The Term Structure of Inflation Expectations

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This Paper:

- estimates a no arbitrage affine term structure model incorporating information from surveys of inflation expectations
- allows for differing expectations across forecasters (surveys)
- asks how effective monetary policy is in the US by exploring the stability of (objective or model consistent) inflation expectations

State Variables

$$m_t = \begin{bmatrix} g_t \\ \pi_t \end{bmatrix} \tag{1}$$

$$z_t = \begin{bmatrix} m_t \\ x_t \end{bmatrix} \tag{2}$$

$$z_t = \mu + \Phi z_{t-1} + \Sigma \varepsilon_t \tag{3}$$

objective expectations:

$$E_t(z_{t+\tau}) = \sum_{s=0}^{\tau-1} \Phi^s \mu + \Phi^\tau z_t \tag{4}$$

The Short Rate and Yields

$$r_t = \delta_0 + \delta_z z_t \quad (5)$$

$$\log \xi_t = -r_{t-1} - \frac{1}{2}\Lambda_{t-1}^{\top}\Lambda_{t-1} - \Lambda_{t-1}\varepsilon_t \quad (6)$$

$$\Lambda_t = \Lambda_0 + \Lambda_z z_t \quad (7)$$

$$(8)$$

$$\begin{aligned} y_t(\tau) &= a^{\mathbb{Q}}(\tau) + b^{\mathbb{Q}}(\tau) z_t && (9) \\ &= \underbrace{a^{\mathbb{P}}(\tau) + b^{\mathbb{P}}(\tau) z_t}_{\text{short rate expectations}} + \underbrace{a^{TP}(\tau) + b^{TP}(\tau) z_t}_{\text{term premium}} && (10) \end{aligned}$$

Survey Expectations

Modelling the change of measure from the physical measure to the subjective probability measures (one for each survey):

$$\log \xi_t^i = -\frac{1}{2}\Lambda_{t-1}^{i\top}\Lambda_{t-1}^i - \Lambda_{t-1}^i \varepsilon_t \tag{11}$$

$$\Lambda_t^i = \Lambda_0^i + \Lambda_z^i z_t \tag{12}$$

The forecasters only differ in their view on inflation: only second element of Λ_0^i and second row of Λ_z^i are different form 0.
This leads to subjective VAR dynamics of the form:

$$z_t = \mu^i + \Phi^i z_{t-1} + \Sigma\varepsilon_t^i \tag{13}$$

Survey Expectations

$$\bar{p}^i_{t,s}(\tau) = E^i_t(\bar{\pi}_{t,s,\tau}) = E^i_t\left(\frac{1}{\tau}\sum_{j=1}^{\tau}\pi_{t+s+j}\right) \quad (14)$$

$$= \underbrace{a^{\mathbb{P}}(s,\tau) + b^{\mathbb{P}}(s,\tau)z_t}_{\text{objective forecasts}} + \underbrace{a^{TBi}(s,\tau) + b^{TBi}(s,\tau)z_t}_{\text{term bias}} \quad (15)$$

Inflation Premium

Real SDF:

$$\log \xi_t^R = -r_{t-1} + \pi_t - \frac{1}{2}\Lambda_{t-1}^\top \Lambda_{t-1} - \Lambda_{t-1}\varepsilon_t \tag{16}$$

Inflation Premium:

$$y_t(\tau) = y_t^R(\tau) + \overline{p}_{t,0}(\tau) + IP_t(\tau) \tag{17}$$

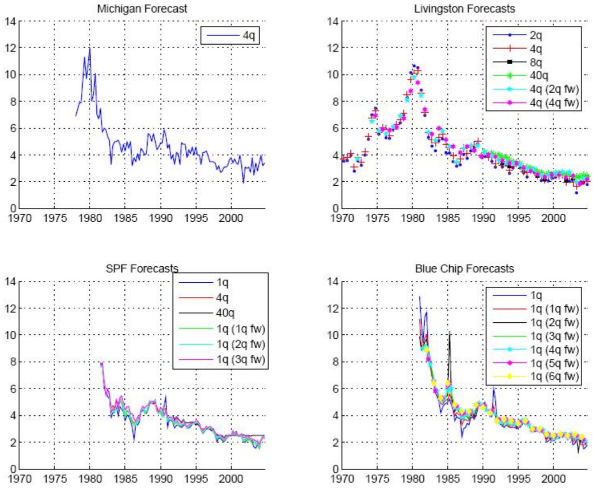
Data and Measurement Equations

- inflation and GDP (assumed to observed without measurement error)
- nominal yields with measurement equation

