Exit, selection, and the value of firms
Hugo A. Hopenhayn, JEDC 1992

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March 23, 2010
Tobin’s q

- Tobin’s q and average q:
  - $q = \frac{\text{firm value}}{\text{replacement cost of capital}}$
  - $\bar{q} = \frac{\sum \text{firm value}}{\sum \text{replacement cost of capital}}$, for an industry or the entire market
  - Is $q \neq 1$ an indicator of efficiency of competition?
Variation in $q$

- Systematic difference between industries and firms. Often correlated to static measures of monopolistic power.

- How could average or marginal $q < 1$?
  - Irreversible investment. Declining industry.

- How could $q > 1$? Lindenberg and Ross [1981] suggest:
  - Monopoly Rents: Pricing power.
  - Ricardian Rents: Assets difficult to imitate

- Hopenhayn [1992]: Explore the bias in $\bar{q}$ and distribution of $q$ in a perfectly competitive industry equilibrium
Demand and Environment

- Discrete time
- Consumers and Demand:
  - Single, homogeneous good
  - Aggregate demand $Q$ has inverse demand: $p = D(Q)$
- Firms:
  - Continuum of firms
  - Firms are price competitive price takers
  - $p_t$ is the equilibrium price.
Firm Cost Heterogeneity

- Firms heterogeneous over $\phi \in \mathbb{R}$
- To produce output $x_t$, cost is $c(\phi_t, x_t) \downarrow$ in $\phi_t$
- $\phi_t$ is Markov: $\phi_t \sim F(\phi'|\phi)$
- $F$ is iid across firms
- New entrants draw from distribution $G(\phi)$
Firm Profits

Production is a static decision:

- Solve static problem to convert from cost to profits
- Indirect Profits: $\pi(\phi, p) \uparrow$ in $\phi, p$
- Indirect Profit Maximizing Output: $x(\phi, p)$
Exit Decision

- **Timing:**
  1. Realize $\phi$
  2. Make exit choice
  3. Otherwise produce, collect profits

- **Firm Value**

$$v(\phi; p) = \max \left\{ 0, \pi(\phi, p) + \beta \int v(\phi'; p) F(d\phi'|\phi) \right\}$$

- Under reasonable requirements on intrinsics, for each $p \exists \phi^*$,

$$v(\phi^*; p) = 0, \quad \text{Exit if: } \phi \leq \phi^*$$
Entry Decision

- **Timing:**
  i) Pay fixed cost $c_e$. Interpret as irreversible initial investment.
  ii) Draw $\phi$ from $G$
  iii) Can exit immediately, otherwise $\phi' \sim F(\phi'|\phi)$

- **Value of entry:**

  $$v^e(p) = \int v(\phi; p) G(\phi) - c_e$$

- **Free Entry condition:** $v^e(p) \leq 0$

- **Under technical assumptions, $\exists! p^* \text{ s.t. } v^e(p^*) = 0$**
Industry

- Aggregates:
  - Price $p$
  - $\mu_t$ measures number of firms over $\phi$
- Total mass of firms: $M_t = \mu_t(\mathbb{R})$
- Aggregate Supply:

$$Q(\mu, p) = \int x(\phi, p)\mu(d\phi)$$
Stationary Industry Evolution

Given $\lambda$ mass of entrants each period, $\phi^*$ reservation value:

$$
\mu_{t+1}([\phi^*, s]) = \left[ \int F(s|\phi)\mu_t(d\phi) - \int F(\phi^*|\phi)\mu_t(d\phi) \right] 
+ \lambda [G(s) - G(\phi^*)]
$$

$$
\mu_{t+1}([-\infty, s]) = 0 \quad \forall s < \phi^*
$$

Invariant Measure if $\mu_{t+1} = \mu_t \quad \forall t$
Stopping Times/Decomposition of Distribution

