Sovereign Default under Model Uncertainty

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Reading Group Presentation

February 23, 2010

Introduction

- Literature on default risk: captures main components of the cycle associated with default risk, but fails to reproduce the level of spreads
- "Historical default probabilities are not enough to generate such spreads."
- **This paper**: introduces fear of model misspecification to account for these features
- Fear of model misspecification ⇒ premium on risky debt
- In addition: explains the small difference in spreads across countries

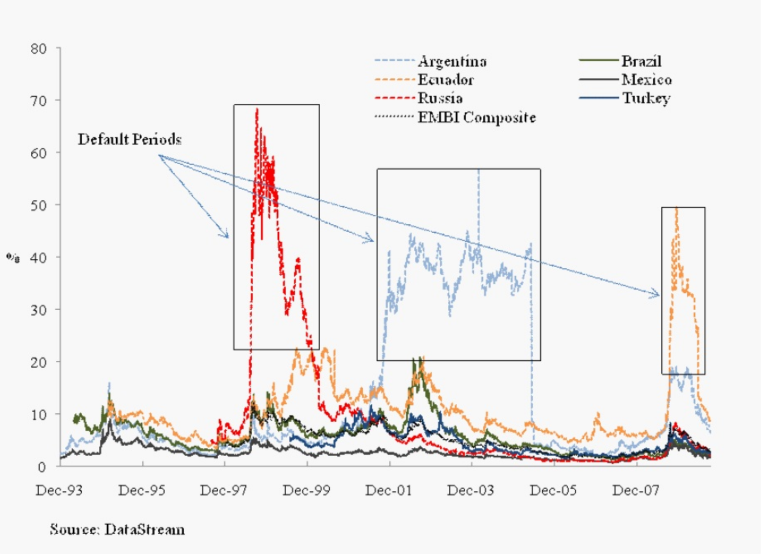


Figure 1. Sovereign bond spreads for selected countries: EMBI+

Setup

- Small open economy
- 2 types of agents:
  - Government: trade debt with foreign investors
  - Foreign investors: continuum, ambiguity averse
- Incomplete markets $\Rightarrow$ non-contingent debt
- No enforcement $\Rightarrow$ default can be optimal

Notation:

- y: stochastic endowment
- $f(\cdot|\cdot)$: approximating density
- $\hat{f}(\cdot|\cdot)$: unknown, true density
- B: bond holdings
- $q(B', y)$: bond price
- Z: debt limit (prevents Ponzi schemes)
- $\delta \in \{0, 1\}$: default (0), no default (1)
- $y^{\text{def}} = h(y)$: output/consumption during default
- η: entropy constraint
- α: Lagrange multiplier on entropy constraint

Government's problem

- Choose default/no default
  No default ⇒ choose next period debt

$$V^O(B,y) = \max_{\delta(B,y)\in\{0,1\}} \{(1-\delta)V^D(y) + \delta V^C(B,y)\}$$

where

$$V^D(y) = u(h(y)) + \beta\Big[(1-\gamma)\int_{y'} V^D(y')f(y'|y)dy' + \gamma\int_{y'} V^O(0,y')f(y'|y)dy'\Big]$$

$$V^C(B,y) = \max_{B'\leq Z}\left\{u(y+q(B')B'-B) + \beta\int_{y'} V^O(B',y')f(y'|y)dy'\right\}$$

Government's decision

- Optimal default policy: sets of values y' where it is optimal to repay, $C(\cdot)$, or default, $D(\cdot)$

$$C(B') = \{y' \in Y : V^C(B', y') \geq V^D(y')\}$$
$$D(B') = \{y' \in Y : V^C(B', y') < V^D(y')\}$$

- No default $\Rightarrow B' = \tilde{B}(B, y)$

$$c(B, y) = \begin{cases} y - B + q\left(\tilde{B}(B, y)\right)\tilde{B}(B, y) & \text{if } \delta(B, y) = 1 \\ y^{\text{def}} & \text{if } \delta(B, y) = 0 \end{cases}$$

Foreign investors

- Continuum of measure 1
- Borrow/lend at risk-free rate r
- Problem:

$$
\begin{aligned}
q(B', y) = \quad & \min_{\hat{f}(y'|y)} \frac{1}{1+r} \int_0^\infty \delta(B', y') \hat{f}(y'|y) dy' \\
\text{s.t.} \quad & \frac{1}{1+r} \int_0^\infty \hat{f}(y'|y) \log \frac{\hat{f}(y'|y)}{f(y'|y)} dy' \leq \eta \\
& \int_0^\infty \hat{f}(y'|y) dy' = 1
\end{aligned}
$$

