"Efficiency in repeated trade with hidden valuations"" by Susan Athey and David Miller

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Questions

- An owner wishes to sell a good.
- Buyers and sellers have private information about own valuations.
- Do gains from trade guarantee trade? (Myerson-Satterthwaith)
- Does the answer to the previous question change when trade is repeated? (Athey-Miller (2007), Miller(2009))

Static Environment

- Two agents meet. $i = s, b$.
- s own's an object valued by both.
- Their valuations are given by $\theta_i \sim F_i$.
- Call θ the vector of valuations.
- F_i has support $[a_i, b_i]$.

Static Environment

- Agents are expected utility maximizer's.
- Utility for s is:

$$\begin{aligned} U_s &= \theta_s \text{ if object is kept} \\ U_s &= t \text{ if object sold at price } t \end{aligned}$$

- Utility for b is:

$$\begin{aligned} U_b &= 0 \text{ if object is not sold} \\ U_s &= \theta_b - t \text{ if object sold at price } t \end{aligned}$$

Incentives to Reveal Valuation

- Under full information: Nash-Bargaining.
- Under asymmetric information: agents agree on a mechanism ex-ante.

Static Mechanism I

- *Direct Revelation Principle*
- A mechanism in this environment is a pair of functions:

$$
\begin{aligned}
p &: [a_s, b_s] \times [a_b, b_b] \longrightarrow [0, 1] \\
t &: [a_s, b_s] \times [a_b, b_b] \longrightarrow R
\end{aligned}
$$

- p is the probability that the object is traded.
- t is a specified transfer.

Static Mechanism II

- Any mechanism defines the following objects:

$$\tilde{p}_s(\theta_1) = \int_{a_b}^{b_b} p(\theta_1, \theta_b)\, dF_b(\theta_b) \;\; ; \; \tilde{p}_b(\theta_2) = \int_{a_b}^{b_b} p(\theta_s, \theta_2)\, dF_s(\theta_s)$$

and

$$\tilde{t}_s(\theta_1) = \int_{a_b}^{b_b} t(\theta_1, \theta_b)\, dF_b(\theta_b) \,; \tilde{t}_b(\theta_2) = \int_{a_b}^{b_b} t(\theta_s, \theta_2)\, dF_s(\theta_s)$$

- The agents' expected utility conditional on their information are:

$$\begin{aligned} v_s(\theta_s) &= \tilde{t}(\theta_s) - \theta_s \tilde{p}(\theta_s) \\ v_b(\theta_b) &= \theta_b \tilde{p}(\theta_b) - t(\theta_t) \end{aligned}$$

- A mechanism is incentive compatible if $\forall \theta$,

$$v_s(\theta_s) \geq \tilde{t}\left(\tilde{\theta}_s\right) - \theta_s \tilde{p}\left(\tilde{\theta}_s\right) \;\; \forall \tilde{\theta}_s \in [a_s, b_s]$$

$$v_b(\theta_b) \geq \theta_b \tilde{p}\left(\tilde{\theta}_b\right) - \tilde{t}\left(\tilde{\theta}_b\right) \;\; \forall \tilde{\theta}_b \in [a_b, b_b]$$

Mechanism - Descriptions

- A mechanism is ex-post IR $\forall\theta$

$$v_b(\theta_b) \geq 0$$

and

$$v_s(\theta_s) \geq \theta_s$$

- A mechanism is efficient if:

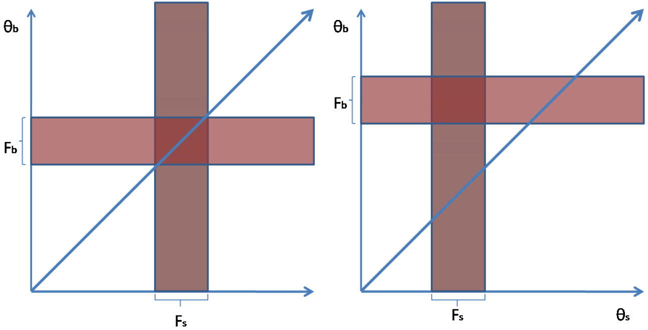
$$p = 1 \text{ if } \theta_b \geq \theta_s \text{ and } 0 \text{ otherwise}$$

- Note that these mechanisms satisfy a budget balance condition.

