Overconfidence and Speculative Bubbles
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Sargent Reading Group

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Specific episodes in the US and abroad where market prices for some classes of assets display significant correlation between:

1. High prices.
2. High trading volume.
3. High price volatility.

Such historical examples include the 1929 boom and crash, the dot-com bubble and the recent housing crisis.
The historical episodes above are referred to as price bubbles. An asset price has a **bubble** component if the price of the asset is different from it’s **fundamental value**.

In this paper there is a bubble in a risky asset:

- Agents disagree about the probability distribution of the dividend streams.
- They go through waves of relative optimism and pessimism with optimists holding the assets.
Motivation

- Such fluctuations will generate trade.
- Price of the asset includes the option to sell to more optimistic agents in the future.
- This drives the asset price above the agents own valuation of the fundamentals (subjective definition of a bubble).
- Short sale constraints on the asset are important to generate such bubbles.

Use the framework to do some comparative statics:
- On the effects of increases in investor confidence.
- On the effectiveness of a Tobin tax on transactions.
- On possibility of bubbles occurring in markets with high transaction costs (i.e. housing)
The Model

Assets

- A single risky asset with cumulative dividend process $D_t$:

$$dD_t = f_t dt + \sigma_D dZ^D_t$$

- The fundamental variable $f_t$ is not observable but follows an Ornstein-Uhlenbeck (OU) process:

$$df_t = -\lambda (f_t - \bar{f}) + \sigma_f dZ^f_t$$

- The discrete time equivalent of the process above is a stationary AR1 process.

- The stationary probability distribution of OU is Gaussian with mean $\bar{f}$ and variance $\frac{\sigma^2_f}{2\lambda}$.

- No short sales allowed.

- Asset is in finite supply.
The Model

Agents:
- Two sets of risk-neutral agents A and B.
- For convenience, if O refers to one group, then the other group is $\bar{O}$.
- They have deep pockets (infinite wealth).
- Same discount factor $r$.

Information Structure
- Agents in both groups observe signals $s^A$ and $s^B$ where for $O \in \{A, B\}$:
  $$ds^O_t = f_t dt + \sigma_s dz^O_t$$
- $Z^D$, $Z^f$, $Z^A$, and $Z^B$ are mutually independent Brownian Motions.
- Agent O believes that his signal is more informative.
The Model

- That is agent O believes (incorrectly) that innovations to $s^O$ are correlated with innovations to $f_t$.

$$ds^O_t = f_t dt + \sigma_s (\phi dZ^f_t + \sqrt{1 - \phi^2} dZ^O_t)$$

- However, he believes (correctly) that innovations to the other agent’s signal are uncorrelated with innovations to $f_t$.

- $\phi$ represents the degree of over-confidence of an agent in his own signal $s^o$.

- They agree to disagree.

Evolution of Beliefs

- Agents cannot infer $f_t$ perfectly due to dividend noise.

- They have to use observations of $D$, $s^A$ and $s^B$ to form beliefs about $f_t$. 
The Model

- Hence they face a filtering problem for $f_t$ with Gaussian initial conditions.
- The discrete time equivalent of this problem is the Kalman filter.
- The only difference is that the prior and posterior variance are the same in continuous time, and they follow an ODE.
- Like in the discrete time counterpart can get a time invariant posterior variance regarding beliefs about $f_t$ given by $\gamma$.
- $\gamma$ decreases with $\phi$. 
The Model

Evolution of Beliefs

- The conditional mean of beliefs about \( f_t \) in A (denote it \( \hat{f}_t^A \)) evolves according to:

\[
d\hat{f}_t^A = -\lambda(\hat{f}_t^A - \bar{f})dt + \frac{\phi \sigma_s \sigma_f + \gamma}{\sigma_s} dW_t^A + \frac{\gamma}{\sigma_s} dW_t^B + \frac{\gamma}{\sigma_D} dW_t^D
\]

- \( dW_t^A, dW_t^B \) and \( dW_t^D \) are surprises from signals \( s^A, s^B \) and D modeled as innovations to standard mutually independent Brownian Motions.

- Since conditional variance of \( f_t \) is constant then refer to conditional means as beliefs.

- Denote \( g^O = \hat{f}^\bar{O} - \hat{f}^O \) the difference in the conditional mean of beliefs.

- \( g^O > 0 \) implies that group O is relatively more pessimistic.
The Model

Evolution of Beliefs

- The evolution of belief differences for group O is given by:
  \[ dg^O = -\rho g^O dt + \sigma_g dW_g^O \]

  \[ \rho = \sqrt{(\lambda + \phi \frac{\sigma_f}{\sigma_s})^2 + (1 - \phi^2)\sigma_f^2\left(\frac{2}{\sigma_s^2} + \frac{1}{\sigma_D^2}\right)} \]

  \[ \sigma_g = \sqrt{2\phi\sigma_f} \]

- \( g^A \) follows a mean-reverting process from A’s perspective.
- Mean-reversion is measured by \(-\rho/2\sigma_g^2\).
- A higher \( \phi \) increases the divergence in opinion and slows mean reversion.
- This is crucial for the results that follow.
The Model

Trader’s Problem

- Seller pays $c \geq 0$ for a unit of asset sold.
- Let $O$ denote the group of current owner of the asset.
- The price of the asset $p_t^O$ is given by:

$$p_t^O = \max_{\tau \geq 0} E_t^O \left[ \int_t^{t+\tau} e^{-r(s-t)} dD_s + e^{-r\tau} (p_{t+\tau}^O - c) \right]$$

where $\tau$ is a stopping time, $p_{t+\tau}^O$ is the buyer’s reservation price at time $t + \tau$.

