"Redistribution, taxes, and the median voter"

Review of Economic Dynamics 2006

by Marco Bassetto and Jess Benhabib

Motivation

- Ideally we would wish to study a model in which households, with heterogeneous wealth distributions, choose a sequence of redistributive tax rates through voting.
- The problem arises that the agents are voting over a sequence of tax rates, and thus the space we are dealing with is infinite dimensional.
- The infinite dimensional space means that the usual single-peakedness assumption for the median voter theorem cannot be used.

Aim of The Paper

- This paper solves the problem proving that when preferences are identical and Gorman aggregable a Condorcet winner exists.
- Furthermore they show that under theses assumptions the policy chosen will have the “bang-bang” property, i.e. capital income taxes remain at the upper bound until they drop to 0.
- This condenses a multi-dimensional problem into just a single choice of an optimal stopping time

Assumptions

- Output y_t is produced by competitive firms with a linearly homogeneous production function

$$y_t = F(k_t, l_t) \text{ where capital depreciates fully}$$

Assumptions

- Output y_t is produced by competitive firms with a linearly homogeneous production function

$$y_t = F(k_t, l_t) \text{ where capital depreciates fully}$$

- We have a continuum of agents, indexed by i. In the initial period they each own a wealth level W_0^i.

Assumptions

- Output y_t is produced by competitive firms with a linearly homogeneous production function

$$y_t = F(k_t, l_t) \text{ where capital depreciates fully}$$

- We have a continuum of agents, indexed by i. In the initial period they each own a wealth level W_0^i.
- In each period the government levies non-negative proportional taxes on labor income ν_t and capital income τ_t (which is subject to upper bound $\bar{\tau}$). It uses this income and one period debt to pay off an exogenous sequence $\{g_t\}$ and to finance lump sum transfer T_t,giving us the budget constraint

$$\tau_t r_t(k_t + b_t) + \nu_t w_t + b_{t+1} = r_t b_t + g_t + T_t$$

The Household

- We assume that the household's preferences over the consumption stream $\{c_t^i\}$ is given by $\sum_{t=0}^{\infty} \beta^t u(c_t^i)$, where

$$u(c) = \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c + B\right)^{1-\sigma}$$

The Household

- We assume that the household's preferences over the consumption stream $\{c_t^i\}$ is given by $\sum_{t=0}^{\infty} \beta^t u(c_t^i)$, where

$$u(c) = \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c + B\right)^{1-\sigma}$$

- The household supplies labor inelastically and maximizes its utility subject to the period-by-period budget constraint

$$(1-\tau_t)r_t W_t^i + (1-\nu_t)w_t + T_t \geq c_t^i + W_{t+1}^i$$

(under no arbitrage) and imposing no ponzi-schemes.

The Household

- We assume that the household's preferences over the consumption stream $\{c_t^i\}$ is given by $\sum_{t=0}^{\infty} \beta^t u(c_t^i)$, where

$$u(c) = \frac{\sigma}{1-\sigma}\left(\frac{A}{\sigma}c + B\right)^{1-\sigma}$$

- The household supplies labor inelastically and maximizes its utility subject to the period-by-period budget constraint

$$(1-\tau_t)r_t W_t^i + (1-\nu_t)w_t + T_t \geq c_t^i + W_{t+1}^i$$

(under no arbitrage) and imposing no ponzi-schemes.
- We can thus write the budget constraint in present value form

$$\sum_{t=0}^{\infty} \beta^t c_t^i q_t(1+\theta_t) = W_0^i + \sum_{t=0}^{\infty}(T_t + w_t(1-\nu_t))\beta^t q_t(1+\theta_t)$$

with $q_t := \beta^{-t}\prod_{s=0}^{t}(r_s)^{-1}$ and $1+\theta_t := \prod_{s=0}^{t}(1-\tau_s)^{-1}$

Competative Equilibirum

Definition

A competitive equilivirium is

$$\left\{\{c_s, k_s, b_s, \tau_s, \nu_s, T_s, r_s, w_s, \{c_s^i, W_s^i\}\right\}_{s=0}^{\infty}$$

such that

(1) *Households are maximizing subject to budget constraints.*

Competative Equilibirum

Definition

A competitive equilivirium is

$$\left\{\{c_s, k_s, b_s, \tau_s, \nu_s, T_s, r_s, w_s, \{c_s^i, W_s^i\}\right\}_{s=0}^{\infty}$$

such that

(1) *Households are maximizing subject to budget constraints.*

(2) *Factor prices are equal to marginal products:* $r_t = F_k(k_t, 1)$ *and* $w_t = F_l(k_t, 1)$.

