“Redistribution, taxes, and the median voter”

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Motivation

• Ideally we would wish to study a model in which households, with heterogeneous wealth distributions, choose a sequence of redistributive tax rates through voting.

• The problem arises that the agents are voting over a sequence of tax rates, and thus the space we are dealing with is infinite dimensional.

• The infinite dimensional space means that the usual single-peakedness assumption for the median voter theorem cannot be used.
Aim of The Paper

- This paper solves the problem proving that when preferences are identical and Gorman aggregable a Condorcet winner exists.
- Furthermore they show that under these assumptions the policy chosen will have the “bang-bang” property, i.e. capital income taxes remain at the upper bound until they drop to 0.
- This condenses a multi-dimensional problem into just a single choice of an optimal stopping time.
Assumptions

- Output $y_t$ is produced by competitive firms with a linearly homogeneous production function

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- Output $y_t$ is produced by competitive firms with a linearly homogeneous production function

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where capital depreciates fully.

- We have a continuum of agents, indexed by $i$. In the initial period they each own a wealth level $W_0^i$.

- In each period the government levies non-negative proportional taxes on labor income $\nu_t$ and capital income $\tau_t$ (which is subject to upper bound $\bar{\tau}$). It uses this income and one period debt to pay off an exogenous sequence $\{g_t\}$ and to finance lump sum transfer $T_t$, giving us the budget constraint

$$\tau_t r_t (k_t + b_t) + \nu_t w_t + b_{t+1} = r_t b_t + g_t + T_t$$
The Household

- We assume that the household’s preferences over the consumption stream \( \{c_t^i\} \) is given by \( \sum_{t=0}^{\infty} \beta^t u(c_t^i) \), where

\[
u(c) = \frac{\sigma}{1-\sigma} \left( \frac{A}{\sigma} c + B \right)^{1-\sigma}\]
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- The household supplies labor inelastically and maximizes its utility subject to the period-by-period budget constraint

\[
(1 - \tau_t) r_t W_t^i + (1 - \nu_t) w_t + T_t \geq c_t^i + W_{t+1}^i
\]

(under no arbitrage) and imposing no ponzi-schemes.
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(under no arbitrage) and imposing no ponzi-schemes.

• We can thus write the budget constraint in present value form

\[
\sum_{t=0}^{\infty} \beta^t c_t q_t (1 + \theta_t) = \mathcal{W}_0^i + \sum_{t=0}^{\infty} (T_t + w_t (1 - \nu_t)) \beta^t q_t (1 + \theta_t)
\]

with \( q_t := \beta^{-t} \prod_{s=0}^{t} (r_s)^{-1} \) and \( 1 + \theta_t := \prod_{s=0}^{t} (1 - \tau_s)^{-1} \)
Competative Equilibrium

Definition

A competitive equilibrium is

\[
\{\{c_s, k_s, b_s, \tau_s, \nu_s, T_s, r_s, w_s, \{c^i_s, W^i_s\}\}\}_{s=0}^{\infty}
\]

such that

(1) Households are maximizing subject to budget constraints.
Competitive Equilibrium

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such that

1. Households are maximizing subject to budget constraints.

2. Factor prices are equal to marginal products:
   \[ r_t = F_k(k_t, 1) \text{ and } w_t = F_l(k_t, 1). \]
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A competitive equilibrium is

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such that

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2. Factor prices are equal to marginal products:
   $$r_t = F_k(k_t, 1) \text{ and } w_t = F_l(k_t, 1).$$

3. Markets clear
   $$\int c_t^i di = c_t \text{ and } \int W_t^i di = k_t + b_t$$

Existence

Theorem
For any sequence \(\{c_s, k_s\}_{s=0}^{\infty}\), there exists a competitive equilibrium if and only if the sequence satisfies

1. 
\[ k_{t+1} + c_t + g_t = F(k_t, 1) \]

2. 
\[ (F_k(k_{t+1}, 1)(1 - \bar{\tau}))^{-1} \leq \frac{\beta^t u'(c_{t+1})}{u'(c_t)} \leq (F_k(k_{t+1}, 1))^{-1} \]
Existence Proof Sketch

- We start by setting \( r_t, w_t \) marginal products and choose taxes

\[
(1 - \tau_{t+1})^{-1} = \frac{\beta r_{t+1} u'(c_{t+1})}{u'(c_t)} = \frac{1 + \theta_{t+1}}{1 + \theta_t}
\]

Choose \( \{\nu_t, T_t, b_{t+1}\} \) to satisfy government constraint.
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Choose \( \{\nu_t, T_t, b_{t+1}\} \) to satisfy government constraint.