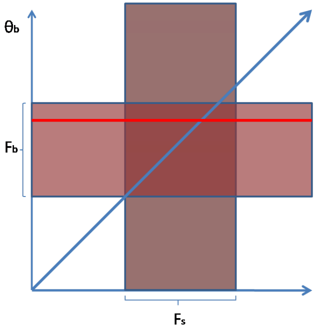
Environments

- There are two cases.
- Case 1: ex-post gains from trade are positive with probability 1.
- Case 2: ex-post gains from trade are positive with probability < 1.

Gains from Trade



Incentives



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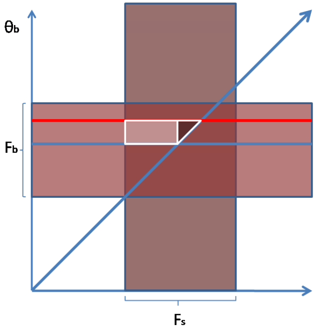

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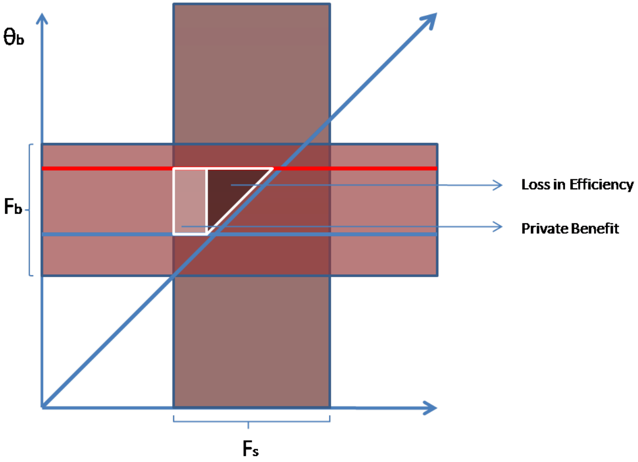

Myerson-Satterthwaite 1

Theorem (Myerson-Satterthwaite)

A mechanism is IC and ex-post IR iff:

1. $\tilde{p}_1(\cdot)$ *is decreasing.*
2. $\tilde{p}_2(\cdot)$ *is increasing.*
3. *The mechanism satisfies:*

$$\begin{aligned} v_s(b_1) + v_b(a_1) &= \min_{\theta_s \in [a_1, b_1]} v_s(\theta_s) + \min_{\theta_b \in [a_1, b_1]} v_b(\theta_b) \\ &= \int_{[a_s, b_s]} \int_{[a_b, b_b]} \left[\theta_s + \frac{F(\theta_s)}{f(\theta_s)}\right] + \left[v_2 - \frac{1 - F(v_2)}{f(v_2)}\right] \\ &\quad \times p(\theta_s, \theta_b) \times f(\theta_s) \times f(\theta_b) \times d\theta_s \times d\theta_b \\ &\geq 0 \end{aligned}$$

- Nature of the proof.

Myerson-Satterthwaite - Impossibility Theorem

- Why is this condition useful?

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  - Useful to characterize efficient mechanisms.

Myerson-Satterthwaite - Impossibility Theorem

- Why is this condition useful?
  - Useful to check if efficient trade is incentive-feasible.
  - Useful to characterize efficient mechanisms.
- Result 1: Efficient trade cannot be supported.
- Any efficient mechanism would yield:

$$v_s(b_1) + v_b(a_1) = -\int_{[a_b, b_b]} (1 - F(t)) F_1(t)\, dt < 0$$

Efficient Mechanisms

- What is the most efficient mechanism?

