“Optimal Fiscal Policy With Redistribution”
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by Iván Werning
Motivation

• In order to avoid the first-best outcome in the representative Ramsey Problem we rule out any form of lump sum taxes, or any tax policies that could generate them.

• This is unsatisfactory, as we seem to have created a seemingly arbitrary second best problem.

• A way to solve this is to heterogeneity among the workers, creating a tension between loss to distortion and gain from redistribution.

• The goal of this paper is to introduce skill heterogeneity to the Ramsey problem, while keeping tractability.
Groundwork

- A continuum of workers is divided into \( I \) types of relative size \( \pi^i \) and skill \( \theta^i \). Preferences of consumption and labor streams are given by the utility function

\[
\sum_{t=0}^{\infty} \beta^t \mathbb{E}[U^i(c_t, L_t)]
\]

where \( U^i(c, L) = U(c, L/\theta^i) \).

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- An allocation is a stream of consumption, labor and capital \( \{c^i(s^t), L^i(s^t), K(s^t)\} \) and we construct the following aggregates \( c(s^t) = \sum_i c^i(s^t)\pi^i \) and \( L(s^t) = \sum_i L^i(s^t)\pi^i \) giving us the resource constraint

\[
c(s^t) + K(s^t) + g_t(s^t) \leq F(L(s^t), K(s^{t-1}), s^t, t) + (1-\delta)K(s^{t-1})
\]
The Households

- We will assume a linear tax rate $\tau(s^t)$ on labor income along with a lump sum tax $T(s^t)$ and a capital income tax $\kappa(s^t)$. When there is no heterogeneity, Lump Sum taxes are removed.
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- Under complete markets the households budget constraint is then:

$$\sum_{t,s^t} p(s^t)[c^i(s^t) + k^i(s^t) - w(s^t)(1 - \tau(s^t))L^i(s^t) - R(s^t)k^i(s^{t-1})] \leq (1 - \kappa_B(s_0))B^i(s_0) - T$$

where $R(s^t) = 1 + (1 - \kappa(s^t))(r(s^t) - \delta)$ and $T = \sum_{t,s^t} p(s^t)T(s^t)$
The Households

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where $R(s^t) = 1 + (1 - \kappa(s^t))(r(s^t) - \delta)$ and $T = \sum_{t,s^t} p(s^t)T(s^t)$

- No arbitrage then gives us that $p(s^t) = \sum_{s^{t+1}} R(s^{t+1})p(s^{t+1})$ which giving us

$$
\sum_{t,s^t} p(s^t)[c^i(s^t) - w(s^t)(1 - \tau(s^t))L^i(s^t)] \\
\leq R(s_0)k^i(s_0) + (1 - \kappa_B(s_0))B^i(s_0) - T
$$
Firms, Government

- We assume constant returns to scale, so that

\[ r(s^t) = F_K(L(s^t), K(s^{t-1}), s^t, t) \quad w(s^t) = F_L(L(s^t), K(s^{t-1}), s^t, t) \]
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- The government's budget constraint is
  \[
  (1 - \kappa B(s_0)) \sum_i B_i(s^0) \pi_i + \sum_{t,s^t} p(s^t) g(s^t) \\
  \leq T + \sum_{t,s^t} p(s^t)[\tau(s^t) w(s^t) L(s^t) + \kappa(s^t)(r(s^t) - \delta) K(s^{t-1})]
  \]
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  \]

- Finally we define a competitive equilibrium given initial capital and bond holdings as a sequence of taxes, prices and an allocation such that
  1. Households maximize utility subject to budget constraint
  2. Firms FOC holds
  3. The governments budget constraint holds.
  4. Market’s clear
Representative Agent

- We will approach this suing the primal approach and create a representative agent with utility function

\[ U^m(c, L, \varphi) \equiv \max_{c^i, L^i} \sum_i \varphi^i U^i(c^i, L^i)\pi^i \]

subject to \( \sum_i c^i\pi^i = c \) and \( \sum_i L^i\pi^i = L \)

for some market weights \( \varphi \equiv \{\varphi^i\} \)
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- We let \( (c^i, L^i) = h^i(c, L; \varphi) \) be the solution to the is problem. The envelope condition then gives us that

\[ U^m_c(c, L; \varphi) = \varphi^i U^i_c(h^i(c, L; \varphi)) \]

and similarly for \( U^m_L \).

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\[ U^m_c(c, L; \varphi) = \varphi^i U^i_c(h^i(c, L; \varphi)) \] and similarly for \( U^m_L \).

