Overconfidence and Speculative Bubbles

by Jose A. Scheinkman and Wei Xiong

Sargent Reading Group

Presenter: Florian Semani

Motivation

Specific episodes in the US and abroad where market prices for some classes of assets display significant correlation between:

1. High prices.
2. High trading volume.
3. High price volatility.

Such historical examples include the 1929 boom and crash, the dot-com bubble and the recent housing crisis.

Motivation

The historical episodes above are referred to as price bubbles.

An asset price has a **bubble** component if the price of the asset is different from it's **fundamental value**.

In this paper there is a bubble in a risky asset:

- Agents disagree about the probability distribution of the dividend streams.
- They go through waves of relative optimism and pessimism with optimists holding the assets.

Motivation

- Such fluctuations will generate trade.
- Price of the asset includes the option to sell to more optimistic agents in the future.
- This drives the asset price above the agents own valuation of the fundamentals (subjective definition of a bubble).
- Short sale constraints on the asset are important to generate such bubbles.

Use the framework to do some comparative statics:

- On the effects of increases in investor confidence.
- On the effectiveness of a Tobin tax on transactions.
- On possibility of bubbles occurring in markets with high transaction costs (i.e. housing)

The Model

Assets

- A single risky asset with cumulative dividend process D_t:

$$dD_t = f_t dt + \sigma_D dZ_t^D$$

- The fundamental variable f_t is not observable but follows an Ornstein-Uhlenbeck (OU) process:

$$df_t = -\lambda(f_t - \bar{f}) + \sigma_f dZ_t^f$$

- The discrete time equivalent of the process above is a stationary AR1 process.
- The stationary probability distribution of OU is Gaussian with mean $\bar{f}$ and variance $\frac{\sigma_f^2}{2\lambda}$.
- No short sales allowed.
- Asset is in finite supply.

The Model

Agents:

- Two sets of risk-neutral agents A and B.
- For convenience, if O refers to one group, then the other group is $\bar{O}$
- They have deep pockets (infinite wealth).
- Same discount factor r.

Information Structure

- Agents in both groups observe signals s^A and s^B where for O $\in \{A, B\}$:

$$ds_t^O = f_t dt + \sigma_s dz_t^O$$

- Z^D, Z^f, Z^A and Z^B are mutually independent Brownian Motions.
- Agent O believes that his signal is more informative.

The Model

- That is agent O believes (incorrectly) that innovations to s^O are correlated with innovations to f_t.

$$ds_t^O = f_t dt + \sigma_s(\phi dZ_t^f + \sqrt{1-\phi^2} dZ_t^O)$$

- However, he believes (correctly) that innovations to the other agent's signal are uncorrelated with innovations to f_t.
- ϕ represents the degree of over-confidence of an agent in his own signal s^o.
- They agree to disagree.

Evolution of Beliefs

- Agents cannot infer f_t perfectly due to dividend noise.
- They have to use observations of D, s^A and s^B to form beliefs about f$_t$.

The Model

- Hence they face a filtering problem for f_t with Gaussian initial conditions.
- The discrete time equivalent of this problem is the Kalman filter.
- The only difference is that the prior and posterior variance are the same in continuous time, and they follow an ODE.
- Like in the discrete time counterpart can get a time invariant posterior variance regarding beliefs about f_t given by γ.
- γ decreases with ϕ.

The Model

Evolution of Beliefs

- The conditional mean of beliefs about f_t in A (denote it $\hat{f}_t^A$) evolves according to:

$$d\hat{f}^A = -\lambda(\hat{f}^A - \bar{f})dt + \frac{\phi\sigma_s\sigma_f + \gamma}{\sigma_s} dW_A^A + \frac{\gamma}{\sigma_s} dW_A^B + \frac{\gamma}{\sigma_D} dW_A^D$$

- dW_A^A, dW_A^B and dW_A^D are surprises from signals s^A, s^B and D modeled as innovations to standard mutually independent Brownian Motions.
- Since conditional variance of f_t is constant then refer to conditional means as **beliefs**.
- Denote $g^O = \hat{f}^{\bar{O}}$ - $\hat{f}^O$ the difference in the conditional mean of beliefs.
- $g^O > 0$ implies that group O is relatively more pessimistic.

The Model

Evolution of Beliefs

- The evolution of belief differences for group O is given by:

$$dg^O = -\rho g^O dt + \sigma_g dW_g^O$$

$$\rho = \sqrt{(\lambda + \phi\frac{\sigma_f}{\sigma_s})^2 + (1-\phi^2)\sigma_f^2(\frac{2}{\sigma_s^2} + \frac{1}{\sigma_D^2})}$$

$$\sigma_g = \sqrt{2}\phi\sigma_f$$

- g^A follows a mean-reverting process from A's perspective.
- Mean-reversion is measured by -$\rho/2\sigma_g^2$.
- A higher ϕ increases the divergence in opinion and slows mean reversion.
- This is crucial for the results that follow.