Foreign investors

- Continuum of measure 1
- Borrow/lend at risk-free rate r
- Problem:

$$q(B', y) = \min_{\hat{f}(y'|y;B')} \frac{1}{1+r} \times$$

$$\int_0^\infty \left(\delta(B', y') + \alpha(B') \log \frac{\hat{f}(y'|y;B')}{f(y'|y)} \right) \hat{f}(y'|y;B') dy'$$

$$\text{s.t.} \quad \int_0^\infty \hat{f}(y'|y;B') dy' = 1$$

Foreign investors "decision"

- Distorted density:

$$\hat{f}^*(y'|y;B') = f(y'|y)\frac{\exp\left(-\frac{\delta(B',y')}{\alpha(B')}\right)}{\mathbb{E}\left[\exp\left(-\frac{\delta(B',y')}{\alpha(B')}\right)\right]}$$

- Bond price determined from distorted density:

$$q(B',y) = \frac{1}{1+r}\int_0^\infty \left[\frac{\hat{f}^*(y'|y;B')}{f(y'|y)}\right]\delta(B',y')f(y'|y)dy'$$

$$= \mathbb{E}\left(m(B',y')\delta(B',y')\right)$$

Foreign investors "decision" (2)

- Where $m(\cdot,\cdot)$ is the modified stochastic discount factor:

$$m(B',y') = \left(\frac{1}{1+r}\right) \frac{\exp\left(-\frac{\delta(B',y')}{\alpha(B')}\right)}{\mathbb{E}\left[\exp\left(-\frac{\delta(B',y')}{\alpha(B')}\right)\right]}$$

- Ambiguity-averse stochastic discount factor depends on B'
- **Key**: model misspecification function $\alpha(B')$

Market price of model uncertainty

- Following Hansen and Sargent (2008): define market price of model uncertainty ρ^u as

$$\rho_t^u = \text{std}_t\left(\frac{\hat{f}^*(y_t)}{f(y_t)}\right) = \text{std}_t\left(\frac{\exp\left(-\frac{\delta(B',y')}{\alpha(B')}\right)}{\mathbb{E}_t\left[\exp\left(-\frac{\delta(B',y')}{\alpha(B')}\right)\right]}\right)$$

- Bond price can be decomposed as

$$\begin{aligned} q(B',y) &= \frac{1-p^D(B',y)}{1+r} + \text{Cov}(m(B',y'),\delta(B',y')) \\ &= \frac{1-p^D(B',y)}{1+r} - \frac{\sigma_\delta(B',y)\rho^u(B',y)}{1+r} \end{aligned}$$

Calibrating η using error detection probabilities

(Following Hansen and Sargent, 2008)

- Start with pair of income and debt (y, B)
- Generate pair next period (y', B'), for both approximating density and distorted density
  $\Rightarrow$ time series for 40 quarters
- Calculate likelihood ratios under nulls of approximating and distorted densities
- Repeat 5000 times to obtain probabilities of model detection error

$$p_{AM} = \text{Prob}\left(\log \frac{L_{AM}}{L_{DM}} < 0 | \text{AM}\right), \quad p_{DM} = \text{Prob}\left(\log \frac{L_{AM}}{L_{DM}} > 0 | \text{DM}\right)$$

- Error detection probability: $p(\eta) = \frac{1}{2}(p_{AM} + p_{DM}) = 0.2$

Calibration of other parameters

Parameters	Values	
Risk-free Rate	r=1.17%	U.S. 5 year bond quarterly yield
Risk-Aversion	ψ=2	
Revenue Process	μ=12.96 ρ=0.99 σ=0.10	Argentina's Revenue process

Table 2. Parameter Values

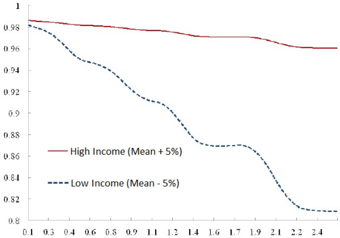
Calibration	Values	Calibrated Moments
Discount Factor	β=0.97	3% Default Probability
Probability of re-entry	γ=0.17	Primary Deficit Volatility 0.68
Entropy Parameter	η=0.01	Error Detection Probability P(η)= 0.2
Revenue Cost	0.9 E(y)	13% Debt Services to Revenue

Table 3. Calibration Moments and Values

- Process for government income (Argentina, 1994-2001):

$$\log(y_{t+1}) = (1-\rho)\mu + \rho \log(y_t) + \varepsilon_{t+1}$$

Results



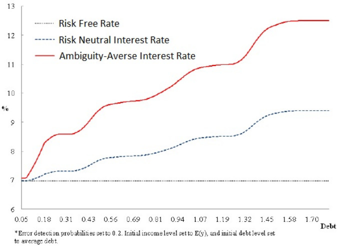


Figure 7. Bond Price as a Function of Debt. Figure 8. Interest rates as a function of debt

- Higher debt $\Rightarrow$ lower bond price
- Higher income $\Rightarrow$ higher bond price
- Captures high spreads through high default probabilities under distorted density