"Optimal Fiscal Policy With Redistribution"

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by Iván Werning

Motivation

- In order to avoid the first-best outcome in the representative Ramsey Problem we rule out any form of lump sum taxes, or any tax policies that could generate them.
- This is unsatisfactory, as we seem to have created a seemingly arbitrary second best problem.
- A way to solve this is to heterogeneity among the workers, creating a tension between loss to distortion and gain from redistribution.
- The goal of this paper is to introduce skill heterogeneity to the Ramsey problem, while keeping tractability.

Groundwork

- A continuum of workers is devided into I types of relative size π^i and skill θ^i. Preferences of consumption and labor streams are given by the utility function

$$\sum_{t=0}^{\infty} \beta^t \mathbb{E}[U^i(c_t, L_t)]$$

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- An allocation is a stream of consumption, labor and capital $\{c^i(s^t), L^i(s^t), K(s^t)\}$ and we construct the following aggregates $c(s^t) = \sum_i c^i(s^t)\pi^i$ and $L(s^t) = \sum_i L^i(s^t)\pi^i$ giving us the resource constraint

$$c(s^t) + K(s^t) + g_t(s^t) \le F(L(s^t), K(s^{t-1}), s^t, t) + (1-\delta)K(s^{t-1})$$

The Households

- We will assume a linear tax rate $\tau(s^t)$ on labor income along with a lump sum tax $T(s^t)$ and a capital income tax $\kappa(s^t)$. When there is no heterogeneity, Lump Sum taxes are removed.

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- Under complete markets the households budget constraint is then

$$\sum_{t,s^t} p(s^t)[c^i(s^t) + k^i(s^t) - w(s^t)(1-\tau(s^t))L^i(s^t) \\ -R(s^t)k^i(s^{t-1})] \leq (1-\kappa_B(s_0))B^i(s_0) - T$$

where $R(s^t) = 1 + (1-\kappa(s^t))(r(s^t) - \delta)$ and
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- No arbitrage then gives us that $p(s^t) = \sum\nolimits_{s^{t+1}} R(s^{t+1})p(s^{t+1})$ which giving us

$$\begin{aligned}\sum\nolimits_{t,s^t} p(s^t)[c^i(s^t) - w(s^t)(1-\tau(s^t))L^i(s^t)]\\ \leq R(s_0)k^i(s_0) + (1-\kappa_B(s_0))B^i(s_0) - T\end{aligned}$$

Firms,Government

- We assume constant returns to scale, so that

$$r(s^t) = F_K(L(s^t), K(s^{t-1}), s^t, t) \quad w(s^t) = F_L(L(s^t), K(s^{t-1}), s^t, t)$$

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- The governments budget constraint is

$$\begin{aligned}&(1 - \kappa_B(s_0)) \textstyle\sum_i B^i(s^0)\pi^i + \sum_{t,s^t} p(s^t)g(s^t) \\ &\quad \leq T + \textstyle\sum_{t,s^t} p(s^t)[\tau(s^t)w(s^t)L(s^t) + \kappa(s^t)(r(s^t) - \delta)K(s^{t-1})]\end{aligned}$$

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- Finally we define a competitive equilibrium given initial capital and bond holdings as a sequence of taxes, prices and an allocation such that
  - (i) Households maximize utility subject to budget constraint
  - (ii) Firms FOC holds
  - (iii) The governments budget constraint holds.
  - (iv) Market's clear

Representative Agent

- We will approach this suing the primal approach and create a representative agent with utility function

$$U^m(c, L, \varphi) \equiv \max_{c^i, L^i} \sum_i \varphi^i U^i(c^i, L^i)\pi^i$$

$$\text{subject to} \sum_i c^i \pi^i = c \text{ and } \sum_i L^i \pi^i = L$$

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- We let $(c^i, L^i) = h^i(c, L; \varphi)$ be the solution to the is problem. The envelope condition then gives us that $U^m_c(c, L; \varphi) = \varphi^i U^i_c(h^i(c, L; \varphi)$ and similarly for U^m_L.

