Affine Term Structure Models and the Forward Premium Anomaly

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Reading Group Presentation

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Forward Premium Anomaly

- No arbitrage $\Rightarrow R_t = \frac{F_t}{S_t} R_t^*$
- Forward premium: $f_t - s_t = r_t - r_t^*$
- Forward premium regression:

$$s_{t+1} - s_t = a_1 + a_2(f_t - s_t) + \text{residual}$$

- Results: US dollar vs. ...

Currency	a_1	a_2	Std Err	R^2
British Pound	-0.0062	-1.840	0.0339	0.0213
	(0.0027)	(0.847)		

- Anomaly: high interest rate currencies tend to appreciate

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Fama (1984)

- Attributes behavior to a time-varying risk-premium
- Decomposition: $f_t - s_t = \underbrace{f_t - \mathbb{E}_t s_{t+1}}_{p_t} + \underbrace{\mathbb{E}_t s_{t+1} - s_t}_{q_t}$
- Population regression coefficient:

$$a_2 = \frac{\text{cov}(q, p+q)}{\text{var}(p+q)} = \frac{\text{cov}(q, p) + \text{var}(q)}{\text{var}(p+q)}$$

- Fama's conditions: $a_2 < 0$ requires
  1. $\text{cov}(p, q) < 0$
  2. $\text{var}(p) > \text{var}(q)$

Pricing kernels

$$\begin{aligned} \text{For dollar returns :} \quad & \mathbb{E}_t(m_{t+1}R_{t+1}) = 1 \\ \text{For pound returns :} \quad & \mathbb{E}_t(m^*_{t+1}R^*_{t+1}) = 1 \\ \text{For converted returns :} \quad & \mathbb{E}_t\left[m_{t+1}(S_{t+1}/S_t)R^*_{t+1}\right] = 1 \end{aligned}$$

- Pound asset and currencies both traded

$$\Rightarrow \ \mathbb{E}_t(m^*_{t+1}R^*_{t+1}) = \mathbb{E}_t\left[m_{t+1}(S_{t+1}/S_t)R^*_{t+1}\right]$$

- Complete markets ⇒ unique choice $m^*_{t+1} = m_{t+1}S_{t+1}/S_t$

$$\Rightarrow \ s_{t+1} - s_t = \log m^*_{t+1} - \log m_{t+1}$$

Forward Rates and Risk Premiums

- Pricing a forward contract at t

$$\mathbb{E}_t[m_{t+1}(F_t - S_{t+1})] = 0$$

- Rearranging $\Rightarrow$

$$(F_t/S_t)\mathbb{E}_t(m_{t+1}) = \mathbb{E}_t(m_{t+1}S_{t+1}/S_t) = \mathbb{E}_t(m^*_{t+1})$$

- $\Rightarrow$ forward premium

$$\begin{aligned} f_t - s_t &= \log \mathbb{E}_t m^*_{t+1} - \log \mathbb{E}_t m_{t+1} \\ &= r_t - r^*_t \end{aligned}$$

and $r_t = -\log \mathbb{E}_t m_{t+1}$

Forward Rates and Risk Premiums (2)

- Expected rate of depreciation:

$$q_t \equiv \mathbb{E}_t s_{t+1} - s_t = \mathbb{E}_t \log m^*_{t+1} - \mathbb{E}_t \log m_{t+1}$$

- $\Rightarrow$ risk premium:

$$p_t = (\log \mathbb{E}_t m^*_{t+1} - \mathbb{E}_t \log m^*_{t+1}) - (\log \mathbb{E}_t m_{t+1} - \mathbb{E}_t \log m_{t+1})$$

Cumulant decomposition

- Expand $\log \mathbb{E}_t m_{t+1}$ in terms of cumulants of the conditional distribution of $\log m_{t+1}$

$$\log \mathbb{E}_t m_{t+1} = \sum_{j=1}^{\infty} \frac{\kappa_{jt}}{j!}$$

- Note that $\kappa_{1t} = \mu_{1t}$

$$\Rightarrow \quad p_t = \kappa^*_{-1,t} - \kappa_{-1,t} \qquad \left(\kappa_{-1,t} = \sum_{j=2}^{\infty} \frac{\kappa_{jt}}{j!} \right)$$

- First moments affect q
- Higher moments affect p

General Affine Model

Duffie and Kan (1996)

- Vector of state variables z:

$$z_{t+1} = (I - \Phi)\theta + \Phi z_t + V(z_t)^{1/2}\varepsilon_{t+1}$$

where $\{\varepsilon_t\} \sim \text{NID}(0, I)$, Φ is stable with positive diagonal elements, and V is diagonal with elements

$$v_i(z) = \alpha_i + \beta_i^\top z$$

- Given z pricing kernel is

$$-\log m_{t+1} = \delta + \gamma^\top z_t + \lambda^\top V(z_t)^{1/2}\varepsilon_{t+1}$$