Efficient Mechanisms

- What is the most efficient mechanism?
- Maximize Expected Gains from trade:

$$\max_{p,t} \int_{[a_s,b_s]} v(\theta_1)\, dF_s(\theta_1) + \int_{[a_b,b_b]} v(\theta_2)\, dF_b(\theta_1)$$

subject to:

$$\begin{aligned} & \int_{[a_s,b_s]} \int_{[a_b,b_b]} \left[\theta_s + \frac{F(\theta_s)}{f(\theta_s)}\right] + \left[v_2 - \frac{1 - F(v_2)}{f(v_2)}\right] \\ & \times p(\theta_s, \theta_b) \times f(\theta_s) \times f(\theta_b) \times d\theta_s \times d\theta_b \\ \geq \quad & 0 \end{aligned} \tag{1}$$

Solution Efficient Mechanisms

- $\alpha-$ virtual valuation.

$$c_1(\theta_s, \alpha) = \left[\theta_s + \alpha \frac{F(\theta_s)}{f(\theta_s)}\right] \text{ and } c_2(\theta_b, \alpha) = \left[\theta_b + \alpha \frac{(1 - F(\theta_b))}{f(\theta_b)}\right]$$

- Let p^α be a mechanism such that,

$$\begin{aligned} p^\alpha(\theta_s, \theta_b) &= 1 \text{ if } c_1(\theta_s, \alpha) < c_2(\theta_b, \alpha) \\ &= 0 \text{ otherwise} \end{aligned}$$

Optimal Mechanisms

- Note two things p^1 is the maximizer of

Theorem (Myerson-Satterthwaite 2)

If there exists a mechanism (p^α, x) some $\alpha \in [0,1]$ such that,

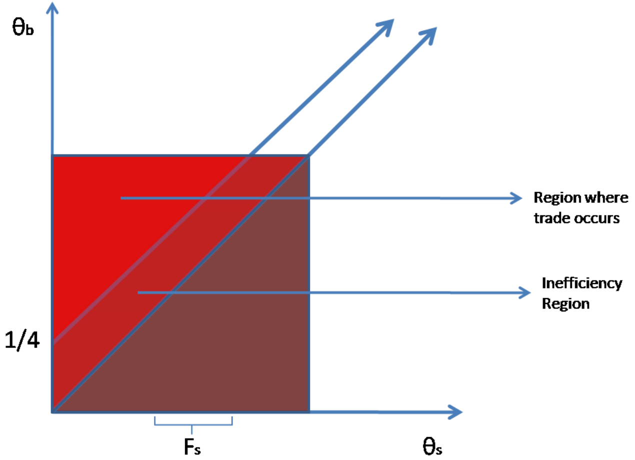
$$\min_{\theta_s \in [a_1, b_1]} v_s(\theta_s) = \min_{\theta_b \in [a_1, b_1]} v_b(\theta_b) = 0$$

then this mechanism is the maximizer of expected profits among all the IC and IR mechanisms. If $c_1(\theta_s, 1)$ and $< c_2(\theta_b, 1)$ are increasing. That mechanism must exist.

Example

Example

$\theta_s, \theta_b \sim U\,[0, 1]$.
$\alpha = 1/3$.


θb
1/4
Fs
θs
Region where
trade occurs
Inefficiency
Region

Athey Miller 2007

- Can improve the outcomes billateral-trade .

Athey Miller 2007

- Can improve the outcomes billateral-trade .
- Answer depends on the balance of transfers.
- With ex-ante balance, efficiency is attainable.
- With ex-post balance, efficiency is unattainable and most mechanism resemble a posted-price mechanism.
- An intermediate case is a bounded account.

Recursive Mechanisms

- Mechanism is a triplet $g = < p, t, w >$
- $p : [a_1, b_1] \times [a_2, b_2] \rightarrow [0, 1]^2$. Let $\mathcal{P}$ be the space of such functions.
- $t : [a_s, b_s] \times [a_b, b_b] \rightarrow R^2$. Let $\mathcal{T}$ be the space of such functions.
- $w : [a_s, b_s] \times [a_b, b_b] \rightarrow R^2$. Let $\mathcal{W}$ be the space of such functions.
- w plays the role of a continuation reward.
- Note that the mechanism here is defined differently.