• Thus, we can compute after-tax prices as

\[ w(s^t)(1 - \tau(s^t)) = -\frac{U^m_L(c(s^t), L(s^t); \varphi)}{U^m_c(c(s^t), L(s^t); \varphi)} \]

\[ \frac{p(s^t)}{p(s_0)} = \frac{\beta_t U^m_c(c(s^t), L(s^t); \varphi)}{U^m_c(c(s_0), L(s_0); \varphi)} \Pr(s^t) \]
Characterizing the Equilibria

- We can substitute those prices into the Households budget constraint (in equilibria) to obtain the implementability conditions

\[
\sum_{t,s} \beta^t \left[ U_m^c(c(s^t), L(s^t); \varphi)h^i,c(c(s^t), L(s^t); \varphi) + U_m^l(c(s^t), L(s^t); \varphi)h^i,l(c(s^t), L(s^t); \varphi) \right] \Pr(s^t)
\]

\[
= U_m^c(c(s_0), L(s_0); \varphi)(R_0k_i^0 + (1 - \kappa_B(s_0))B^i(s_0) - T)
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\[
\sum_{t,s^t} \beta^t [U^m_c(c(s^t), L(s^t); \varphi) h^i, c(c(s^t), L(s^t); \varphi)
\]

\[
+ U^m_L(c(s^t), L(s^t); \varphi) h^i, L(c(s^t), L(s^t); \varphi)] Pr(s^t)
\]

\[
= U^m_c(c(s^0), L(s^0); \varphi)(R^i_0 k^i_0 + (1 - \kappa_B(s_0)) B^i(s_0) - T)
\]

Proposition

Given initial wealth \( \{R^i_0 k^i_0 + (1 - \kappa_B(s_0)) B^i_0\} \) an aggregate allocation \( \{c(s^t), L(s^t), K(s^t)\} \) can be supported by a competitive equilibrium if and only if the resource constraints hold and there exists market weights and a lump sum tax \( T \) so that the implementability conditions hold for all \( i \).
The Planner’s Problem

- Letting $\lambda^i$ be the Pareto weights on each worker type, our governments and $\mu^i \pi^i$ the multipliers on the implementability conditions, the Lagrangian becomes

$$\sum_{t,s} \beta^t W(c(s^t), L(s^t); \varphi, \mu, \lambda) \Pr(s^t)$$

$$- U^m_c(c(s_0), L(s_0); \varphi) \sum_i \mu^i (R_0 k^i + (1 - \kappa_B(s_0)) B^i(s_0) - T) \pi^i$$
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• We define the pseudo utility function as

$$W(c, L; \varphi, \mu, \lambda) \equiv \sum_i \pi^i [\lambda^i U^i(h^i(c, L; \varphi))$$

$$+ \mu^i (U^m_c(c, L; \varphi) h^{i,c}(c, L; \varphi) + U^m_L(c, L; \varphi) h^{i,L}(c, L; \varphi))]$$
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- We will maximize this Lagrangian subject to the resource constraint.
The First Order Conditions

- For $t \geq 1$ the FOC are

$$F_L = \frac{-W_L}{W_C} \quad \text{and} \quad W_C(s^t) = \beta \sum_{s_{t+1}} W_C(s^{t+1}) R^*(s^{t+1}) \Pr(s_{t+1}|s^t)$$

where $R^*(s^t) = F_K(s^t) + 1 - \delta$ is the social return to capital.
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where $R^*(s^t) = F_K(s^t) + 1 - \delta$ is the social return to capital.

- The FOC w.r.t $T$ gives us $\sum_i \mu_i \pi^i = 0$, while our time 0 FOC of $R_0$ and $\kappa_B(s_0)$ give us

$$\sum_i \mu^i k_0^i \pi^i = 0 \quad \text{or} \quad R_0 = 0$$

$$\sum_i \mu^i B_i(s_0) \pi^i = 0 \quad \text{or} \quad \kappa_B(s_0) = 1.$$  

Thus, the FOC above will hold for $t = 0$
Optimal Tax Rates

- Using our FOC, the firms FOC, and our implicit price formula we can derive the tax policy

\[ \tau^*(c, L; \varphi, \mu, \lambda) \equiv 1 - \frac{U^m}{W_L} \frac{W_c}{U^m} \]
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\[ \tau^*(c, L; \varphi, \mu, \lambda) \equiv 1 - \frac{U^m_L W_c}{W_L U^m_c} \]

- We can also rewrite our no arbitrage condition as

\[ U^m_c(s^t) = \beta \sum_{s^{t+1}} U^m_c(s^{t+1}) R(s^{t+1}) \Pr(s_{t+1}|s^t) \]