- Plugging for $dD = \hat{f}^O + \sigma_D dW_D^O$, using Ito’s lemma to express $\hat{f}_t^O$ and cancelling the martingale term:

$$p_t^O = \max_{\tau \geq 0} E_t^O \left[ \int_t^{t+\tau} e^{-r(s-t)} [\bar{f} + e^{-\lambda(s-t)}(\hat{f}_t^O - \bar{f})] ds + e^{-r\tau} (p_{t+\tau}^O - c) \right]$$
The Model

Trader’s Problem

• Conjecture an equilibrium price function:

\[ p_t^O = p^O(\hat{f}_t^O, g_t^O) = \frac{\bar{f}}{r} + \frac{\hat{f}_t^O - \bar{f}}{r + \lambda} + q(g_t^O) \]

**Fundamental Valuation**

**Resale Option Value**

• Combining the last two equations above:

\[ q(g_t^O) = \max_{\tau \geq 0} E_t^O \left\{ \frac{g_t^O}{r + \lambda} + q(g_{t+\tau}) - c \right\} e^{-r\tau} \]

**Buyer’s Excess Optimism**

**Future Resale Value**
The Model

Equilibrium Price Solution

- Trade occurs at the region of $x$ (stopping region SR) such that:

$$q(x) = \frac{x}{r + \lambda} + q(-x) - c$$

- The complement of the stopping region is the continuation region (CR).

- In CR $e^{-rt}q(g_t^0)$ follows a martingale, while in SR it follows a supermartingale.

- Using Ito’s lemma to expand $e^{-rt}q(g_t^0)$, then $q$ in CR must satisfy the ODE:

$$\frac{1}{2}\sigma_g^2 q'' - \rho x q' - rq = 0$$

- In SR replace $=$ with $<$. 
Equilibrium Price Solution

- To find such $q$ guess that $CR = (-\infty, k^*)$.
- $k^*$ is the minimal amount of difference in beliefs that generates a trade.
- Can find a function $h(x)$ that solves the ODE above in $CR$. Then:

\[
q(x) = \begin{cases} 
\beta_1 h(x) & x < k^* \\
\frac{x}{r+\lambda} + \beta_1 h(-x) - c & x \geq k^*
\end{cases}
\]

\[
\beta_1 = \frac{1}{(r + \lambda)[h'(k^*) + h'(-k^*)]}
\]

- $h$ and its first three derivatives are positive everywhere. Also, $\lim_{x \to -\infty} h(x) = 0$. 
The Model

Equilibrium Price Solution

- Can show that $q$ above is an equilibrium option value function.
- Optimal policy is given by:

$$\begin{cases} 
\text{Immediate sale if } g^O > k^* \\
\text{Otherwise wait until first time when } g^O \geq k^*
\end{cases}$$

- Define a bubble as the difference between the price one is willing to pay for an asset and the price if asset is kept forever and never sold.
- When $g^O = k^*$ ownership switches to $\tilde{O}$.
- Therefore the bubble at that time is given by:

$$b = p^O_t - \left[ \frac{\tilde{f}}{r} + \frac{\hat{f}^O_t - \tilde{f}}{r + \lambda} \right] = q(-k^*)$$
Some Results

Price Volatility

- Volatility of the option value is given by:

\[
\eta(x) = \frac{\sqrt{2\phi \sigma_f}}{r + \lambda} \frac{h'(x)}{h'(k^*) + h'( -k^*)} \quad \forall x < k^*
\]

- This is increasing in \( x \).
- The volatility of the fundamental valuation increases with \( \phi \).
- For \( \lambda = 0 \), the volatility of the fundamental valuation is given by \( \frac{\sigma_f}{r} \).
Some Results

Small Trading Costs

- $k^*$ depends continuously on cost $c$.
- If $c = 0$, then $k^* = 0$. In this case trade occurs whenever beliefs cross.
- The expected duration between trades is zero.
- Therefore even at $c=0$ can sustain a bubble. Agents expect to make infinitesimal gains at very high trading frequencies that compound to a bubble.

$$b = \frac{1}{2(r + \lambda)} \frac{h(0)}{h'(0)}$$

For small $c$, size of bubble depends positively on $\sigma_g$ and $\rho$. The effect of a change in $k^*$ is second order.

- As $\sigma_g$ increases, beliefs oscillate more leading to faster trading opportunities.
Some Results

- As $\rho$ increases, the resale option will be exercised quicker.
- Both factors increase in the size of the bubble.
- Understand the effects of an increase in investor confidence through the effect on $\sigma_g$ and $\rho$.
- Figure 1 plots the effect of an increase in confidence on:
  - a. The trading barrier $k^*$.
  - b. Expected duration of trades.
  - d. Volatility of option value.
- Numbers are in multiple of $\sigma_f/(r + \lambda)$.
Figure I
Some Results

Increases in Trading Costs

- As $c$ increases, the trading barrier $k^*$ increases as owners want to be compensated for the higher cost of sale.
- Expected duration between trades increases.
- However, there is an increase in profits for each trade.
- Therefore, the level of the bubble and volatility do not fall by much.
- Implies that a Tobin tax (tax levied on speculative trading) will not have a considerable effect on lowering the bubble and reducing volatility.
- Bubbles can exist in markets with high transaction costs.
Conclusion

- Developed a model with price bubbles due to belief heterogeneity and short sale constraints.
- Bubbles arises due to the right of the owner to sell the asset in the future to more optimistic agents.
- Characterized the response of the bubble, prices, asset volatility and turnover to changes in confidence and trading costs.
- Showed that bubbles can persist in markets with high transaction costs.
- A Tobin tax reduces trading volume but does not affect the bubble and price volatility considerably.
- Limitations of the model:
  1. No differences or shifts in risk-aversion of agents.
  2. No limits on agents wealth.