Standard Cox-Ingersoll-Ross

$$-\log m_{t+1} = (1+\lambda_1^2/2)z_{1t} + \lambda_1 z_{1t}^{1/2}\varepsilon_{1,t+1}$$
$$-\log m^*_{t+1} = (1+\lambda_2^2/2)z_{2t} + \lambda_2 z_{2t}^{1/2}\varepsilon_{2,t+1}$$

- Regression coefficient:

$$a_2 = 1 + \frac{\lambda_1^2}{2}$$

- The model is inconsistent with the anomaly for any value of λ_1

Independent Factor Model

$$-\log m_{t+1} = (1+\lambda_0^2/2)z_{0t} + (-1+\lambda_1^2/2)z_{1t} + \lambda_0 z_{0t}^{1/2}\varepsilon_{0t+1} + \lambda_1 z_{1t}^{1/2}\varepsilon_{1t+1}$$
$$-\log m^*_{t+1} = (1+\lambda_0^2/2)z_{0t} + (-1+\lambda_2^2/2)z_{2t} + \lambda_0 z_{0t}^{1/2}\varepsilon_{0t+1} + \lambda_2 z_{2t}^{1/2}\varepsilon_{2t+1}$$

- Regression coefficient:

$$a_2 = 1 - \frac{\lambda_1^2}{2}$$

- Model *can* generate $a_2 < 0$ with large enough λ_1
- However, parameter estimates $\Rightarrow$ extreme distributional properties of z_1

Interdependent Factor Model

$$-\log m_{t+1} = (1+\lambda_1^2/2) z_{1t} + (\gamma^* + \lambda_2^2/2) z_{2t} + \lambda_1 z_{1t}^{1/2} \varepsilon_{1t+1} + \lambda_2 z_{2t}^{1/2} \varepsilon_{2t+1}$$
$$-\log m_{t+1}^* = (\gamma^* + \lambda_2^2/2) z_{1t} + (1+\lambda_1^2/2) z_{2t} + \lambda_2 z_{1t}^{1/2} \varepsilon_{1t+1} + \lambda_1 z_{2t}^{1/2} \varepsilon_{2t+1}$$

- Regression coefficient:

$$a_2 = 1 + \frac{\lambda_1^2 - \lambda_2^2}{2(1-\gamma^*)}$$

- Model *can* generate $a_2 < 0$ with $\lambda_2^2 > \lambda_1^2$
- Short rates:

$$r_t = z_{1t} + \gamma^* z_{2t} \qquad r_t^* = \gamma^* z_{1t} + z_{2t}$$

Estimating the model

- Data: dollar-pound exchange rate, dollar short rate, and the dollar forward premium
- Use GMM to match
  - mean and variance of the dollar short rate
  - variance and autocorrelation of the dollar-pound forward premium
  - variance of the dollar-pound depreciation rate
  - regression coefficient a_2

Estimates

Parameter	Estimate	Standard Error
θ_1	0.005	4.950×10^-4
σ_1	0.017	0.004
ϕ_1	0.919	0.037
γ^*	0.331	0.080
λ_1	−5.541	2.184
λ_2	−5.886	2.202

$$z_{it+1} = (1 - \phi_1)\theta_1 + \phi_1 z_{it} + \sigma_1 z_{it}^{1/2} \varepsilon_{it+1}$$

- "Price of risk" coefficients λ_i are huge!
- Counterfactual implications for other risk premiums
  Eg. long yields in the implied curve reach as high as 80 percent per annum

Conclusion

- Possible to parameterize some affine term structure models to fit the forward premium anomaly
- However, with the necessary parameters the models have serious shortcomings

If Exchange Rates Are Random Walks, Then Almost Everything We Say About Monetary Policy Is Wrong

- How do changes in short-term interest rates affect the economy?
- Standard DSGE: through conditional means

$$r_t = -\log \mathbb{E}_t \left[\beta \frac{U_{c,t+1}}{U_{c,t}} \frac{1}{\pi_{t+1}} \right]$$

- Two countries:

$$r_t - r_t^* = q_t - p_t = \mathbb{E}_t s_{t+1} - s_t - p_t$$

- Exchange rate random walks $\Rightarrow$ q_t constant

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$$
\begin{aligned}
r_t &= \mathbb{E}_t \left[-\log \frac{U_{c,t+1}}{U_{c,t}} \frac{1}{\pi_{t+1}} \right] - \frac{1}{2} \text{var}_t \left[\log \frac{U_{c,t+1}}{U_{c,t}} \frac{1}{\pi_{t+1}} \right] \\
&= -\mathbb{E}_t \log \frac{U_{c,t+1}}{U_{c,t}} + \mathbb{E}_t \log \pi_{t+1} + \text{constant}
\end{aligned}
$$

- Two countries:

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