Recursive Mechanisms I

- $u_i(\theta) = [\theta_i p_i(\theta) + t_i(\theta)]$
- $U_i(\theta) = (1-\delta)[\theta_i p_i(\theta) + t_i(\theta)] + \delta w_i(\theta)$
- $q^*(\theta) = 1_{[\theta_b > \theta_s]}$ efficient allocation.
- $v^{GET}(\theta) = (\theta_b - \theta_s) q^*(\theta)$
- $v^{ETU} = \theta_s + E\left[v^{GET}(\theta)\right] = E[\max_i \theta_i]$
- Set of feasible promises:

$$\mathbf{V} = \left\{v = (v_s, v_b) \in R^2 : v_b + v_s \leq v^{ETU} \text{ and } E[\theta_s] \leq v_s\right\}$$

Recursive Mechanisms II

Definition (Recursive Mechanism)

A recursive mechanism is a triplet $< V, \gamma, \mathbf{v}_o >$ suche that $V \subset \mathbf{V}, \gamma : V \to \mathcal{P} \times \mathcal{T} \times \mathcal{W}$ such that the following conditions are satisfied:

1. Promise Keeping: $\forall i$ and $\forall \mathbf{v}, E\left[U_i\left(\theta, \gamma\left(\mathbf{v}\right)\right)\right] = \mathbf{v}_i$.
2. Coherence: $\forall \theta$ and $\forall v \in V, w\left(\cdot; v\right) \in V$.

Recursive Mechanisms III

Definition

A mechanism γ is **ex-post incentive compatible** if:

$$\theta_i \in \arg\max_{\theta_i} (1-\delta) u_i (\theta_i, \theta_{-i}, v) + \delta w_i (\theta_i, \theta_{-i}, v)$$

for $\forall \mathbf{v} \in V . \forall \theta$, and $i \in \{s, b\}$.

- Note that this definition is stronger that *ex*-ante IC.

Definition

A mechanism is **ex-post Individually rational** if:

$$U_b (\theta, \gamma(\mathbf{v})) \geq 0$$

and

$$U_s (\theta, \gamma(\mathbf{v})) \geq \delta \theta_s + (1-\delta) E \left[\tilde{\theta}_s \right]$$

Definition

A mechanism is stationary if V is a singleton.

Restrictions on Budgets

- There are three environments.
- Ex-ante Budget Balance:

$$E\left[t_s(\theta) + t_b(\theta)\right] = 0$$

- Ex-post Budget Balance:

$$t_s(\theta) + t_b(\theta) = 0, \forall\theta$$

- The third one is in the middle.

Equilibria with Ex-ante Budget Balance

Proposition

There exists a stationary efficient mechanism satisfying ex-post IR, ex-post IC and **ex-ante** *budget balance iff* $\delta > \frac{1}{2}$.

- Agents accept negative transfers in exchange for repeating the relationship.

Equilibria with Ex-post Budget Balance (Miller 2007)

- No major results for this paper.
- Miller 2007 shows that:

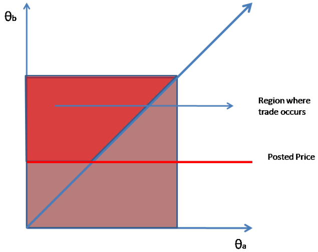
Proposition

For every recursive mechanism γ, that satisfies ex-post bb and ex-post IC, there exists ε such that $v_s + v_b < v^{ETU} - \varepsilon$, for any $\delta < 1$ and $\mathbf{v} \in V(\delta, \gamma)$.

- In addition, the solution to the efficient problem yields a linear programming problem.

Example

$U[0,1], U[0,1]$



Equilibria with a Bounded Budget Account

- Now the environment admits the possibility that

$$t_s + t_b \neq 0$$

- Note that this condition implies negative expeted transfers.
- Here the environment imposes amount of accumulated transfers overt time $\bar{A}$.

Account Recursive Mechanism II

Definition (Account Recursive Mechanism)

An account recursive mechanism is a triplet $\langle V_a, \gamma_a, \mathbf{v}_o \rangle$ such that $V_a \subset R \times \mathbf{V}, \gamma_a : R \times V \longrightarrow \mathcal{P} \times \mathcal{T} \times \mathcal{W}$ such that the following conditions are satisfied:

1. Promise Keeping: $\forall i$ and $\forall (A, \mathbf{v}) \in V_a, E\left[U_i\left(\theta, \gamma_a(A, \mathbf{v})\right)\right] = \mathbf{v}_i$.
2. Coherence: $\forall \theta$ and $\forall (A, \mathbf{v}) \in V_a$:

$$\begin{aligned} \alpha(\cdot; A, v) &\equiv A - \left(t_a(\theta; A, v) + t_b(\theta; A, v)\right) \\ \left(\alpha(\cdot; A, v), w(\cdot; A, v)\right) &\in V_a \end{aligned}$$