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- Thus, we can compute after-tax prices as
$$w(s^t)(1 - \tau(s^t) = \frac{-U^m_L(c(s^t), L(s^t); \varphi)}{U^m_c(c(s^t), L(s^t); \varphi)}$$
$$\frac{p(s^t)}{p(s_0)} = \beta^t \frac{U^m_c(c(s^t), L(s^t); \varphi)}{U^m_c(c(s_0), L(s_0); \varphi)} \Pr(s^t)$$

Characterizing the Equilibria

- We can substitute those prices into the Households budget constraint (in equilibria) to obtain the impementability conditions

$$\sum_{t,s^t}\beta^t[U_c^m(c(s^t),L(s^t);\varphi)h^{i,c}(c(s^t),L(s^t);\varphi)$$

$$+U_L^m(c(s^t),L(s^t);\varphi)h^{i,L}(c(s^t),L(s^t);\varphi)]\Pr(s^t)$$

$$=U_c^m(c(s_0),L(s_0);\varphi)(R_0k_0^i+(1-\kappa_B(s_0))B^i(s_0)-T)$$

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Proposition

Given initial wealth $\{R_0k_0^i+(1-\kappa_B(s_0)B_0^i\}$ *an aggregate allocation* $\{c(s^t),L(s^t),K(s^t)\}$ *can be supported by a competitive equilibrium if and only if the resource constraints hold and there exists market weights and a lump sum tax* T *so that the implementability conditions hold for all* i*.*

The Planner's Problem

- Letting λ^i be the Pareto weights on each worker type, our governments and $\mu^i \pi^i$ the multipliers on the implementability conditions, the Lagrangian becomes

$$\sum_{t,s^t} \beta^t W(c(s^t), L(s^t); \varphi, \mu, \lambda) \Pr(s^t)$$

$$-U_c^m(c(s_0), L(s_0); \varphi) \sum_i \mu^i (R_0 k_0^i + (1 - \kappa_B(s_0))B^i(s_0) - T)\pi^i$$

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- We define the pseudo utility function as

$$W(c, L; \varphi, \mu, \lambda) \equiv \sum_i \pi^i [\lambda^i U^i(h^i(c, L; \varphi))$$

$$+\mu^i (U_c^m(c, L; \varphi) h^{i,c}(c, L; \varphi) + U_L^m(c, L; \varphi) h^{i,L}(c, L; \varphi))]$$

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- We will maximize this Lagrangian subject to the resource constraint.

The First Order Conditions

- For $t \geq 1$ the FOC are

$$F_L = \frac{-W_L}{W_c} \text{ and } W_c(s^t) = \beta \sum_{s_{t+1}} W_c(s^{t+1}) R^*(s^{t+1}) \Pr(s_{t+1}|s^t)$$

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- The FOC w.r.t T gives us $\sum_i \mu_i \pi^i = 0$, while our time 0 FOC of R_0 and $\kappa_B(s_0)$ give us

$$\sum_i \mu^i k_0^i \pi^i = 0 \text{ or } R_0 = 0$$

$$\sum_i \mu^i B^i(s_0) \pi^i = 0 \text{ or } \kappa_B(s_0) = 1.$$

Thus, the FOC above will hold for $t = 0$

Optimal Tax Rates

- Using our FOC, the firms FOC, and our implicit price formula we can derive the tax policy

$$\tau^*(c, L; \varphi, \mu, \lambda) \equiv 1 - \frac{U_L^m}{W_L}\frac{W_c}{U_c^m}$$

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- We can also rewrite our no arbitrage condition as

$$U_c^m(s^t) = \beta \sum_{s_{t+1}} U_c^m(s^{t+1})R(s^{t+1})\Pr(s_{t+1}|s^t)$$

In general there are many ways $R(s^{t+1})$ which make no-arbitrage and our FOC compatible, will choose

$$R(s^{t+1}) = R^*(s^{t+1})\frac{U_c^m(s^t)}{W_c(s^t)}\frac{W_c(s^{t+1})}{U_c^m(s^{t+1})}$$

