Asset Pricing in a Production Economy with Chew-Dekel Preferences

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Asset Pricing in Production Economies

Goal: Generate realistic asset prices (moments) along with quantities in a production economy
Difficulties:

- Endogenous variables
- Multiple channels to smooth consumption
- Marginal product of capital doesnt look like asset returns (not volatile enough)

Early Progress: Jermann (1998), Boldrin, Christiano, Fisher (2001), Tallarini (2000)

This Paper: Use Disappointment Aversion in a production economy

Disappointment Aversion Primer

Finite Set of payoffs

$$\{x_1, \ldots, x_N\}$$

Expected Utillity Certainty Equivalent:

$$u(\mu) = \sum_{i=1}^{N} \pi_i u(x_i)$$

Disappointment Aversion Certainty Equivalent:

$$\begin{aligned} u(\mu) &= \sum_{i=1}^{N} \pi_i u(x_i) - \theta \sum_{x_i \leq \mu} \pi_i (u(\mu) - u(x_i)) \\ &= \sum_{i=1}^{N} \tilde{\pi}_i u(x_i) \\ \tilde{\pi}_i &= \pi_i \frac{1 + \theta I(x_i, \mu)}{1 + \theta \sum_{x_i \leq \mu} \pi_i} \end{aligned}$$

Disappointment Aversion Indifference Curves

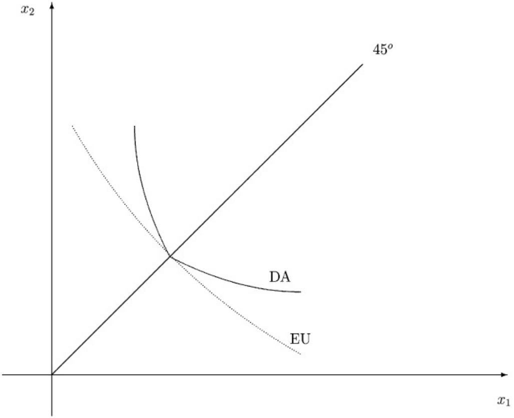

Fig. 10. Indifference curves.

Consider the following gamble:
Initial wealth w_0
$\frac{u''(w_0)w_0}{u'(w_0)} = \eta - 1$
Payoff $w_0(1-\kappa)$ or $w_0(1+\kappa)$ each with equal probability
Risk Premium, $P(\kappa)$ solves

$$\frac{1+\theta}{2+\theta} u(w_0(1-\kappa)) + \frac{1}{2+\theta} u(w_0(1+\kappa)) = u(w_0(1-P(\kappa)))$$

Taylor Expansion:

$$P(k) = \frac{\theta}{2+\theta} d\kappa + \frac{1}{2}(1-\eta)\left[\frac{4(1+\theta)}{(2+\theta)^2}\right] d\kappa^2$$

What we'll see: at levels of κ faced by agents in the economy, a little θ substitutes for a lot of η

CES Time Aggregator:

$$v(S_t) = [C_t^{\gamma} + \beta \mu^{\gamma}(S_t)]^{\frac{1}{\gamma}}$$

Risk Aggregator:

$$\begin{aligned} \mu^{\eta}(S_t) &= \sum_{S_{t+1}} \pi(S_{t+1}|S_t) v^{\eta}(S_{t+1}) \\ &\quad -\theta \sum_{S_{t+1} \in \Delta_{t+1}} \pi(S_{t+1}|S_t)(\mu^{\eta}(S_t) - v^{\eta}(S_{t+1})) \\ \Delta_{t+1} &= \{S_{t+1} : v(S_{t+1}) < \mu(S_t)\} \end{aligned}$$

Cases

$\theta = 0, \gamma = \eta \rightarrow$ Expected Utillity

$\theta = 0, \gamma \neq \eta \rightarrow$ Kreps-Porteus/Epstein-Zin

$\theta > 0 \rightarrow$ Disappointment Aversion

Technology

$$\begin{aligned} y_t &= k_t^{\alpha}(z_t l_t)^{1-\alpha} \\ z_t &= e^{\lambda t+\epsilon_t} \\ \epsilon_{t+1} &= \rho\epsilon_t + \xi_{t+1} \\ \xi_{t+1} &\sim N(0,\sigma^2) \end{aligned}$$

Capital Accumulation

$$e^{\varphi}k_{t+1} = x_t + (1-\delta)k_t$$

Aggregate resource constraint

$$y_t = c_t + x_t + g(k_t, k_{t+1})$$

Adjustment Costs

$$g(k_t, k_{t+1}) = \left| e^{\lambda+\varphi}\frac{k_{t+1}}{k_t} - \psi \right|^{\iota} k_t$$