In general there are many ways \( R(s^{t+1}) \) which make no-arbitrage and our FOC compatible, will choose

\[ R(s^{t+1}) = R^*(s^{t+1}) \frac{U^m_c(s^t) W_c(s^{t+1})}{W_c(s^t) U^m_c(s^{t+1})} \]
Separable and Isoelastic Preferences

We begin by assuming that

\[ U(c, L) = u(c) - v(L) = \frac{c^{1-\sigma}}{1 - \sigma} - \alpha \frac{L^\gamma}{\gamma} \]
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- Under these preferences it is possible to derive that

\[ U^m = \Phi^m_u u(c) - \Phi^m_v v(L) \text{ and } W = \Phi^W_u u(c) - \Phi^W_v v(L) \]

where the \(\Phi\)'s depend on \(\varphi\), \(\mu\), and \(\lambda\).
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where the \( \Phi \)'s depend on \( \varphi, \mu, \) and \( \lambda \)

- This gives us that

\[ \tau^*(c, L) = \bar{\tau} \equiv 1 - \frac{\Phi^m_v \Phi^W_u}{\Phi^m_u \Phi^W_v} \]

Note that \( R = R^* \) and thus the tax on capital will be set to 0.
Balanced Growth Path Preferences

- We let $U(c, n) = \frac{1}{1-\sigma} (c^\alpha (1-n)^{1-\alpha})^{1-\sigma}$. Under these preferences, it is possible to show that $U^m$ is proportional to $U$ and that

$$W(c, L) = \Phi_U^W U(c, L) + \Phi_{UL}^W U_L(c, L)$$

where the $\Phi$’s are determined by $\varphi$ and $\mu$.
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where the $\Phi$’s are determined by $\varphi$ and $\mu$.

- From this we can derive that

$$\tau^*(L) = \frac{1}{(1 - L)\Phi_U^W / \Phi_U^L + \sigma(1 - \alpha) + \alpha}$$

and that

$$\frac{R(s^{t+1})}{R^*(s^{t+1})} = \frac{1 - L(s^t)}{1 - L(s^{t+1})} \frac{\tau^*(L(s^{t+1}))^{-1} - 1}{\tau^*(L(s^t))^{-1} - 1}$$
Ramsey Equivalence

- We note that in both of these cases we could show that $W$ is proportional to

$$U^m(c, L) + \hat{\mu}(U^m_c(c, L)c + U^m_L(c, L)L)$$

where $\hat{\mu}$ summarized the models primitives.
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- We also note that taxes were a function of the allocation, indexed by this parameter $\hat{\mu}$

- This gives us the equivalence: For any heterogeneous economy with separable isoelastic or balanced growth path preferences there exists a representative agent economy with preferences given by $U^m$ with same structure of uncertainty and technology, such that for some initial debt level we have the same tax policy and FOC’s
Shocks to The Distributions of Skills

- We will consider the separable isoelastic case with $\theta_t(s_t)$ this one can show

$$U^m = \Phi^m_u u(c) - \Phi^m_{v,t}(s_t) v(L) \quad \text{and} \quad W = \Phi^W_u u(c) - \Phi^W_{v,t}(s_t) v(L)$$

where the variation comes solely from $\theta^i_t(s_t)$. 
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- Thus we obtain that

$$\tau(s^t) = \bar{\tau}_t(s_t) = 1 - \frac{\Phi^m_{v,t}(s_t)}{\Phi^m_u} \frac{\Phi^W_{v,t}(s_t)}{\Phi^W_u}$$

and $R^* = R$ so a zero capital tax is optimal.
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where the variation comes solely from $\theta^i_t(s_t)$.

- Thus we obtain that

$$\tau(s^t) = \tilde{\tau}_t(s_t) = 1 - \frac{\Phi^m_{v,t}(s_t)}{\Phi^m_u} \frac{\Phi^W_u}{\Phi^W_{v,t}(s_t)}$$

and $R^* = R$ so a zero capital tax is optimal.