3. Account Keeping: $\forall (A, \mathbf{v}) \in V_a$

$$v_s + v_b = (1-\delta) E\left[\sum_{\tau=1}^{\infty} \delta^{\tau-1}\left(\left(\sum_{i=b,s} q_i\left(\tilde{\theta}^{(\tau)}, \tilde{A}^{(\tau)}, \tilde{v}^{(\tau)}\right)\theta_i\right) + \tilde{A}^{(\tau)} - \tilde{A}^{(\tau+1)}\right)\right]$$

where $\tilde{\theta}^{(\tau)}, \tilde{A}^{(\tau)}, \tilde{v}^{(\tau)}$ are the random sequences generated by the mechanism, starting from (A, v_s, v_b).

Athey-Miller 1

Definition

A mechanism is Balanced Budget Account (BBA) if:

$$\alpha(\cdot; A, v) \in [0, \bar{A}] \text{ for } \forall (A, \mathbf{v}) \in V_a$$

Proposition (Athey Miller 1)

There $\nexists$ stationary account recursive mechanism satisfying ex-post IR, ex-post IC and BBA.

Account Recursive Mechanism II

Condition

There exist functions $h_b : [a_1, b_1] \to R$ *and* $h_s : [a_2, b_2] \to R$ *such that*

$$h_b(\theta_s) \geq 0 \text{ and } h_s(\theta_b) \geq \frac{\delta}{1-\delta} E\left[\tilde{\theta}_s\right]$$

and

$$v^{GFT}(\theta) + h_b(\theta_s) + h_s(\theta_b) \leq \delta E\left[\tilde{\theta}_s + 2v^{GFT}\left(\tilde{\theta}\right) + h_b\left(\tilde{\theta}_s\right) + h_s\left(\tilde{\theta}_b\right)\right]$$

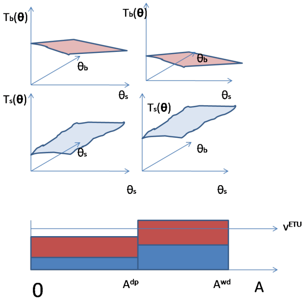
Proposition

Under condition 1, there exists an account recursive mechanism satisfying ex-post IR, ex-post IC and BBA and where the **allocation is efficient in every period!**

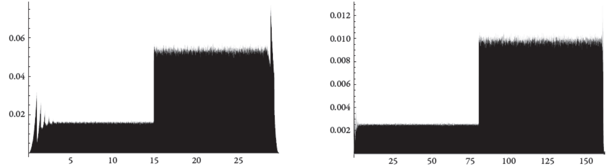
Implementation

- Optimal Allocation Rule: $q^* (\theta)$
- Trigger Accounts: $[0, A_{dp}] \cup (A_{dp}, A_{wp}]$
- Deposit Regime: $[0, A_{dp}]$
  - $t_s(\theta) + t_b(\theta) \leq 0$
  - $v^{dp} + v^{dp} < v^{ETU}$
- Withdrawal Regime: $(A_{dp}, A_{wp}]$
  - $t_s(\theta) + t_b(\theta) \geq 0$
  - $v^{dp} + v^{dp} > v^{ETU}$
- Promise utility does not vary within regimes.
- Varies only across Regimes.

Incentives



Mechanism	δ	$\mathbb{E}[v_b + v_s]$	%GFT	$\frac{1}{1-\delta}\mathbb{E}[v_b + v_s]$	$\bar{A}$	$\max A$
Almost efficient	0.67	0.650	90.1	1.97	1.70	2.65
	0.80	0.655	93.0	3.27	2.02	3.21
	0.90	0.661	96.7	6.61	2.81	4.65
	0.95	0.665	98.9	13.30	4.30	7.52
	0.99	0.667	99.998	66.67	15.88	30.51
Exact efficient	0.95	0.667	100	13.33	18.76	29.69
	0.99	0.667	100	66.67	105.34	162.94
Optimal posted price	any δ	0.625	75.0		0	0



(a) $\delta = 0.95$ (b) $\delta = 0.99$