Labor fixed $l_t = l$

Planners Problem

$$v(k,\epsilon) = max_{k'}\left\{c^{\gamma} + \beta e^{[\varphi+\gamma\lambda]}\mu[v(k',\epsilon')]^{\gamma}\right\}^{\frac{1}{\gamma}}$$

s.t

$$\begin{aligned} c + e^{\lambda+\varphi}k' &= k^{\alpha}(e^{\epsilon}l)^{1-\alpha} + (1-\delta)k - \left|e^{\lambda+\varphi}\frac{k'}{k} - \psi\right|^{\iota} k \\ \epsilon' &= \rho\epsilon + \xi' \\ \xi' &\sim N(0,\sigma^2) \end{aligned}$$

where $\mu = \mu[v(k',\epsilon')]$ solves:

$$\mu^{\eta} = E\left\{\frac{(1+\theta I(k',\epsilon',\mu))v^{\eta}(k',\epsilon')}{1+\theta E[I(k',\epsilon',\mu)]}\right\}$$

$$\begin{aligned} I(k',\epsilon',\mu) &= 1, v(k',\epsilon') < \mu \\ & \quad 0, otherwise \end{aligned}$$

Asset Prices, definitions

Risk Free Rate:

$$\sum_i \pi(\epsilon_i|\epsilon) m(\epsilon_i|k,\epsilon) R^f(k,\epsilon) = 1$$

Stochastic Discount Factor:

$$m(\epsilon_i|k,\epsilon) \quad = \quad \beta e^{\lambda(\gamma-1)} \left[\frac{1+\theta I(k',\epsilon_i,\mu)}{1+\theta \sum_i \pi(\epsilon_i|\epsilon) I(k',\epsilon_i,\mu)} \right] \left[\frac{c(k',\epsilon_i)}{c(k,\epsilon)} \right]^{\eta-1} \left[\frac{v(k',\epsilon_i)/\mu(k',\epsilon)}{c(k',\epsilon_i)/c(k,\epsilon)} \right]^{\eta-\gamma}$$

Gross Return on Equity:

$$R^e(k,\epsilon,\epsilon_i) = \frac{\alpha k'^{(\alpha-1)} (e^{\epsilon_i} l)^{(1-\alpha)} + (1-\delta) - \frac{\partial}{\partial k'} g(k', k''(k',\epsilon_i))}{1 + e^{-(\lambda+\varphi)} \frac{\partial}{\partial k'} g(k,k')}$$

Calibration Parameters

Parameter	DA	EZ
η	0.5	-55
γ	-28	-21
θ	0.25	0
ι	1.23	1.26?
α	0.36	same
δ	0.015	same
λ	0.00387	same
φ	0.00298	same
β^*	0.9919	same
ρ	0.95	same
σ	0.0164	same

Quantitity IRFS with DA

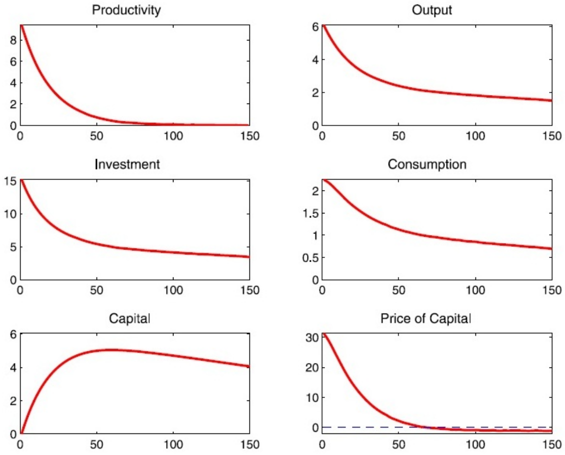

Fig. 1. Impulse responses of quantities – percentage deviations from trend.

Pricing IRFS with DA

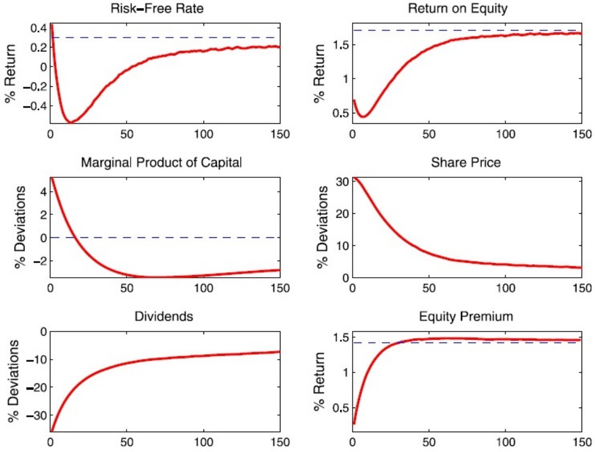

Fig. 2. Impulse responses of financial variables.

