Sovereign Default under Model Uncertainty

Job market paper, Alejo Costa

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Reading Group Presentation

February 23, 2010
Introduction

- Literature on default risk: captures main components of the cycle associated with default risk, but fails to reproduce the level of spreads.

- “Historical default probabilities are not enough to generate such spreads.”

- **This paper**: introduces fear of model misspecification to account for these features.

- Fear of model misspecification $\Rightarrow$ premium on risky debt.

- In addition: explains the small difference in spreads across countries.
Figure 1. Sovereign bond spreads for selected countries: EMBI+
Setup

- Small open economy
- 2 types of agents:
  - Government: trade debt with foreign investors
  - Foreign investors: continuum, ambiguity averse
- Incomplete markets $\Rightarrow$ non-contingent debt
- No enforcement $\Rightarrow$ default can be optimal
Notation:

- $y$: stochastic endowment
- $f(\cdot|\cdot)$: approximating density
- $\hat{f}(\cdot|\cdot)$: unknown, true density
- $B$: bond holdings
- $q(B', y)$: bond price
- $Z$: debt limit (prevents Ponzi schemes)
- $\delta \in \{0, 1\}$: default (0), no default (1)
- $y^{\text{def}} = h(y)$: output/consumption during default
- $\eta$: entropy constraint
- $\alpha$: Lagrange multiplier on entropy constraint
Government’s problem

- Choose default/no default
  No default ⇒ choose next period debt

\[ V^O(B, y) = \max_{\delta(B,y) \in \{0,1\}} \{(1 - \delta)V^D(y) + \delta V^C(B, y)\} \]

where

\[ V^D(y) = u(h(y)) + \beta \left[ (1 - \gamma) \int_{y'} V^D(y')f(y'|y)dy' \right. \]
\[ \left. + \gamma \int_{y'} V^O(0, y')f(y'|y)dy' \right] \]

\[ V^C(B, y) = \max_{B' \leq Z} \left\{ u(y + q(B')B' - B) + \beta \int_{y'} V^O(B', y')f(y'|y)dy' \right\} \]
Government’s decision

- Optimal default policy: sets of values $y'$ where it is optimal to repay, $C(\cdot)$, or default, $D(\cdot)$

$$C(B') = \{ y' \in Y : V_C(B', y') \geq V^D(y') \}$$
$$D(B') = \{ y' \in Y : V_C(B', y') < V^D(y') \}$$

- No default $\Rightarrow B' = \tilde{B}(B, y)$

$$c(B, y) = \begin{cases} y - B + q(\tilde{B}(B, y)) \tilde{B}(B, y) & \text{if } \delta(B, y) = 1 \\ y_{\text{def}} & \text{if } \delta(B, y) = 0 \end{cases}$$
Foreign investors

- Continuum of measure 1
- Borrow/lend at risk-free rate $r$
- Problem:

$$q(B', y) = \min_{\hat{f}(y'|y)} \frac{1}{1 + r} \int_{0}^{\infty} \delta(B', y') \hat{f}(y'|y) dy'$$

s.t. $$\frac{1}{1 + r} \int_{0}^{\infty} \hat{f}(y'|y) \log \frac{\hat{f}(y'|y)}{f(y'|y)} dy' \leq \eta$$

$$\int_{0}^{\infty} \hat{f}(y'|y) dy' = 1$$
Foreign investors

- Continuum of measure 1
- Borrow/lend at risk-free rate $r$
- Problem:

$$q(B', y) = \min_{\hat{f}(y'|y;B')} \frac{1}{1 + r} \times$$

$$\int_{0}^{\infty} \left( \delta(B', y') + \alpha(B') \log \frac{\hat{f}(y'|y;B')}{f(y'|y)} \right) \hat{f}(y'|y;B') dy'$$

s.t. $$\int_{0}^{\infty} \hat{f}(y'|y;B') dy' = 1$$
Foreign investors “decision”

- Distorted density:

\[ \hat{f}^*(y' | y; B') = f(y' | y) \exp \left(- \frac{\delta(B', y')}{\alpha(B')} \right) \frac{\exp \left(- \frac{\delta(B', y')}{\alpha(B')} \right)}{\mathbb{E} \left[ \exp \left(- \frac{\delta(B', y')}{\alpha(B')} \right) \right]} \]