$$y_t(\tau)^{obs} = a^{\mathbb{Q}}(\tau) + b^{\mathbb{Q}}(\tau)z_t + \omega_t$$

- survey measures of inflation expectations:

$$\bar{p}^{i,obs}_{t,s}(\tau) = a^{\mathbb{P}}(s,\tau) + b^{\mathbb{P}}(s,\tau)z_t + \chi^i_{t,s}(\tau)$$


Michigan Forecast
4q
Livingston Forecasts
2q
4q
8q
40q
4q (2q fw)
4q (4q fw)
SPF Forecasts
1q
4q
40q
1q (1q fw)
1q (2q fw)
1q (3q fw)
Blue Chip Forecasts
1q
1q (1q fw)
1q (2q fw)
1q (3q fw)
1q (4q fw)
1q (5q fw)
1q (6q fw)
0 2 4 6 8 10 12 14
1970 1975 1980 1985 1990 1995 2000

Estimation

Penalized Maximum Likelihood:

$$L_p = L - \frac{1}{2\sigma_p^2}\sum_{\tau}\left[a^{TP}(\tau)^2 + b^{TP}(\tau)^\top Diag(var(z_t))b^{TP}(\tau)\right]$$
$$-\frac{1}{2\sigma_p^2}\sum_{i,s,\tau}\left[a^{TBi}(s,\tau)^2 + b^{TBi}(s,\tau)^\top Diag(var(z_t))b^{TBi}(s,\tau)\right]$$

σ_p^2=300

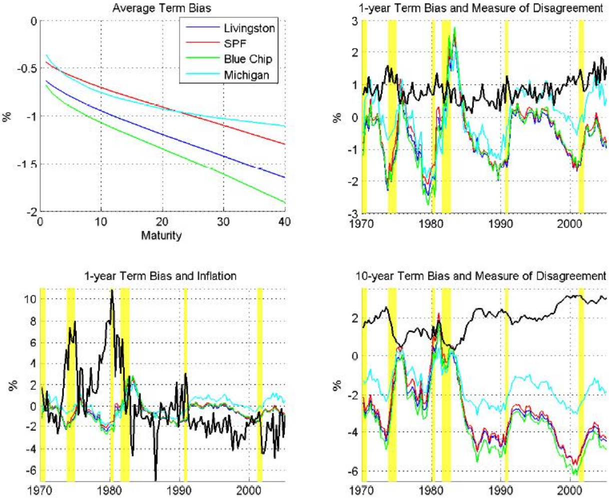
Estimated Models

- AS: All data, Subjective expectations
- AO: All data, Objective expectations
- NF: No Forecasts used in estimation (subjective expectations can't be identified)
- OF: No yields used in estimation (risk neutral probability measure can't be identified)