- The household first order condition, plus iteration, then gives us

\[
\frac{A \left( \frac{A}{\sigma} c_t^i + B \right)^{-\sigma}}{A \left( \frac{A}{\sigma} c_0^i + B \right)^{-\sigma}} = \frac{q_t(1 + \theta_t)}{1 + \theta_0} = \frac{A \left( \frac{A}{\sigma} c_t + B \right)^{-\sigma}}{A \left( \frac{A}{\sigma} c_0 + B \right)^{-\sigma}}
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- The household first order condition, plus iteration, then gives us

$$A \left( \frac{A}{\sigma} c^i_t + B \right)^{-\sigma} = \frac{q_t (1 + \theta_t)}{1 + \theta_0} = \frac{A \left( \frac{A}{\sigma} c_t + B \right)^{-\sigma}}{A \left( \frac{A}{\sigma} c_0 + B \right)^{-\sigma}}$$

- From this we can conclude that

$$\frac{A}{\sigma} c^i_t + B = \alpha^i \left( \frac{A}{\sigma} c_t + B \right)$$

we choose $\alpha$ to satisfy house holds budget constraint. The paper then proves that $\int \alpha^i di = 1$ and hence $\int c^i_t di = c_t$
Preferences over equilibria

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Lemma

For each household $i$ there exists a function $G : \mathbb{R}^4 \to \mathbb{R}$ such that the utility of the household in a competitive equilibrium is $G(V, c_0, \tau_0, W_0^i - W_0)$ where $V = \sum_{t=0}^{\infty} \beta^t u(c_t)$. Also,

$$\text{sign} \left( \frac{\partial G}{\partial c_0} \right) = \text{sign} \left( \frac{\partial G}{\partial \tau_0} \right) = \text{sign}(W_0 - W_0^i)$$
Sketch of Proof

- We begin by subtraction the average budget constraint from the household’s constraint to get

\[
\sum_{t=0}^{\infty} \beta^t (c_t^i - c_t) \left( \frac{A}{\sigma} c_t + B \right)^{-\sigma} = r_0 (1 - \tau_0) (W_0^i - W_0) \left( \frac{A}{\sigma} c_0 + B \right)^{-\sigma}
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- Substituting our aggregation equation and then solving for \( \alpha_i \) we obtain

\[ \alpha^i = 1 + \frac{A r_0 (1 - \tau_0) (W^i_0 - W_0) (\frac{A}{\sigma} c_0 + B)^{-\sigma}}{V(1 - \sigma)} \]
**Sketch of Proof**

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$$

- Thus we can get a formula for $G$ as follows

$$
G = (\alpha^i)^{1-\sigma} V = \left[ 1 + \frac{A r_0 (1 - \tau_0) (W_0^i - W_0) (\frac{A}{\sigma} c_0 + B)^{-\sigma}}{V(1 - \sigma)} \right]^{1-\sigma} V
$$
The Median Voter

Theorem

*The tax sequence preferred by the household with median wealth is a Condorcet winner*
The Median Voter

Theorem

The tax sequence preferred by the household with median wealth is a Condorcet winner

- Consider two equilibria given by \( \{c_t\}, \tau_0 \) and \( \{\hat{c}_t\}, \hat{\tau}_0 \). We can define

\[
H := \left\{ W^i_0 : G(V, c_0, \tau_0, W^i_0 - W_0) \geq G(\hat{V}, \hat{c}_0, \hat{\tau}_0, W^i_0 - W_0) \right\}
\]

as households \( W^0_i \) who (weakly) prefer the first equilibria similarly

\[
\hat{H} := \left\{ W^i_0 : G(\hat{V}, \hat{c}_0, \hat{\tau}_0, W^i_0 - W_0) \geq G(V, c_0, \tau_0, W^i_0 - W_0) \right\}
\]
The Median Voter cont.

- One can show that the sign of the derivative of

$$\ln \left( \frac{G(V, c_0, \tau_0, W_0^i - W_0)}{G(\hat{V}, \hat{c}_0, \hat{\tau}_0, W_0^i - W_0)} \right)$$

is independent of $W_0^i$ and thus there exists a $W_0^*$ such that for all $W_0^i \leq W_0^*$ we have $W_0^i$ belongs to one of $\hat{H}$ or $H$ and if $W_0^i \geq W_0^*$ then it belongs to the other.
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- Thus whichever preference the median voter has, it is the same as the majority.
"Bang-Bang" Tax Policy

We begin by assuming that the median voter’s wealth $W_0^m$ is below the mean and show the following theorem.

**Theorem**

The capital tax sequence $\{\tau_t\}_{0}^{\infty}$ preferred by the median voter has the bang-bang property: if $\tau_t < \bar{\tau}$ then $\tau_s = 0$ for $s > t$. 
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- We then split the theorem into cases where $G$ is increasing, decreasing or constant in $V$.
- In each case we will use our theorem to construct a new equilibrium from a modified $\{c_t, k_t\}$.
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- We then split the theorem into cases where $G$ is increasing, decreasing or constant in $V$.
- In each case we will use our theorem to construct a new equilibrium from a modified $\{c_t, k_t\}$
- When $G$ is decreasing or constant taxes will be at their upper bound for all periods
Increasing in V

- Let $t$ be the first period where $\tau_t \neq \bar{\tau}$ and assume that $\tau_s \neq 0$ for some $s > t$. 

