > 1$ but $E(R) > r$. The Bank's optimal optimal contract solves:

$$max_{L,X,c_1(),c_2()} E[u(c_1(R)) + u(c_2(R))]$$

s.t.

$$L + X \leq E$$

$$c_1(R) \leq L$$

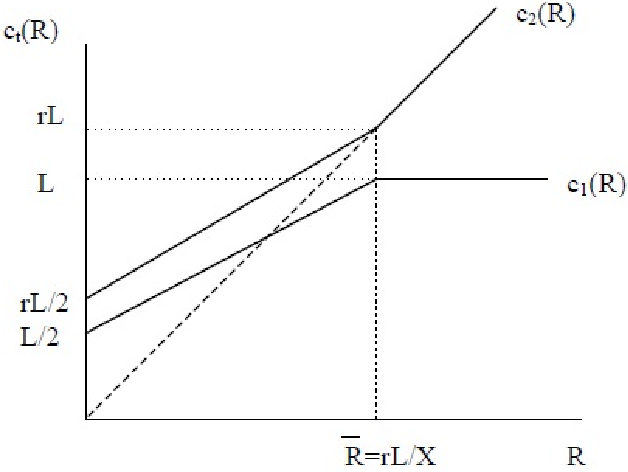
$$c_2(R) \leq r(L - c_1(R)) + RX$$

$$c_1(R) \leq c_2(R)$$

Taking FOC's w.r.t. $c_1(R)$ and $C_2(R)$ in the relaxed problem in which I.C. is not imposed shows again that the solution achieves first best:

$$u'(c_1(R)) \geq ru'(c_2(R))$$

Solution



Deposit contracts

With deposits the Bank's problem is:

$$max_{L,X,\overline{c},\alpha(R)} E[u(c_1(R)) + u(c_2(R))]$$

s.t.

$$L + X \leq E$$

$$c_1(R) + \alpha(R)c_{21}(R) \leq L$$

$$(1 - \alpha(R))c_2(R) = r(L - c_1(R) - \alpha(R)c_2) + RX$$

$$c_1(R) \leq \overline{c}$$

$$c_1(R) + \alpha(R)c_2(R) = L if c_1(R) < \overline{c}$$

$$c_1(R) = c_2(R) if \alpha(R) > 0$$

$$c_1(R) \leq c_2(R)$$

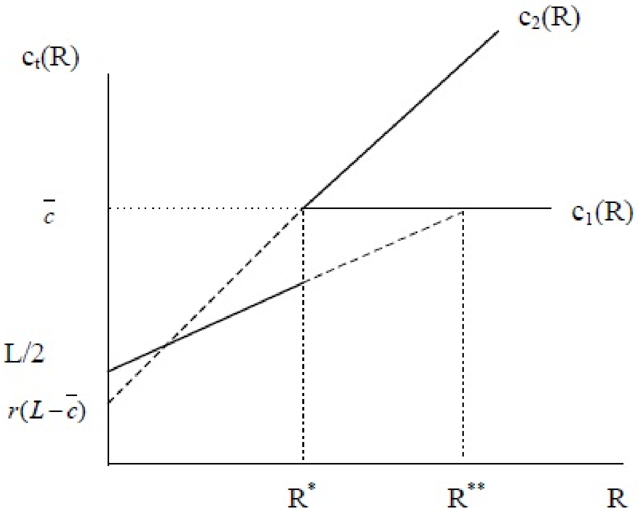
Solutions

- In this case it is important to stress that the Bank is choosing $\alpha(R)$.
- For any choice of L and $\overline{c}$ s.t. $L > \overline{c}$ there are different functions $\alpha(R)$ satisfying the equilibrium restrictions
- It is possible that if r is sufficiently high the bank chooses $L > \overline{c}$ so to exploit the higher returns r and stop runs at lower values of R
- However when extra liquidity is provided the system is subject to "unexpected runs" in the region $(R*, R**)$

$$\overline{c} = r(L - \overline{c}) + R * X$$

$$2\overline{c} < L + R * *X$$

Solutions



Optimal monetary policy

Consider now nominal contracts and a cental bank providing a line of credit to banks

- Deposit promise a nominal value D at time 1
- Central bank provides an interest free line of credit M to the bank at time 1 which must be repaid at time 2
- $p_1 R, p_2(R)$ are state contingent prices

Optimal Monetary Policy

With this setup it is possible to implement the optimal allocation

Fix M and $D > M$ and let $c_1(R) and c_2(R)$ be the optimal risk sharing allocation. Let prices and the proportion of early withdrawers satisfy

$$p_1(R)c_1(R) = D$$

$$\alpha(R)D = M$$

$$\alpha(R)p_2(R)c_2(R) = M$$

When $R < \overline{R}$ the bank draws on the line of credit and gives each depositor $\frac{M}{1+\alpha(R)}$ in money and $\frac{c_1(R)}{1+\alpha(R)}$ in goods. Market clearing requires:

$$\frac{M}{1+\alpha(R)} = p_1(R)\alpha(R)\frac{c_1(R)}{1+\alpha(R)}$$

Optimal Monetary policy

Theorem

Suppose that the central bank makes available to the representative bank an interest free line of credit of M units of money at date 1 which must be repaid at date 2. Then there exists equilibrium price levels $p_1(R)$ and $p_2(R)$ and an equilibrium fraction of early withdrawers $\alpha(R)$ for each value of R, which will implement the incentive efficient allocation.