- Let $\alpha_\tau$ be the fraction of entrants remaining after $\tau$ periods.
- Let $\tilde{\mu}_\tau$ be the distribution over $\phi$ of firms entering $\tau$ periods ago.
- Let $\tilde{\mu}_0$ be $G$ conditional on $\phi > \phi^*$. i.e. enter and stay.
- Assume finite expectation of stopping time: $\sum_{\tau=0}^{\infty} \alpha_\tau$

Decomposition of Distribution:

$$\mu = \lambda \sum_{\tau=0}^{\infty} \alpha_\tau \tilde{\mu}_\tau$$
Stationary Equilibrium

$(p^*, \phi^*, \lambda^*, \mu^*(\phi))$, such that:

i) Market Clearing: $p^* = D(Q(\mu^*, p^*))$

ii) Solve Exit Problem: $v(\phi^*; p^*) = 0$

iii) Free Entry, Zero Profit: $v^e(p^*) = 0$

iv) Invariant measure $\mu^*(\phi)$

Proposition

*Given a stationary equilibrium with entry and exit (and 11 'reasonable' technical assumptions), $\bar{q} > 1$*
No Ex-post Uncertainty

- $\phi_{t+1} = \phi_t$
- Value function:
  \[
  v(\phi; p) = \max \left\{ 0, \frac{1}{1 - \beta} \pi(\phi, p) \right\}
  \]
- Entry remains the same:
  \[
  v^e(p) = \int v(\phi; p) G(d\phi) - c_e
  \]
- Decision is still reservation $\phi^*$.
- After successful entry, always remain, so distribution: $\mu$ is $G$ conditional on $\phi \geq \phi^*$
Stationary Equilibrium

\((p^*, \phi^*, M^*)\), such that:

i) Market Clearing: \( p^* = D(Q(\mu^*, p^*)) \)

ii) Solve Exit Problem: \( v(\phi^*; p^*) = 0 \)

iii) Free Entry, Zero Profit: \( v^e(p^*) = 0 \)

iv) \( M^* = \mu(\mathbb{R}) = 1 - G(\phi^*) \)
Bias of $\bar{q}$

- Entry gives expression for 'replacement cost of capital':

$$c_e = \frac{1}{1 - \beta} \int_{\phi \geq \phi^*} \pi(\phi, p^*) G(d\phi)$$

- Value function conditional on post-entry continuation gives average value:

$$\bar{v} = \frac{1}{1 - \beta} \frac{\int_{\phi \geq \phi^*} \pi(\phi, p^*) G(d\phi)}{1 - G(\phi^*)}$$

$$\implies \bar{q} = \frac{\bar{v}}{c_e} = \frac{1}{1 - G(\phi^*)} > 1$$
Higher Barriers to Entry

- Interpret $c_e$ as containing barriers to entry
- A higher $c_e$ yields a higher $p^*$:
  - From free Entry: $\nu^e(p) \leq \int \nu(\phi; p)G(d\phi) - c_e$
- Which gives a lower reservation:
  - From $\nu(\phi; p) = \max\left\{0, \frac{1}{1-\beta}\pi(\phi, p)\right\}$
- Giving a lower average $q$ value:
  - From $\bar{q} = \frac{1}{1-G(\phi^*)}$
- ... contrary to 'standard intuition'
Increasing Differences

Proposition

If $\pi$ is strictly increasing in $p$ and $\phi$ then for $p' > p,$ $v(\phi, p') - v(\phi, p)$ is strictly increasing in $\phi.$ i.e. higher $p$ gives a value function with a higher slope (over $\phi$)
Higher Fixed Costs

- Assume $c(\phi, q) = c_f + v c(\phi, q)$ for fixed and variable costs
- A larger $c_f$ yields a higher $p^*$ from free entry
- Which gives a higher reservation $\phi^*$ (see diagram)
- Giving a higher $\bar{q}$ value

![Diagram showing the effect of an increase in fixed cost on selection.](image)
Summary of Results

- Looking at only surviving firms gives biased estimates of 'ex ante' profitability.
- Unless industry is declining, $q$ values will tend to be biased above 1.
- Selection, rather than monopoly rents, can generate many of the same results on $\bar{q}$.
- Entry and exit alone can generate a distribution of $q$ values.