- Bond price determined from distorted density:

\[ q(B', y) = \frac{1}{1 + r} \int_0^\infty \left[ \hat{f}^*(y' | y; B') \right] \delta(B', y') f(y' | y) dy' \]

\[ = \mathbb{E} \left( m(B', y') \delta(B', y') \right) \]
Foreign investors “decision” (2)

Where \( m(\cdot, \cdot) \) is the modified stochastic discount factor:

\[
m(B', y') = \left( \frac{1}{1 + r} \right) \frac{\exp \left( - \frac{\delta(B', y')}{\alpha(B')} \right)}{\mathbb{E} \left[ \exp \left( - \frac{\delta(B', y')}{\alpha(B')} \right) \right]}
\]

- Ambiguity-averse stochastic discount factor depends on \( B' \)

- **Key**: model misspecification function \( \alpha(B') \)
Market price of model uncertainty

- Following Hansen and Sargent (2008): define market price of model uncertainty $\rho_u$ as

$$\rho_u^t = \text{std}_t \left( \frac{\hat{f}^*(y_t)}{f(y_t)} \right) = \text{std}_t \left( \frac{\exp \left( -\frac{\delta(B', y')}{\alpha(B')} \right)}{\mathbb{E}_t \left[ \exp \left( -\frac{\delta(B', y')}{\alpha(B')} \right) \right]} \right)$$

- Bond price can be decomposed as

$$q(B', y) = \frac{1 - p^D(B', y)}{1 + r} + \text{Cov}(m(B', y'), \delta(B', y'))$$

$$= \frac{1 - p^D(B', y)}{1 + r} - \frac{\sigma\delta(B', y)\rho_u(B', y)}{1 + r}$$
Calibrating $\eta$ using error detection probabilities

(Following Hansen and Sargent, 2008)

- Start with pair of income and debt ($y, B$)
- Generate pair next period ($y', B'$), for both approximating density and distorted density
  $\Rightarrow$ time series for 40 quarters
- Calculate likelihood ratios under nulls of approximating and distorted densities
- Repeat 5000 times to obtain probabilities of model detection error

\[
p_{AM} = \text{Prob} \left( \log \frac{L_{AM}}{L_{DM}} < 0 \mid AM \right), \quad p_{DM} = \text{Prob} \left( \log \frac{L_{AM}}{L_{DM}} > 0 \mid DM \right)
\]

- Error detection probability: $p(\eta) = \frac{1}{2}(p_{AM} + p_{DM}) = 0.2$
Calibration of other parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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<tbody>
<tr>
<td>Risk-free Rate</td>
<td>$r=1.17%$</td>
<td>U.S. 5 year bond quarterly yield</td>
</tr>
<tr>
<td>Risk-Aversion</td>
<td>$\psi=2$</td>
<td></td>
</tr>
<tr>
<td>Revenue Process</td>
<td>$\mu=12.96, \rho=0.99, \sigma=0.10$</td>
<td>Argentina's Revenue process</td>
</tr>
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Table 2. Parameter Values

<table>
<thead>
<tr>
<th>Calibration</th>
<th>Values</th>
<th>Calibrated Moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$\beta=0.97$</td>
<td>3% Default Probability</td>
</tr>
<tr>
<td>Probability of re-entry</td>
<td>$\gamma=0.17$</td>
<td>Primary Deficit Volatility 0.68</td>
</tr>
<tr>
<td>Entropy Parameter</td>
<td>$\eta=0.01$</td>
<td>Error Detection Probability $P(\eta)=0.2$</td>
</tr>
<tr>
<td>Revenue Cost</td>
<td>0.9 $E(\gamma)$</td>
<td>13% Debt Services to Revenue</td>
</tr>
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Table 3. Calibration Moments and Values


$$\log(y_{t+1}) = (1 - \rho)\mu + \rho \log(y_t) + \varepsilon_{t+1}$$
Results

Figure 7. Bond Price as a Function of Debt. Figure 8. Interest rates as a function of debt

- Higher debt $\Rightarrow$ lower bond price
- Higher income $\Rightarrow$ higher bond price
- Captures high spreads through high default probabilities under distorted density