Asset Pricing in a Production Economy with Chew-Dekel Preferences

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Presentation by Matt Smith

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Asset Pricing in Production Economies

Goal: Generate realistic asset prices (moments) along with quantities in a production economy

Difficulties:
- Endogenous variables
- Multiple channels to smooth consumption
- Marginal product of capital doesn't look like asset returns (not volatile enough)


This Paper: Use Disappointment Aversion in a production economy
Finite Set of payoffs

\[ \{ x_1, \ldots, x_N \} \]

Expected Utility Certainty Equivalent:

\[
\begin{align*}
u(\mu) &= \sum_{i=1}^{N} \pi_i u(x_i) \\
&= \sum_{i=1}^{N} \tilde{\pi}_i u(x_i)
\end{align*}
\]

Disappointment Aversion Certainty Equivalent:

\[
\begin{align*}
u(\mu) &= \sum_{i=1}^{N} \pi_i u(x_i) - \theta \sum_{x_i \leq \mu} \pi_i (u(\mu) - u(x_i)) \\
&= \sum_{i=1}^{N} \tilde{\pi}_i u(x_i) \\
\tilde{\pi}_i &= \pi_i \frac{1+\theta I(x_i; \mu)}{1+\theta \sum_{x_i \leq \mu} \pi_i}
\end{align*}
\]
Fig. 10. Indifference curves.
Consider the following gamble:
Initial wealth $w_0$

$$\frac{u''(w_0)w_0}{u'(w_0)} = \eta - 1$$

Payoff $w_0(1 - \kappa)$ or $w_0(1 + \kappa)$ each with equal probability

Risk Premium, $P(\kappa)$ solves

$$\frac{1 + \theta}{2 + \theta} u(w_0(1 - \kappa)) + \frac{1}{2 + \theta} u(w_0(1 + \kappa)) = u(w_0(1 - P(\kappa)))$$

Taylor Expansion:

$$P(k) = \frac{\theta}{2 + \theta} d\kappa + \frac{1}{2} (1 - \eta) \left[ \frac{4(1 + \theta)}{(2 + \theta)^2} \right] d\kappa^2$$

What we’ll see: at levels of $\kappa$ faced by agents in the economy, a little $\theta$ substitutes for a lot of $\eta$
Recursive Preferences

CES Time Aggregator:

\[\nu(S_t) = \left[C_t^\gamma + \beta \mu^\gamma(S_t)\right]^{\frac{1}{\gamma}}\]

Risk Aggregator:

\[\mu^\eta(S_t) = \sum_{S_{t+1}} \pi(S_{t+1}|S_t)\nu^\eta(S_{t+1})\]

\[\Delta_{t+1} = \{S_{t+1} : \nu(S_{t+1}) < \mu(S_t)\}\]

Cases

\[\theta = 0, \gamma = \eta \rightarrow \text{Expected Utility}\]
\[\theta = 0, \gamma \neq \eta \rightarrow \text{Kreps-Porteous/Epstein-Zin}\]
\[\theta > 0 \rightarrow \text{Disappointment Aversion}\]
Technology

\[ y_t = k_t^\alpha (z_t l_t)^{1-\alpha} \]
\[ z_t = e^{\lambda t + \epsilon_t} \]
\[ \epsilon_{t+1} = \rho \epsilon_t + \xi_{t+1} \]
\[ \xi_{t+1} \sim N(0, \sigma^2) \]

Capital Accumulation

\[ e^{\varphi} k_{t+1} = x_t + (1 - \delta) k_t \]

Aggregate resource constraint

\[ y_t = c_t + x_t + g(k_t, k_{t+1}) \]

Adjustment Costs

\[ g(k_t, k_{t+1}) = \left| e^{\lambda + \varphi \frac{k_{t+1}}{k_t}} - \psi \right|^\iota k_t \]