Competative Equilibirum

Definition

A competitive equilivirium is

$$\left\{\{c_s, k_s, b_s, \tau_s, \nu_s, T_s, r_s, w_s, \{c_s^i, W_s^i\}\right\}_{s=0}^{\infty}$$

such that

(1) *Households are maximizing subject to budget constraints.*

(2) *Factor prices are equal to marginal products:* $r_t = F_k(k_t, 1)$ *and* $w_t = F_l(k_t, 1)$.

(3) *Markets clear*

$$\int c_t^i di = c_t \text{ and } \int W_t^i di = k_t + b_t$$

(4) *Government budget and transversality condition satisfied.*

Existence

Theorem

For any sequence $\{c_s, k_s\}_{s=0}^{\infty}$, there exists a competitive equilibrium if and only if the sequence satisfies

1. $$k_{t+1} + c_t + g_t = F(k_t, 1)$$

2. $$(F_k(k_{t+1}, 1)(1 - \bar{\tau}))^{-1} \leq \frac{\beta^t u'(c_{t+1})}{u'(c_t)} \leq (F_k(k_{t+1}, 1))^{-1}$$

Existence Proof Sketch

- We start by setting r_t, w_t marginal products and choose taxes

$$(1-\tau_{t+1})^{-1} = \frac{\beta r_{t+1} u'(c_{t+1})}{u'(c_t)} = \frac{1+\theta_{t+1}}{1+\theta_t}$$

Choose $\{\nu_t, T_t, b_{t+1}\}$ to satisfy government constraint.

Existence Proof Sketch

- We start by setting r_t, w_t marginal products and choose taxes

$$(1-\tau_{t+1})^{-1} = \frac{\beta r_{t+1} u'(c_{t+1})}{u'(c_t)} = \frac{1+\theta_{t+1}}{1+\theta_t}$$

  Choose $\{\nu_t, T_t, b_{t+1}\}$ to satisfy government constraint.

- The household first order condition, plus iteration, then gives us

$$\frac{A(\frac{A}{\sigma}c_t^i + B)^{-\sigma}}{A(\frac{A}{\sigma}c_0^i + B)^{-\sigma}} = \frac{q_t(1+\theta_t)}{1+\theta_0} = \frac{A(\frac{A}{\sigma}c_t + B)^{-\sigma}}{A(\frac{A}{\sigma}c_0 + B)^{-\sigma}}$$

Existence Proof Sketch

- We start by setting r_t, w_t marginal products and choose taxes

$$(1-\tau_{t+1})^{-1} = \frac{\beta r_{t+1} u'(c_{t+1})}{u'(c_t)} = \frac{1+\theta_{t+1}}{1+\theta_t}$$

Choose $\{\nu_t, T_t, b_{t+1}\}$ to satisfy government constraint.

- The household first order condition, plus iteration, then gives us

$$\frac{A(\frac{A}{\sigma}c_t^i + B)^{-\sigma}}{A(\frac{A}{\sigma}c_0^i + B)^{-\sigma}} = \frac{q_t(1+\theta_t)}{1+\theta_0} = \frac{A(\frac{A}{\sigma}c_t + B)^{-\sigma}}{A(\frac{A}{\sigma}c_0 + B)^{-\sigma}}$$

- From this we can conclude that

$$\frac{A}{\sigma}c_t^i + B = \alpha^i\left(\frac{A}{\sigma}c_t + B\right)$$

we choose α to satisfy house holds budget constraint. The paper then proves that $\int \alpha^i di = 1$ and hence $\int c_t^i di = c_t$

Preferences over equilibria

We can't use single peakness to give us a Condorcet winner, but we will use the Gorman aggreagation to simplify the households preferences over equilibria.

Preferences over equilibria

We can't use single peakness to give us a Condorcet winner, but we will use the Gorman aggreagation to simplify the households preferences over equilibria.

Lemma

For each household i there exists a function $G : \mathbb{R}^4 \to \mathbb{R}$ such that the utility of the household in a competitive equilibrium is $G(V, c_0, \tau_0, W_0^i - W_0)$ where $V = \sum_{t=0}^{\infty} \beta^t u(c_t)$. Also,

$$sign\left(\frac{\partial G}{\partial c_0}\right) = sign\left(\frac{\partial G}{\partial \tau_0}\right) = sign(W_0 - W_0^i)$$

Sketch of Proof

- We begin by subtraction the average budget constraint from the household's constraint to get

$$\sum_{t=0}^{\infty} \beta^t (c_t^i - c_t) \left(\frac{A}{\sigma} c_t + B \right)^{-\sigma} = r_0 (1 - \tau_0)(W_0^i - W_0) \left(\frac{A}{\sigma} c_0 + B \right)^{-\sigma}$$