The Model

Trader's Problem

- Seller pays c $\geq$ 0 for a unit of asset sold.
- Let O denote the group of current owner of the asset.
- The price of the asset p_t^O is given by:

$$p_t^O = \max_{\tau \geq 0} \quad E_t^O \left[\int_t^{t+\tau} e^{-r(s-t)} dD_s + e^{-r\tau}(p_{t+\tau}^{\bar{O}} - c) \right]$$

- where τ is a stopping time, $p_{t+\tau}^{\bar{O}}$ is the buyer's reservation price at time $t + \tau$.
- Plugging for dD $= \hat{f}^O + \sigma_D dW_D^O$, using Ito's lemma to express $\hat{f}_t^O$ and cancelling the martingale term:

$$p_t^O = \max_{\tau \geq 0} E_t^O \left[\int_t^{t+\tau} e^{-r(s-t)} [\bar{f} + e^{-\lambda(s-t)}(\hat{f}_t^O - \bar{f})] ds + e^{-r\tau}(p_{t+\tau}^{\bar{O}} - c) \right.$$

The Model

Trader's Problem

- Conjecture an equilibrium price function:

$$p_t^O = p^O(\hat{f}_t^O, g_t^O) = \underbrace{\frac{\bar{f}}{r} + \frac{\hat{f}_t^O - \bar{f}}{r+\lambda}}_{Fundamental\ Valuation} + \underbrace{q(g_t^O)}_{Resale\ Option\ Value}$$

- Combining the last two equations above:

$$q(g_t^O) = \max_{\tau \geq 0} E_t^O \left\{ \left[\underbrace{\frac{g_{t+\tau}^O}{r+\lambda}}_{Buyer's\ Excess\ Optimism} + \underbrace{q(g_{t+\tau}^{\bar{O}})}_{Future\ Resale\ Value} - c \right] e^{-r\tau} \right\}$$

The Model

Equilibrium Price Solution

- Trade occurs at the region of x (stopping region SR) such that:

$$\underbrace{q(x)}_{Option\,Value} = \underbrace{\frac{x}{r+\lambda} + q(-x) - c}_{Value\,of\,Immediate\,Sale}$$

- The complement of the stopping region is the continuation region (CR).
- In CR $e^{-rt}q(g_t^0)$ follows a martingale, while in SR it follows a supermartingale.
- Using Ito's lemma to expand $e^{-rt}q(g_t^0)$, then q in CR must satisfy the ODE:

$$\frac{1}{2}\sigma_g^2 q^{''} - \rho x q^{'} - rq = 0$$

- In SR replace $=$ with $<$.

The Model

Equilibrium Price Solution

- To find such q guess that CR $= (-\infty, k^*)$.
- k* is the minimal amount of difference in beliefs that generates a trade.
- Can find a function h(x) that solves the ODE above in CR. Then:

$$q(x) = \begin{cases} \beta_1 h(x) & x < k^* \\ \frac{x}{r+\lambda} + \beta_1 h(-x) - c & x \geq k^* \end{cases}$$

$$\beta_1 = \frac{1}{(r+\lambda)[h'(k^*) + h'(-k^*)]}$$

- h and its first three derivatives are positive everywhere. Also, $\lim_{x \to -\infty} h(x) = 0$.

The Model

Equilibrium Price Solution

- Can show that q above is an equilibrium option value function.
- Optimal policy is given by:

$$\begin{cases} \textit{Immediate sale if } g^{O} > k^{*} \\ \\ \textit{Otherwise wait until first time when } g^{O} \geq k^{*} \end{cases}$$

- Define a bubble as the difference between the price one is willing to pay for an asset and the price if asset is kept forever and never sold.
- When $g^{O} = \mathrm{k}^{*}$ ownership switches to $\bar{O}$.
- Therefore the bubble at that time is given by:

$$b = p_t^{\bar{O}} - [\frac{\bar{f}}{r} + \frac{\hat{f}_t^{\bar{O}} - \bar{f}}{r + \lambda}] = q(-k^{*})$$

Some Results

Price Volatility

- Volatility of the option value is given by:

$$\eta(x) = \frac{\sqrt{2}\phi\sigma_f}{r+\lambda}\frac{h^{'}(x)}{h^{'}(k^*)+h^{'}(-k^*)} \quad \forall x < k^*$$

- This is increasing in x.
- The volatility of the fundamental valuation increases with ϕ.
- For $\lambda = 0$, the volatility of the fundamental valuation is given by $\frac{\sigma_f}{r}$

