Optimal Financial Crises

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Overview of the results

Three environments

- Baseline Model: deposit contracts with bank runs allow optimal risk sharing
- Costly liquidation: nominal deposit contracts with runs and accommodative monetary policy achieve first best
- Endogenous liquidation: same policy as above
Baseline Model

- $t = 0, 1, 2$
- 1 good
- 2 Assets: safe one period storage technology $L$, and risky long term asset $X$
- $X$ cannot be liquidated at time 1 and each unit invested at 0 yields a stochastic payoff of $R$ at time 2, $E(R) > 1$
- The realization of $R$ becomes publicly known at time 1
- One bank and a continuum of mass 2 of ex-ante identical agents endowed with $E$ units of the good at time 0, facing idiosyncratic unobservable shocks to preferences:
  \[ U(c_1, c_2) = \frac{1}{2} u(c_1) + \frac{1}{2} u(c_2) \]
- No sequential service constraint
Bank

If the bank can write contracts contingent on $R$, then the Bank’s problem is:

$$
\max_{L, X, c_1(), c_2()} E[u(c_1(R)) + u(c_2(R))]
$$

s.t.

$$
L + X \leq E
$$

$$
c_1(R) \leq L
$$

$$
c_2(R) \leq L - c_1(R) + RX
$$

$$
c_1(R) \leq c_2(R)
$$

Taking FOC’s w.r.t. $c_1(R)$ and $C_2(R)$ in the relaxed problem in which I.C. is not imposed shows that the solution achieves first best:

$$
u'(c_1(R)) \geq u'(c_2(R))
$$
Solution

Theorem

The solution \((L, X, c_1(), c_2())\) to the optimal incentive compatible risk sharing problem is uniquely characterized by:

\[
c_1(R) = \min \{L, \frac{L + RX}{2}\}
\]

\[
c_2(R) = \max \{RX, \frac{L + RX}{2}\}
\]

\[
L + X = E
\]

\[
E[u'(c_1(R))] = E[u'(c_2(R))R]
\]

The allocation is first best efficient.
Solution cont’d

\[ R = \frac{L}{X} \]
Deposits and Runs

Assume now that the bank can only offer a "noncontingent" contract which pays $\bar{c}$ if possible at time 1 and whatever it’s left at time 2.

- $\alpha(R)$ is the proportion of late consumers withdrawing early.
- $c_{21}(R), c_{22}(R)$ are equilibrium consumption of late consumers that run and wait respectively
The problem of the bank is now:

\[ \max_{L,X,\bar{c},\alpha(R)} E[u(c_1(R)) + u(c_2(R))] \]

s.t.

\[ L + X \leq E \]

\[ c_1(R) + \alpha(R)c_{21}(R) \leq L \]

\[ (1 - \alpha(R))c_{22}(R) = L - c_1(R) - \alpha(R)c_{21} + RX \]

\[ c_1(R) \leq \bar{c} \]

\[ c_1(R) + \alpha(R)c_{21}(R) = L \text{ if } c_1(R) < \bar{c} \]

\[ c_{21}(R) = c_{22}(R) = c_2(R) \]

\[ c_1(R) = c_2(R) \text{ if } \alpha(R) > 0 \]

\[ c_1(R) \leq c_2(R) \]
From the constraints one gets:

\[
c_1(R) = \min \{ \bar{c}, \frac{L}{1 + \alpha(R)} \}
\]

\[
c_2(R) = \max \{ RX + L - \bar{c}, RX + \alpha(r) \frac{L}{1 + \alpha(R)} \}
\]

Solving for \( \alpha(R) \)

\[
c_1(R) = \min \{ \bar{c}, \frac{L}{2} \}
\]

\[
c_2(R) = \max \{ RX + L - \bar{c}, \frac{L + RX}{2} \}
\]