Sketch of Proof

- We begin by subtraction the average budget constraint from the household's constraint to get

$$\sum_{t=0}^{\infty}\beta^t(c_t^i - c_t)\left(\frac{A}{\sigma}c_t + B\right)^{-\sigma} = r_0(1-\tau_0)(W_0^i - W_0)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}$$

- Substituting our aggregation equation and then solving for α_i we obtain

$$\alpha^i = 1 + \frac{Ar_0(1-\tau_0)(W_0^i - W_0)(\frac{A}{\sigma}c_0 + B)^{-\sigma}}{V(1-\sigma)}$$

Sketch of Proof

- We begin by subtraction the average budget constraint from the household's constraint to get

$$\sum_{t=0}^{\infty}\beta^t(c_t^i - c_t)\left(\frac{A}{\sigma}c_t + B\right)^{-\sigma} = r_0(1-\tau_0)(W_0^i - W_0)\left(\frac{A}{\sigma}c_0 + B\right)^{-\sigma}$$

- Substituting our aggregation equation and then solving for α_i we obtain

$$\alpha^i = 1 + \frac{Ar_0(1-\tau_0)(W_0^i - W_0)(\frac{A}{\sigma}c_0 + B)^{-\sigma}}{V(1-\sigma)}$$

- Thus we can get a formula for G as follows

$$G = (\alpha^i)^{1-\sigma}V = \left[1 + \frac{Ar_0(1-\tau_0)(W_0^i - W_0)(\frac{A}{\sigma}c_0 + B)^{-\sigma}}{V(1-\sigma)}\right]^{1-\sigma} V$$

The Median Voter

Theorem

The tax sequence preferred by the household with median wealth is a Condorcet winner

The Median Voter

Theorem

The tax sequence preferred by the household with median wealth is a Condorcet winner

- Consider two equilibiria given by $\{c_t\}, \tau_0$ and $\{\hat{c}_t\}, \hat{\tau}_0$. We can define

$$H := \left\{ W_0^i : G(V, c_0, \tau_0, W_0^i - W_0) \geq G(\hat{V}, \hat{c}_0, \hat{\tau}_0, W_0^i - W_0) \right\}$$

as households W_i^0 who (weakly) prefer the first equilibria similarly

$$\hat{H} := \left\{ W_0^i : G(\hat{V}, \hat{c}_0, \hat{\tau}_0, W_0^i - W_0) \geq G(V, c_0, \tau_0, W_0^i - W_0) \right\}$$

The Median Voter cont.

- One can show that the sign of the derivative of

$$\ln\left(\frac{G(V, c_0, \tau_0, W_0^i - W_0)}{G(\hat{V}, \hat{c}_0, \hat{\tau}_0, W_0^i - W_0)}\right)$$

is independent of W_0^i and thus there exists a W_0^* such that for all $W_0^i \leq W_0^*$ we have W_0^i belongs to one of $\hat{H}$ or H and if $W_i^0 \geq W_0^*$ then it belongs to the other.

The Median Voter cont.

- One can show that the sign of the derivative of

$$\ln\left(\frac{G(V, c_0, \tau_0, W_0^i - W_0)}{G(\hat{V}, \hat{c}_0, \hat{\tau}_0, W_0^i - W_0)}\right)$$

  is independent of W_0^i and thus there exists a W_0^* such that for all $W_0^i \leq W_0^*$ we have W_0^i belongs to one of $\hat{H}$ or H and if $W_i^0 \geq W_0^*$ then it belongs to the other.
- Thus whichever preference the median voter has, it is the same as the majority.

"Bang-Bang" Tax Policy

We begin by assuming that the median voter's wealth W_0^m is below the mean and show the following theorem

Theorem

The capital tax sequence $\{\tau_t\}_0^\infty$ *preferred by the median voter has the bang-bang property: if* $\tau_t < \bar{\tau}$ *then* $\tau_s = 0$ *for* $s > t$.

"Bang-Bang" Tax Policy

We begin by assuming that the median voter's wealth W_0^m is below the mean and show the following theorem

Theorem

The capital tax sequence $\{\tau_t\}_0^\infty$ preferred by the median voter has the bang-bang property: if $\tau_t < \bar{\tau}$ then $\tau_s = 0$ for $s > t$.

- We first note that our lemma implies that the initial tax will be $\bar{\tau}$.

