The Term Structure of Inflation Expectations
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This Paper:

- estimates a no arbitrage affine term structure model incorporating information from surveys of inflation expectations
- allows for differing expectations across forecasters (surveys)
- asks how effective monetary policy is in the US by exploring the stability of (objective or model consistent) inflation expectations
State Variables

$$m_t = \begin{bmatrix} g_t \\ \pi_t \end{bmatrix}$$  

(1)

$$z_t = \begin{bmatrix} m_t \\ x_t \end{bmatrix}$$  

(2)

$$z_t = \mu + \Phi z_{t-1} + \Sigma \varepsilon_t$$  

(3)

objective expectations:

$$E_t(z_{t+\tau}) = \sum_{s=0}^{\tau-1} \Phi^s \mu + \Phi^\tau z_t$$  

(4)
The Short Rate and Yields

\[ r_t = \delta_0 + \delta_z z_t \]  

(5)

\[ \log \xi_t = -r_{t-1} - \frac{1}{2} \Lambda_{t-1}^\top \Lambda_{t-1} - \Lambda_{t-1} \varepsilon_t \]  

(6)

\[ \Lambda_t = \Lambda_0 + \Lambda_z z_t \]  

(7)

\[ y_t(\tau) = a^Q(\tau) + b^Q(\tau) z_t \]  

(9)

\[ = \underbrace{a^P(\tau) + b^P(\tau) z_t}_{\text{short rate expectations}} + \underbrace{a^{TP}(\tau) + b^{TP}(\tau) z_t}_{\text{term premium}} \]  

(10)
Survey Expectations

Modelling the change of measure from the physical measure to the subjective probability measures (one for each survey):

\[
\log \xi^i_t = -\frac{1}{2} \Lambda^i_{t-1} \Lambda^i_{t-1} - \Lambda^i_{t-1} \varepsilon_t \quad (11)
\]

\[
\Lambda^i_t = \Lambda^i_0 + \Lambda^i_z z_t \quad (12)
\]

The forecasters only differ in their view on inflation: only second element of \( \Lambda^i_0 \) and second row of \( \Lambda^i_z \) are different from 0. This leads to subjective VAR dynamics of the form:

\[
z_t = \mu^i + \Phi^i z_{t-1} + \Sigma \varepsilon^i_t \quad (13)
\]
Survey Expectations

$$
\overline{p}^i_{t,s}(\tau) = E_t(\overline{\pi}_{t,s,\tau}) = E_t\left(\frac{1}{\tau} \sum_{j=1}^{\tau} \pi_{t+s+j}\right) \\
= a^P(s, \tau) + b^P(s, \tau)z_t + a^{TBi}(s, \tau) + b^{TBi}(s, \tau)z_t
$$
Inflation Premium

Real SDF:

$$\log \xi_t^R = -r_{t-1} + \pi_t - \frac{1}{2} \Lambda_{t-1}^\top \Lambda_{t-1} - \Lambda_{t-1} \varepsilon_t$$  \hspace{1cm} (16)

Inflation Premium:

$$y_t(\tau) = y_t^R(\tau) + \bar{p}_{t,0}(\tau) + IP_t(\tau)$$ \hspace{1cm} (17)
Data and Measurement Equations

- inflation and GDP (assumed to observed without measurement error)
- nominal yields with measurement equation

\[ y_t(\tau)^{obs} = a^Q(\tau) + b^Q(\tau)z_t + \omega_t \]

- survey measures of inflation expectations:

\[ \overline{p}_{t,s}^{i,obs}(\tau) = a^P(s, \tau) + b^P(s, \tau)z_t + \chi^i_{t,s}(\tau) \]
Estimation

Penalized Maximum Likelihood:

\[ L_p = L - \frac{1}{2\sigma^2_p} \sum_{\tau} \left[ a^{TP}(\tau)^2 + b^{TP}(\tau)^\top \text{Diag}(\text{var}(z_t))b^{TP}(\tau) \right] \]

\[ -\frac{1}{2\sigma^2_p} \sum_{i,s,\tau} \left[ a^{TBi}(s, \tau)^2 + b^{TBi}(s, \tau)^\top \text{Diag}(\text{var}(z_t))b^{TBi}(s, \tau) \right] \]

\[ \sigma^2_p = 300 \]
Estimated Models

- **AS**: All data, Subjective expectations
- **AO**: All data, Objective expectations
- **NF**: No Forecasts used in estimation (subjective expectations can’t be identified)
- **OF**: No yields used in estimation (risk neutral probability measure can’t be identified)
Subjective Expectations

\[ TB_t^i(\tau) = E_t^i(\pi_{t+\tau}) - E_t(\pi_{t+\tau}) \]  \hspace{1cm} (18)

\[ D_t(\tau) = \max_i(TB_t^i(\tau)) - \min_i(TB_t^i(\tau)) \]  \hspace{1cm} (19)
(a) Spot rate loadings on macro factors

(b) Long-run mean

(c) Eigenvalues (highest and lowest)

(d) Standard deviation of g and π