David Evans
Increasing in $V$

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- If we wanted to maximize $V$ we would set $\tau_0 = \bar{\tau}$ and $\tau_t = 0$ for all $t \geq 1$ thus we can conclude that this equilibrium does not maximize

$$
\sum_{s=t+1}^{\infty} \beta^s \frac{\sigma}{1-\sigma} \left( \frac{A}{\sigma} c_t + B \right)^{1-\sigma}
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  give $k_{t+1}$
- We can therefore construct a new sequence of $c_t$’s that follows the old sequence for $s \leq t$ and follows a new sequence for $s > t$, but that also increases $V$.
- Thus as $c_0, \tau_0$ stay the same and $V$ increases we conclude that $G$ increases so this sequence is not preferred by the median voter.

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Decreasing in $V$

- Let $N$ be the first period where $\tau_N < \bar{\tau}$ and $M$ be the first period after $N$ where $\tau_{M+1} > 0$. 
Decreasing in V

- Let $N$ be the first period where $\tau_N < \bar{\tau}$ and $M$ be the first period after $N$ where $\tau_{M+1} > 0$.
- We will change $u'(c_{N-1})$ by a factor of $d\Psi$ and $u'(c_t)$ by a factor of $d\Phi$ for $t = 1, \ldots, M$. These changes take the form of $\frac{dc_t}{d\Phi} = \frac{u'(c_t)}{u''(c_t)} = -(\sigma^{-1}c_tA^{-1}B)$. 
Decreasing in V

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- We will change $u'(c_{N-1})$ by a factor of $d\Psi$ and $u'(c_t)$ by a factor of $d\Phi$ for $t = 1, \ldots, M$. These changes take the form of $\frac{dc_t}{d\Phi} = \frac{u'(c_t)}{u''(c_t)} = - (\sigma^{-1}c_t A^{-1} B)$.
- Imposing feasibility we then obtain

$$dk_t = d\Psi \left( \prod_{j=N}^{t-1} F_k(k_j, 1) \right) (\sigma^{-1}c_{N-1} + A^{-1} B)$$

$$+ d\Phi \sum_{s=N}^{t-1} \left( \prod_{j=s+1}^{t-1} F_k(k_j, 1) \right) (\sigma^{-1}c_s + A^{-1} B)$$
Decreasing in V cont

- We note that \( dk_{M+1} = 0 \) implies that

\[
0 = d\Psi(A\sigma^{-1}c_{N-1} + B) + d\Psi \sum_{s=N}^{t-1} \left( \prod_{j=N}^{s} F_k(k_j, 1)^{-1} \right) (\sigma^{-1}c_s + A^{-1}B)
\]
Decreasing in V cont

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- Combining this with non-negative taxes:

$$\beta F_k(k_t, 1)(A\sigma^{-1}c_t + B)^{-\sigma} \geq (A\sigma^{-1}c_{t-1} + B)^{-\sigma}$$

we obtain (when $d\Psi < 0$)

$$-d\Psi(A\sigma^{-1}c_{N-1} + B)^{1-\sigma} \geq d\Phi \sum_{s=N}^{M} \beta^{s-N+1}(A\sigma^{-1} c_s + B)^{1-\sigma}$$
Decreasing in $V$ cont

• We note that $dk_{M+1} = 0$ implies that

$$0 = d\psi (A\sigma^{-1}c_{N-1}+B) + d\psi \sum_{s=N}^{t-1} \left( \prod_{j=N}^{s} F_k(k_j, 1)^{-1} \right) (\sigma^{-1}c_s + A^{-1}B)$$

• Combining this with non-negative taxes:

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we obtain (when $d\psi < 0$)

$$-d\psi (A\sigma^{-1}c_{N-1} + B)^{1-\sigma} \geq d\Phi \sum_{s=N}^{M} \beta^{s-N+1}(A\sigma^{-1}c_s + B)^{1-\sigma}$$

• This tells us that $dV \leq 0$ with equality only if $\tau_N = 0$. 

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One Example

Corollary

If preferences are CRRA and production is linear, the capital tax preferred by the median voter is \( \bar{\tau} \) forever if

\[
1 + \frac{\sigma r (1 - \bar{\tau})(R - 1)}{\left(1 - \beta^{\frac{1}{\sigma}} r \frac{1 - \sigma}{\sigma} (1 - \tau)^{\frac{1}{\sigma}}\right) r \left(1 - \beta^{\frac{1}{\sigma}} (r(1 - \tau))^{\frac{1 - \sigma}{\sigma}}\right)^{-1}} \leq 0
\]

where \( R = W_0^m / W_0 \) and \( y = rk \). This can only happen if \( \sigma > 1 \).
Corollary

If preferences are CRRA and production is linear, the capital tax preferred by the median voter is $\bar{\tau}$ forever if

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where $R = \frac{W^m_0}{W_0}$ and $y = rk$. This can only happen if $\sigma > 1$. This is done by checking that $\frac{\partial G}{\partial V}$ is negative in the equilibrium when $\tau_t = \bar{\tau}$ for all $t \geq 0$. This maximizes $c_0$ and $\tau_0$ and minimizes $V$. $\frac{\partial G}{\partial V}$ is increasing in $c_0, \tau_0$ and decreasing in $V$ when $\sigma < 1$. 