"Bang-Bang" Tax Policy

We begin by assuming that the median voter's wealth W_0^m is below the mean and show the following theorem

Theorem

The capital tax sequence $\{\tau_t\}_0^\infty$ *preferred by the median voter has the bang-bang property: if* $\tau_t < \bar{\tau}$ *then* $\tau_s = 0$ *for* $s > t$*.*

- We first note that our lemma implies that the initial tax will be $\bar{\tau}$.
- We then split the theorem into cases where G is increasing, decreasing or constant in V.

"Bang-Bang" Tax Policy

We begin by assuming that the median voter's wealth W_0^m is below the mean and show the following theorem

Theorem

The capital tax sequence $\{\tau_t\}_0^\infty$ *preferred by the median voter has the bang-bang property: if* $\tau_t < \bar{\tau}$ *then* $\tau_s = 0$ *for* $s > t$.

- We first note that our lemma implies that the initial tax will be $\bar{\tau}$.
- We then split the theorem into cases where G is increasing, decreasing or constant in V.
- In each case we will use our theorem to construct a new equilibrium from a modified $\{c_t, k_t\}$

"Bang-Bang" Tax Policy

We begin by assuming that the median voter's wealth W_0^m is below the mean and show the following theorem

Theorem

The capital tax sequence $\{\tau_t\}_0^\infty$ *preferred by the median voter has the bang-bang property: if* $\tau_t < \bar{\tau}$ *then* $\tau_s = 0$ *for* $s > t$.

- We first note that our lemma implies that the initial tax will be $\bar{\tau}$.
- We then split the theorem into cases where G is increasing, decreasing or constant in V.
- In each case we will use our theorem to construct a new equilibrium from a modified $\{c_t, k_t\}$
- When G is decreasing or constant taxes will be at their upper bound for all periods

Increasing in V

- Let t be the first period where $\tau_t \neq \bar{\tau}$ and assume that $\tau_s \neq 0$ for some $s > t$.

Increasing in V

- Let t be the first period where $\tau_t \neq \bar{\tau}$ and assume that $\tau_s \neq 0$ for some $s > t$.
- If we wanted to maximize V we would set $\tau_0 = \bar{\tau}$ and $\tau_t = 0$ for all $t \geq 1$ thus we can conclude that this equilibrium does not maximize

$$\sum_{s=t+1}^{\infty} \beta^s \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c_t + B \right)^{1-\sigma}$$

give k_{t+1}

Increasing in V

- Let t be the first period where $\tau_t \neq \bar{\tau}$ and assume that $\tau_s \neq 0$ for some $s > t$.
- If we wanted to maximize V we would set $\tau_0 = \bar{\tau}$ and $\tau_t = 0$ for all $t \geq 1$ thus we can conclude that this equilibrium does not maximize

$$\sum_{s=t+1}^{\infty} \beta^s \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c_t + B \right)^{1-\sigma}$$

  give k_{t+1}
- We can therefore construct a new sequence of c_t's that follows the old sequence for $s \leq t$ and follows a new sequence for $s > t$, but that also increases V.

Increasing in V

- Let t be the first period where $\tau_t \neq \bar{\tau}$ and assume that $\tau_s \neq 0$ for some $s > t$.
- If we wanted to maximize V we would set $\tau_0 = \bar{\tau}$ and $\tau_t = 0$ for all $t \geq 1$ thus we can conclude that this equilibrium does not maximize

$$\sum_{s=t+1}^{\infty} \beta^s \frac{\sigma}{1-\sigma} \left(\frac{A}{\sigma} c_t + B \right)^{1-\sigma}$$

  give k_{t+1}
- We can therefore construct a new sequence of c_t's that follows the old sequence for $s \leq t$ and follows a new sequence for $s > t$, but that also increases V.
- Thus as c_0,τ_0 stay the same and V increases we conclude that G increases so this sequence is not preferred by the median voter.

Decreasing in V

- Let N be the first period where $\tau_N < \bar{\tau}$ and M be the first period after N where $\tau_{M+1} > 0$.

Decreasing in V

- Let N be the first period where $\tau_N < \bar{\tau}$ and M be the first period after N where $\tau_{M+1} > 0$.
- We will change $u'(c_{N-1})$ by a factor of $d\Psi$ and $u'(c_t)$ by a factor of $d\Phi$ for $t = 1, \ldots, M$. These changes take the form of $\frac{dc_t}{d\Phi} = \frac{u'(c_t)}{u''(c_t)} = -(\sigma^{-1} c_t A^{-1} B)$.