Labor fixed \( l_t = l \)
Planners Problem

\[ v(k, \epsilon) = \max_{k'} \left\{ c^\gamma + \beta e^{[\varphi+\gamma \lambda]} \mu[v(k', \epsilon')]^{\gamma} \right\}^{\frac{1}{\gamma}} \]

s.t
\[
\begin{align*}
    c + e^{\lambda+\varphi} k' &= k^\alpha (e^\epsilon l)^{1-\alpha} + (1 - \delta) k - \left| e^{\lambda+\varphi} \frac{k'}{k} - \psi \right|^\epsilon k \\
    \epsilon' &= \rho \epsilon + \xi' \\
    \xi' &\sim N(0, \sigma^2)
\end{align*}
\]

where \( \mu = \mu[v(k', \epsilon')] \) solves:

\[
\mu^n = E \left\{ \frac{(1 + \theta I(k', \epsilon', \mu)) v^n(k', \epsilon')}{1 + \theta E[I(k', \epsilon', \mu)]} \right\}
\]
\[
I(k', \epsilon', \mu) = \begin{cases} 1, & v(k', \epsilon') < \mu \\ 0, & \text{otherwise} \end{cases}
\]
Risk Free Rate:
\[
\sum_i \pi(\epsilon_i|\epsilon) m(\epsilon_i|k, \epsilon) R^f(k, \epsilon) = 1
\]

Stochastic Discount Factor:
\[
m(\epsilon_i|k, \epsilon) = \beta e^{\lambda(\gamma-1)} \left[ \frac{1+\theta l(k', \epsilon_i, \mu)}{1+\theta \sum_i \pi(\epsilon_i|\epsilon) l(k', \epsilon_i, \mu)} \right] \left[ \frac{c(k', \epsilon_i)}{c(k, \epsilon)} \right]^{\eta-1} \left[ \frac{v(k', \epsilon_i)/\mu(k', \epsilon)}{c(k', \epsilon_i)/c(k, \epsilon)} \right]^{\eta-\gamma}
\]

Gross Return on Equity:
\[
R^e(k, \epsilon, \epsilon_i) = \frac{\alpha k'(\alpha-1)(e^{\epsilon l})(1-\alpha) + (1 - \delta) - \frac{\partial}{\partial k'} g(k', k''(k', \epsilon_i))}{1 + e^{-(\lambda+\varphi) \frac{\partial}{\partial k'} g(k, k')}}
\]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>DA</th>
<th>EZ</th>
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<tbody>
<tr>
<td>$\eta$</td>
<td>0.5</td>
<td>-55</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>-28</td>
<td>-21</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.25</td>
<td>0</td>
</tr>
<tr>
<td>$\iota$</td>
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<td>1.26?</td>
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<tr>
<td>$\alpha$</td>
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</tr>
<tr>
<td>$\delta$</td>
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</tr>
<tr>
<td>$\lambda$</td>
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<td>same</td>
</tr>
<tr>
<td>$\varphi$</td>
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</tr>
<tr>
<td>$\beta^*$</td>
<td>0.9919</td>
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</tr>
<tr>
<td>$\rho$</td>
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</tr>
<tr>
<td>$\sigma$</td>
<td>0.0164</td>
<td>same</td>
</tr>
</tbody>
</table>
Fig. 1. Impulse responses of quantities – percentage deviations from trend.
Fig. 2. Impulse responses of financial variables.
### Asset Pricing Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>DA</th>
<th>EZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(r^f)$</td>
<td>1.008%</td>
<td>1.182%</td>
<td>1.146%</td>
</tr>
<tr>
<td>Std($r^f$)</td>
<td>1.668%</td>
<td>4.457%</td>
<td>3.352%</td>
</tr>
<tr>
<td>$E(r^e - r^f)$</td>
<td>7.572%</td>
<td>6.838%</td>
<td>4.4848%</td>
</tr>
<tr>
<td>Std($r^e - r^f$)</td>
<td>15.16%</td>
<td>20.309%</td>
<td>15.323%</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>0.499</td>
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<td></td>
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<tr>
<td>MPR</td>
<td>0.34</td>
<td>0.384</td>
<td></td>
</tr>
</tbody>
</table>

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Fig. 3. Comparative statics. Statistics are annualized.
Fig. 6. Asset pricing with Epstein–Zin preferences ($\theta = 0$).
DA vs EZ

Compare Certainty equivalent for a Disappointment Aversion \((\theta, \eta = 1/2)\) vs Certainty Equivalent EU \((\eta)\)

\[
\left[ \frac{1 + \theta}{2 + \theta} \sqrt{1 - \kappa} + \frac{1}{2 + \theta} \sqrt{1 + \kappa} \right]^2 = \left[ \frac{1}{2} (1 - \kappa)^{\eta} + \frac{1}{2} (1 + \kappa)^{\eta} \right]^{1/\eta}
\]
Fig. 7. Comparison between risk preferences.
Conclusions

- Able to generate fairly reasonable asset pricing moments with either DA or EZ
- DA parameterization consistent with experimental evidence, EZ is not
- Constant labor supply helps