- Consider an example with $\sigma = 1$, $\delta = 1$, Cobb-Douglass production and two types of workers with $\theta^H(s_t) \geq 1 \geq \theta^L(s_t) = 2 - \theta^H(s_t)$, such that inequality $(\theta^H/\theta^L)$ varies from 1 to around 4.8. We look at the Utilitarian case
Figure II
Simulated Sample Path for Inequality, Taxes, and Capital
Comparisons to Ramsey

- Time inconsistency arises when more productive workers accumulate more assets over time, not from using capital in time 0 as a lump sum tax.
Comparisons to Ramsey

- Time inconsistency arises when more productive workers accumulate more assets over time, not from using capital in time 0 as a lump sum tax.
- Ricardian Equivalence (solution to tax problem no longer determines debt policy) is recovered because we now have lump sum taxes.
The Planning Problem

• Let $U$ be separable and $v$ be isoelastic then the planners problem is to maximize

$$
\sum_{t,s^t,i} \beta^t \lambda^i \left( u(c^i(s^t)) - v \left( \frac{L^i(s^t)}{\theta^i_t(s^t)} \right) \right) \Pr(s^t) \pi^i
$$

subject to the incentive compatibility constraint

$$
\sum_{t,s^t} \beta^t \left( u(c^i(s^t)) - v \left( \frac{L^i(s^t)}{\theta^i_t(s^t)} \right) \right) \Pr(s^t) \\
\geq \sum_{t,s^t} \beta^t \left( u(c^j(s^t)) - v \left( \frac{L^j(s^t)}{\theta^j_t(s^t)} \right) \right) \Pr(s^t)
$$

• Letting $\psi^i,j \pi^i$ be multipliers and exploiting $v(L/\theta^i) = (\theta^i/\theta^j)^\gamma v(L/\theta^i)$ one can rewrite the Lagrangian as

$$
\sum_{i,t,s^t} \beta^t \left( \phi^i_c u(c^i(s^t)) - \phi^i_{L,t}(s^t)v \left( \frac{L^i(s^t)}{\theta^i_t(s^t)} \right) \right) \Pr(s^t) \pi^i
$$

where the uncertainty of $\phi^i_{L,t}(s^t)$ only comes from $\theta^i_t(s^t)$.  

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The Tax Rates

- From the first order conditions maximizing the Lagrangian subject to the resource constraint one obtains

\[ u'(c^i(s^t)) = \beta \sum_{s_{t+1}} u'(c^i(s^t, s_{t+1}) R^*(s^t, s_{t+1}) \Pr(s_{t+1}|s^t) \]

and

\[ \frac{1}{\theta^i_t(s_t)} \frac{v'(L^i(s^t)/\theta^i_t(s_t))}{u'(c^i(s^t))} = \frac{\phi^i_c}{\phi^i_{L,t}(s_t)} F_L(L(s^t), K(s^{t-1}), s^t, t) \]
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and

\[ \frac{1}{\theta^i_t(s^t)} \frac{v'(L^i(s^t)/\theta^i_t(s^t))}{u'(c^i(s^t))} = \frac{\phi^i_c}{\phi^i_{L,t}(s^t)} F_L(L(s^t), K(s^{t-1}), s^t, t) \]

- If we define the implicit marginal tax by

\[ \frac{1}{\theta^i_t(s^t)} \frac{v'(L^i(s^t)/\theta^i_t(s^t))}{u'(c^i(s^t))} = (1 - \tau^i(s^t)) F_L(L(s^t), K(s^{t-1}), s^t, t) \]

the we obtain

\[ \tau^i(s^t) = 1 - \frac{\phi^i_c}{\phi^i_{L,t}(s^t)} \]
Decentralization

• The question remains can one implement this with a tax policy.
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If we modify the consumers budget constraint to be

\[
\sum_{t,s^t} p(s^t)c^i(s^t) \leq \sum_{t,s^t} p(s^t)L^i(s^t) - \psi \left( \sum_{t,s^t} p(s^t)w(s^t)L^i(s^t) \right)
\]

then the first order condition yields.

\[
\frac{1}{\theta^i} \frac{v'(L^i(s^t)/\theta^i)}{u'(c^i(s^t))} = \left( 1 - \psi' \left( \sum_{t,s^t} p(s^t)w(s^t)L^i(s^t) \right) \right) w(s^t)
\]
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- If we modify the consumers budget constraint to be

$$
\sum_{t,s^t} p(s^t)c^i(s^t) \leq \sum_{t,s^t} p(s^t)L^i(s^t) - \Psi \left( \sum_{t,s^t} p(s^t)w(s^t)L^i(s^t) \right)
$$

then the first order condition yields.

$$
\frac{1}{\theta^i} \frac{v'(L^i(s^t)/\theta^i)}{u'(c^i(s^t))} = \left( 1 - \Psi' \left( \sum_{t,s^t} p(s^t)w(s^t)L^i(s^t) \right) \right) w(s^t)
$$

- It is possible to show that when $\theta^i_t(s^t) = \theta^i$ then the constrained-efficient allocation can be implemented by a competitive equilibrium with no tax on capital and a non-linear tax on the present value of labor income.