Decreasing in V

- Let N be the first period where $\tau_N < \bar{\tau}$ and M be the first period after N where $\tau_{M+1} > 0$.
- We will change $u'(c_{N-1})$ by a factor of $d\Psi$ and $u'(c_t)$ by a factor of $d\Phi$ for $t = 1, \ldots, M$. These changes take the form of $\frac{dc_t}{d\Phi} = \frac{u'(c_t)}{u''(c_t)} = -(\sigma^{-1} c_t A^{-1} B)$.
- Imposing feasibility we then obtain

$$
\begin{aligned}
dk_t =& d\Psi \left(\prod_{j=N}^{t-1} F_k(k_j, 1) \right) (\sigma^{-1} c_{N-1} + A^{-1} B) \\
&+ d\Phi \sum_{s=N}^{t-1} \left(\prod_{j=s+1}^{t-1} F_k(k_j, 1) \right) (\sigma^{-1} c_s + A^{-1} B)
\end{aligned}
$$

Decreasing in V cont

- We note that $dk_{M+1} = 0$ implies that

$$0 = d\Psi(A\sigma^{-1}c_{N-1}+B)+d\Psi\sum_{s=N}^{t-1}\left(\prod_{j=N}^{s}F_k(k_j,1)^{-1}\right)(\sigma^{-1}c_s+A^{-1}B)$$

Decreasing in V cont

- We note that $dk_{M+1} = 0$ implies that

$$0 = d\Psi(A\sigma^{-1}c_{N-1}+B)+d\Psi\sum_{s=N}^{t-1}\left(\prod_{j=N}^{s}F_k(k_j,1)^{-1}\right)(\sigma^{-1}c_s+A^{-1}B)$$

- Combining this with non-negative taxes:

$$\beta F_k(k_t,1)(A\sigma^{-1}c_t+B)^{-\sigma} \geq (A\sigma^{-1}c_{t-1}+B)^{-\sigma}$$

we obtain (when $d\Psi < 0$)

$$-d\Psi(A\sigma^{-1}c_{N-1}+B)^{1-\sigma} \geq d\Phi\sum_{s=N}^{M}\beta^{s-N+1}(A\sigma^{-1}c_s+B)^{1-\sigma}$$

Decreasing in V cont

- We note that $dk_{M+1} = 0$ implies that

$$0 = d\Psi(A\sigma^{-1}c_{N-1}+B)+d\Psi\sum_{s=N}^{t-1}\left(\prod_{j=N}^{s}F_k(k_j,1)^{-1}\right)(\sigma^{-1}c_s+A^{-1}B)$$

- Combining this with non-negative taxes:

$$\beta F_k(k_t,1)(A\sigma^{-1}c_t + B)^{-\sigma} \geq (A\sigma^{-1}c_{t-1} + B)^{-\sigma}$$

we obtain (when $d\Psi < 0$)

$$-d\Psi(A\sigma^{-1}c_{N-1} + B)^{1-\sigma} \geq d\Phi\sum_{s=N}^{M}\beta^{s-N+1}(A\sigma^{-1}c_s + B)^{1-\sigma}$$

- This tells us that $dV \leq 0$ with equality only if $\tau_N = 0$.

One Example

Corollary

If preferences are CRRA and production is linear, the capital tax preferred by the median voter is $\bar{\tau}$ forever if

$$1+\frac{\sigma r(1-\bar{\tau})(R-1)}{\left(1-\beta^{\frac{1}{\sigma}}r^{\frac{1-\sigma}{\sigma}}(1-\tau)^{\frac{1}{\sigma}}\right)r\left(1-\beta^{\frac{1}{\sigma}}(r(1-\tau))^{\frac{1-\sigma}{\sigma}}\right)^{-1}}\leq 0$$

where $R=W_0^m/W_0$ and $y=rk$. This can only happen if $\sigma>1$.

One Example

Corollary

If preferences are CRRA and production is linear, the capital tax preferred by the median voter is $\bar{\tau}$ forever if

$$1+\frac{\sigma r(1-\bar{\tau})(R-1)}{\left(1-\beta^{\frac{1}{\sigma}}r^{\frac{1-\sigma}{\sigma}}(1-\tau)^{\frac{1}{\sigma}}\right)r\left(1-\beta^{\frac{1}{\sigma}}(r(1-\tau))^{\frac{1-\sigma}{\sigma}}\right)^{-1}}\leq 0$$

where $R = W_0^m/W_0$ and $y = rk$. This can only happen if $\sigma > 1$.

This is done by checking that $\frac{\partial G}{\partial V}$ is negative in the equilibrium when $\tau_t = \bar{\tau}$ for all $t \geq 0$. This maximizes c_0 and τ_0 and minimizes V. $\frac{\partial G}{\partial V}$ is increasing in c_0, τ_0 and decreasing in V when $\sigma < 1$.