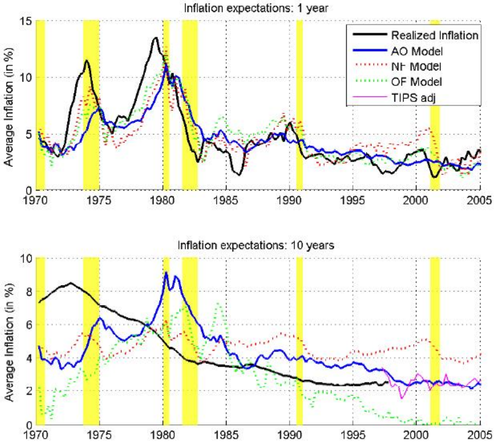
Subjective Expectations

$$TB_t^i(\tau) = E_t^i(\pi_{t+\tau}) - E_t(\pi_{t+\tau}) \tag{18}$$

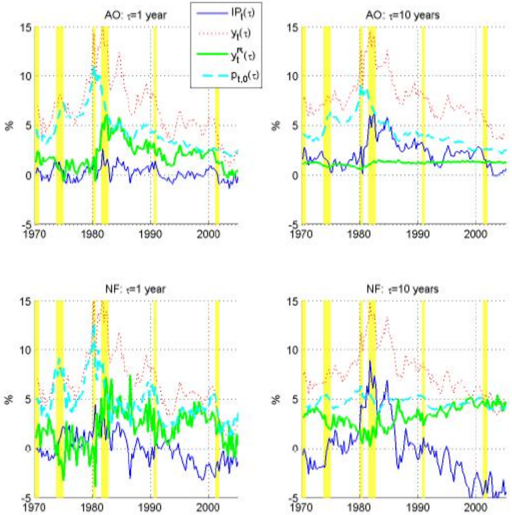
$$D_t(\tau) = \max_i(TB_t^i(\tau)) - \min_i(TB_t^i(\tau)) \tag{19}$$

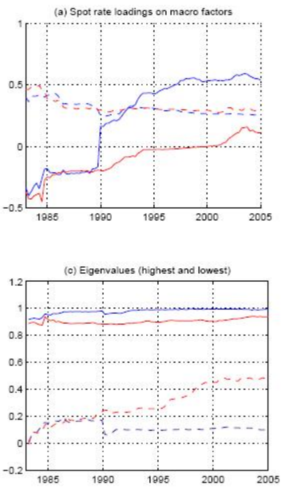


Average Term Bias
Livingston
SPF
Blue Chip
Michigan
%
Maturity
0
-0.5
-1
-1.5
-2
0
10
20
30
40
1-year Term Bias and Measure of Disagreement
3
2
1
0
-1
-2
-3
1970
1980
1990
2000
1-year Term Bias and Inflation
10
8
6
4
2
0
-2
-4
-6
10-year Term Bias and Measure of Disagreement

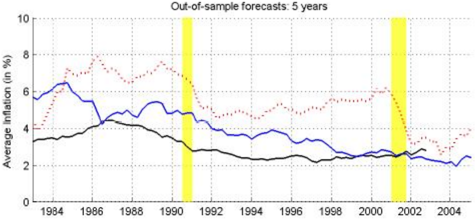


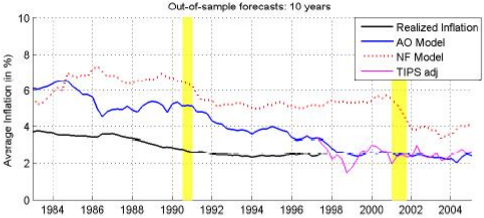
Inflation expectations: 1 year
Realized Inflation
AO Model
NF Model
OF Model
TIPS adj
Average Inflation (in %)
15
10
5
0
1970
1975
1980
1985
1990
1995
2000
2005
Inflation expectations: 10 years
Average Inflation (in %)
10
8
6
4
2
0


AO: τ=1 year
AO: τ=10 years
NF: τ=1 year
NF: τ=10 years
IP_t(τ)
y_t(τ)
y_t^R(τ)
p_t,0(τ)
%
15
10
5
0
-5
1970
1980
1990
2000



(a) Spot rate loadings on macro factors
(b) Long−run mean
AO
NF
(c) Eigenvalues (highest and lowest)
(d) Standard deviation of g and π
1985
1990
1995
2000
2005


Out-of-sample forecasts: 5 years
Average Inflation (in %)
10
8
6
4
2
0
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004


Out-of-sample forecasts: 10 years
Average Inflation in %)
Realized Inflation
AO Model
NF Model
TIPS adj
10
8
6
4
2
0
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004