Comparing this with the first best:

\[
c_1^*(R) = \min \{ L^*, \frac{L^* + RX^*}{2} \}
\]

\[
c_2^*(R) = \max \{ RX^*, \frac{L^* + RX^*}{2} \}
\]
Theorem

A banking system subject to runs can achieve first-best efficiency using standard deposit contracts

Theorem

An equilibrium in which runs are prevented is worse than the first best and strictly worse whenever the support of \( R \) contains zero (provided \( u'(0) > E[u'(ER)R] \))
Costly runs

Banks can now invest at time 1 in a riskless asset with return $r > 1$ but $E(R) > r$. The Bank’s optimal optimal contract solves:

$$\max_{L,X,c_1(),c_2()}E[u(c_1(R)) + u(c_2(R))]$$

s.t.

$$L + X \leq E$$
$$c_1(R) \leq L$$
$$c_2(R) \leq r(L - c_1(R)) + RX$$
$$c_1(R) \leq c_2(R)$$

Taking FOC’s w.r.t. $c_1(R)$ and $C_2(R)$ in the relaxed problem in which I.C. is not imposed shows again that the solution achieves first best:

$$u'(c_1(R)) \geq ru'(c_2(R))$$
Solution
Deposit contracts

With deposits the Bank’s problem is:

$$\max_{L, X, \bar{c}, \alpha(R)} E[u(c_1(R)) + u(c_2(R))]$$

s.t.

$$L + X \leq E$$

$$c_1(R) + \alpha(R)c_2(R) \leq L$$

$$(1 - \alpha(R))c_2(R) = r(L - c_1(R) - \alpha(R)c_2) + RX$$

$$c_1(R) \leq \bar{c}$$

$$c_1(R) + \alpha(R)c_2(R) = Lifc_1(R) < \bar{c}$$

$$c_1(R) = c_2(R) \text{ if } \alpha(R) > 0$$

$$c_1(R) \leq c_2(R)$$
In this case it is important to stress that the Bank is choosing \( \alpha(R) \).

For any choice of \( L \) and \( \bar{c} \) s.t. \( L > \bar{c} \) there are different functions \( \alpha(R) \) satisfying the equilibrium restrictions.

It is possible that if \( r \) is sufficiently high the bank chooses \( L > \bar{c} \) so to exploit the higher returns \( r \) and stop runs at lower values of \( R \).

However when extra liquidity is provided the system is subject to "unexpected runs" in the region \((R^*, R^{**})\)

\[
\bar{c} = r(L - \bar{c}) + R^*X
\]

\[
2\bar{c} < L + R^{**}X
\]
Solutions
Optimal Financial Crises

Optimal monetary policy

Consider now nominal contracts and a central bank providing a line of credit to banks

- Deposit promise a nominal value $D$ at time 1
- Central bank provides an interest free line of credit $M$ to the bank at time 1 which must be repaid at time 2
- $p_1 R, p_2(R)$ are state contingent prices
Optimal Monetary Policy

With this setup it is possible to implement the optimal allocation

Fix $M$ and $D > M$ and let $c_1(R)$ and $c_2(R)$ be the optimal risk sharing allocation. Let prices and the proportion of early withdrawers satisfy

$$p_1(R)c_1(R) = D$$
$$\alpha(R)D = M$$
$$\alpha(R)p_2(R)c_2(R) = M$$

When $R < \bar{R}$ the bank draws on the line of credit and gives each depositor $\frac{M}{1+\alpha(R)}$ in money and $\frac{c_1(R)}{1+\alpha(R)}$ in goods. Market clearing requires:

$$\frac{M}{1 + \alpha(R)} = p_1(R)\alpha(R)\frac{c_1(R)}{1 + \alpha(R)}$$
Theorem

Suppose that the central bank makes available to the representative bank an interest free line of credit of $M$ units of money at date $1$ which must be repaid at date $2$. Then there exists equilibrium price levels $p_1(R)$ and $p_2(R)$ and an equilibrium fraction of early withdrawers $\alpha(R)$ for each value of $R$, which will implement the incentive efficient allocation.