Separable and Isoelastic Preferences

- We begin by assuming that

$$U(c, L) = u(c) - v(L) = \frac{c^{1-\sigma}}{1-\sigma} - \alpha\frac{L^{\gamma}}{\gamma}$$

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$$U^m = \Phi_u^m u(c) - \Phi_v^m v(L) \text{ and } W = \Phi_u^W u(c) - \Phi_v^W v(L)$$

where the Φ's depend on φ, μ, and λ

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- This gives us that

$$\tau^*(c,L) = \bar{\tau} \equiv 1 - \frac{\Phi_v^m}{\Phi_u^m}\frac{\Phi_u^W}{\Phi_v^W}$$

Note that $R = R^*$ and thus the tax on capital will be set to 0.

Balanced Growth Path Preferences

- We let $U(c,n) \equiv \frac{1}{1-\sigma}(c^{\alpha}(1-n)^{1-\alpha})^{1-\sigma}$. Under these preferences, it is possible to show that U^m is proportional to U and that

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- From this we can derive that

$$\tau^*(L) = \frac{1}{(1-L)\Phi_U^W/\Phi_{U_L}^W + \sigma(1-\alpha)+\alpha}$$

and that

$$\frac{R(s^{t+1})}{R^*(s^{t+1})} = \frac{1-L(s^t)}{1-L(s^{t+1})}\frac{\tau^*(L(s^{t+1}))^{-1}-1}{\tau^*(L(s^t))^{-1}-1}$$

Ramsey Equivalence

- We note that in both of these cases we could show that W is proportional to

$$U^m(c, L) + \hat{\mu}(U_c^m(c, L)c + U_L^m(c, L)L)$$

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- We also note that taxes were a function of the allocation, indexed by this parameter $\hat{\mu}$
- This gives us the equivalence: For any heterogeneous economy with separable isoelastic or balanced growth path preferences there exists a representative agent economy with preferences given by U^m with same structure of uncertainty and technology, such that for some initial debt level we have the same tax policy and FOC's

Shocks to The Distributions of Skills

- We will consider the separable isoelastic case with $\theta_t(s_t)$ this one can show

$$U^m = \Phi_u^m u(c) - \Phi_{v,t}^m(s_t)v(L) \text{ and } W = \Phi_u^W u(c) - \Phi_{v,t}^W(s_t)v(L)$$

where the variation comes solely from $\theta_t^i(s_t)$.

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- Thus we obtain that

$$\tau(s^t) = \bar{\tau}_t(s_t) = 1 - \frac{\Phi_{v,t}^m(s_t)}{\Phi_u^m}\frac{\Phi_u^W}{\Phi_{v,t}^W(s_t)}$$

  and $R^* = R$ so a zero capital tax is optimal.

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- Consider an example with $\sigma = 1$, $\delta = 1$, Cobb-Douglass production and two types of workers with $\theta^H(s_t) \geq 1 \geq \theta^L(s_t) = 2 - \theta^H(s_t)$, such that inequality (θ^H/θ^L) varies from 1 to around 4.8. We look at the Utilitarian case