Some Results

Small Trading Costs

- k* depends continuously on cost c.
- if c= 0, then k* = 0. In this case trade occurs whenever beliefs cross.
- The expected duration between trades is zero.
- Therefore even at c=0 can sustain a bubble. Agents expect to make infinitesimal gains at very high trading frequencies that compound to a bubble.

$$b = \frac{1}{2(r+\lambda)} \frac{h(0)}{h'(0)}$$

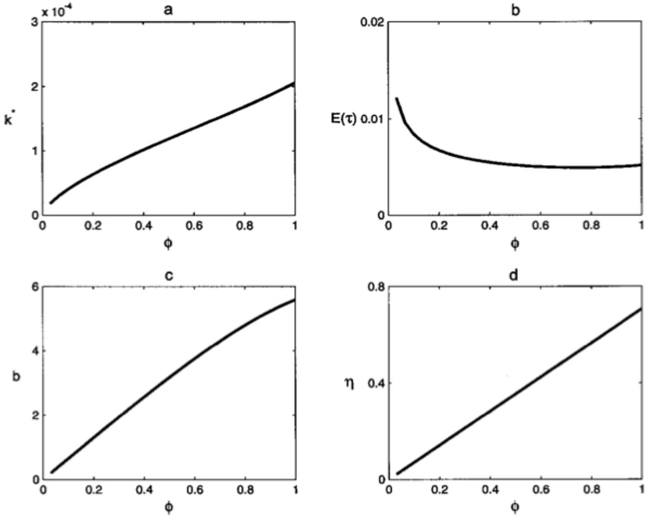
For small c, size of bubble depends positively on σ_g and ρ. The effect of a change in k^* is second order.

- As σ_g increases, beliefs oscillate more leading to faster trading opportunities.

Some Results

- As ρ increases, the resale option will be exercised quicker.
- Both factors increase in the size of the bubble.
- Understand the effects of an increase in investor confidence through the effect on σ_g and ρ.
- Figure 1 plots the effect of an increase in confidence on:
    a. The trading barrier k^*.
    b. Expected duration of trades.
    c. Size of bubble.
    d. Volatility of option value.
    - Numbers are in multiple of $\sigma_f/(r+\lambda)$.

Figure I

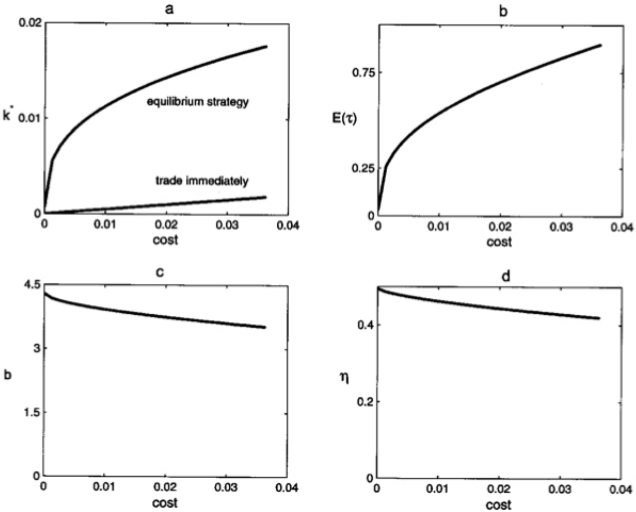


Some Results

Increases in Trading Costs

- As c increases, the trading barrier k^* increases as owners want to be compensated for the higher cost of sale.
- Expected duration between trades increases.
- However, there is an increase in profits for each trade.
- Therefore, the level of the bubble and volatility do not fall by much
- Implies that a Tobin tax (tax levied on speculative trading) will not have a considerable effect on lowering the bubble and reducing volatility.
- Bubbles can exist in markets with high transaction costs.

Figure 2



a
0.02
k*
0.01
0
equilibrium strategy
trade immediately
0
0.01
0.02
0.03
0.04
cost
b
E(τ)
0.75
0.25
0
0
0.01
0.02
0.03
0.04
cost
c
4.5
3
b
1.5
0
0
0.01
0.02
0.03
0.04
cost
d
0.4
η
0.2
0
0
0.01
0.02
0.03
0.04
cost

Conclusion

- Developed a model with price bubbles due to belief heterogeneity and short sale constraints.
- Bubbles arises due to the right of the owner to sell the asset in the future to more optimistic agents.
- Characterized the response of the bubble, prices, asset volatility and turnover to changes in confidence and trading costs.
- Showed that bubbles can persist in markets with high transaction costs.
- A Tobin tax reduces trading volume but does not affect the bubble and price volatility considerably.
- Limitations of the model:
    1. No differences or shifts in risk-aversion of agents.
    2. No limits on agents wealth.