Asset Pricing Moments

Moment	Data	DA	EZ
$E(r^f)$	1.008%	1.182%	1.146%
$Std(r^f)$	1.668%	4.457%	3.352%
$E(r^e - r^f)$	7.572%	6.838%	4.4848%
$Std(r^e - r^f)$	15.16%	20.309%	15.323%
Sharpe Ratio	0.499		
MPR		.34	.384

Slide for Comparative Statics with DA

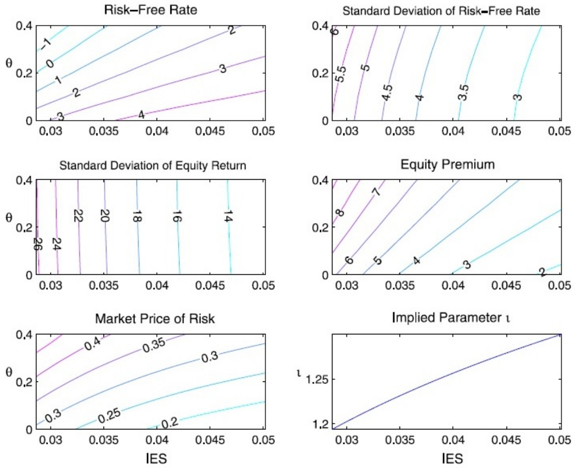

Fig. 3. Comparative statics. Statistics are annualized.

Slide for Comparative Statics with EZ

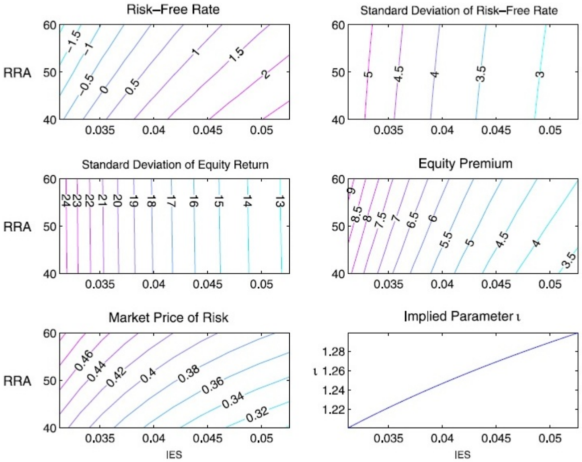

Fig. 6. Asset pricing with Epstein–Zin preferences ($\theta = 0$).

DA vs EZ

Compare Certainty equivalent for a Disappointment Aversion $(\theta, \eta = 1/2)$ vs Certainty Equivalent EU (η)

$$\left[\frac{1+\theta}{2+\theta}\sqrt{1-\kappa}+\frac{1}{2+\theta}\sqrt{1+\kappa}\right]^2=\left[\frac{1}{2}(1-\kappa)^\eta+\frac{1}{2}(1+\kappa)^\eta\right]^{1/\eta}$$

DA vs RA Picture

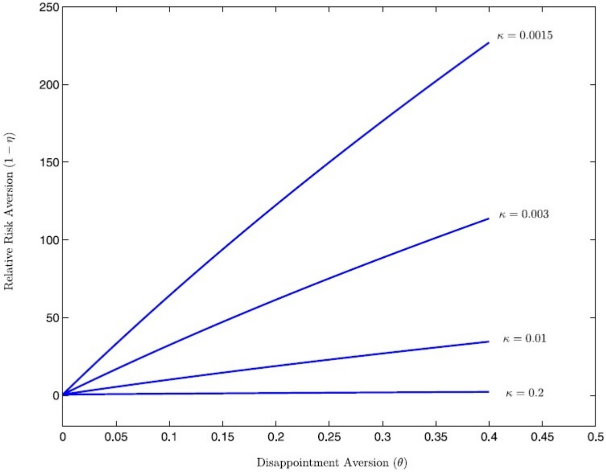

Fig. 7. Comparison between risk preferences.

Conclusions

- Able to generate fairly reasonable asset pricing moments with either DA or EZ
- DA parameterization consistent with experimental evidence, EZ is not
- Constant labor supply helps