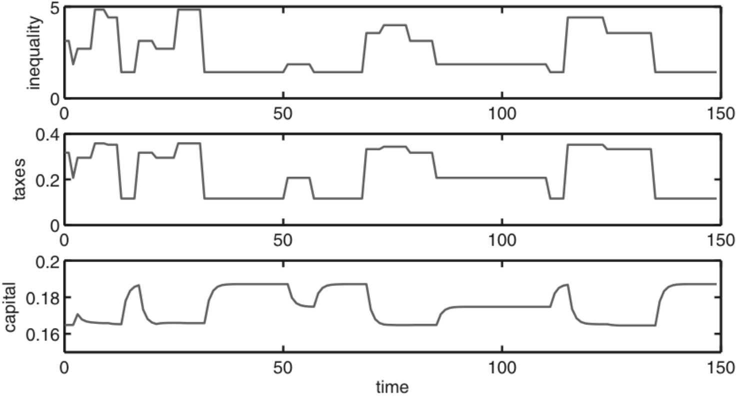

FIGURE II
Simulated Sample Path for Inequality, Taxes, and Capital

Comparisons to Ramsey

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- Time inconsistency arises when more productive workers accumulate more assets over time, not from using capital in time 0 as a lump sum tax.
- Ricardian Equivalence (solution to tax problem no longer determines debt policy) is recovered because we now have lump sum taxes.

The Planning Problem

- Let U be separable and v be isoelastic then the planners problem is to maximize

$$\sum_{t,s^t,i} \beta^t \lambda^i \left(u(c^i(s^t)) - v\left(\frac{L^i(s^t)}{\theta^i_t(s_t)}\right)\right) \Pr(s^t)\pi^i$$

subject to the incentive compatibility constraint

$$\sum_{t,s^t} \beta^t \left(u(c^i(s^t)) - v\left(\frac{L^i(s^t)}{\theta^i_t(s_t)}\right)\right) \Pr(s^t) \geq \sum_{t,s^t} \beta^t \left(u(c^j(s^t)) - v\left(\frac{L^j(s^t)}{\theta^i_t(s_t)}\right)\right) \Pr(s^t)$$

- Letting $\psi^{i,j}\pi^i$ be multipliers and exploiting $v(L/\theta^j) = (\theta^i/\theta^j)^\gamma v(L/\theta^i)$ one can rewrite the Lagrangian as

$$\sum_{i,t,s^t} \beta^t \left(\phi^i_c u(c^i(s^t)) - \phi^i_{L,t}(s_t) v\left(\frac{L^i(s^t)}{\theta^i_t(s_t)}\right)\right) \Pr(s^t)\pi^i$$

where the uncertainty of $\phi^i_{L,t}(s_t)$ only comes from $\theta^j_t(s_t)$.

The Tax Rates

- From the first order conditions maximizing the Lagrangian subject to the resource constraint one obtains

$$u'(c^i(s^t) = \beta \sum_{s_{t+1}} u'(c^i(s^t, s_{t+1})R^*(s^t, s_{t+1}) \Pr(s_{t+1}|s^t)$$

and

$$\frac{1}{\theta_t^i(s_t)} \frac{v'(L^i(s^t)/\theta_t^i(s_t))}{u'(c^i(s^t))} = \frac{\phi_c^i}{\phi_{L,t}^i(s_t)} F_L(L(s^t), K(s^{t-1}), s^t, t)$$

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- If we define the implicit marginal tax by

$$\frac{1}{\theta_t^i(s_t)} \frac{v'(L^i(s^t)/\theta_t^i(s_t))}{u'(c^i(s^t))} = (1 - \tau^i(s^t)) F_L(L(s^t), K(s^{t-1}), s^t, t)$$

the we obtain

$$\tau^i(s^t) = 1 - \frac{\phi_c^i}{\phi_{L,t}^i(s_t)}$$

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- If we modify the consumers budget constraint to be

$$\sum_{t,s^t} p(s^t)c^i(s^t) \leq \sum_{t,s^t} p(s^t)L^i(s^t) - \Psi\left(\sum_{t,s^t} p(s^t)w(s^t)L^i(s^t)\right)$$

then the first order condition yields.

$$\frac{1}{\theta^i}\frac{v'(L^i(s^t)/\theta^i)}{u'(c^i(s^t))} = \left(1 - \Psi'\left(\sum_{t,s^t} p(s^t)w(s^t)L^i(s^t)\right)\right)w(s^t)$$

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- It is possible to show that when $\theta^i_t(s^t) = \theta^i$ then the constrained-efficient allocation can be implemented by a competitive equilibrium with no tax on capital and a non linear tax on the present value of labor income