Joint-Search Theory:
New Opportunities and New Frictions

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(based on slides provided by Gianluca Violante)

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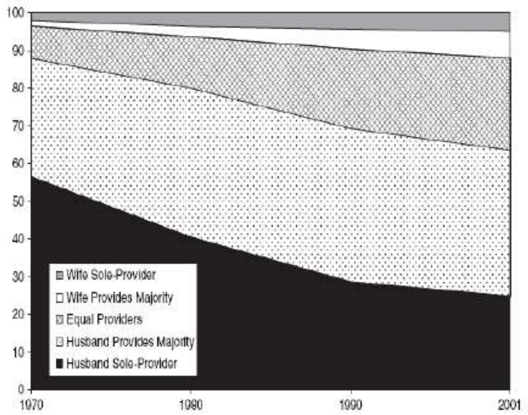
Bottom Line: For many households, job search is increasingly becoming a joint decision process.

Female Labor Force Participation in the US

The Rise of Dual Career Couples

FIGURE 1. PERCENT DISTRIBUTION OF MARITAL HOUSEHOLDS BY COUPLE'S INCOME CONTRIBUTIONS: 1970–2001



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- Study familiar economic environments of McCall (1970) and Burdett (1978)
- Study two cases where joint decision leads to different outcomes from single-agent search:
  1. Couple has concave utility over pooled income
  2. Couple receives job offers from multiple locations and faces a cost of living apart

Single-Search Value Functions: A Review

- Flow value for unemployed worker:

$$rV = u(b) + \alpha \int \max\{W(w) - V, 0\}\, dF(w)$$

- Flow value for employed worker:

$$rW(w) = u(w)$$

Baseline Joint Search Model

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- Flow value for dual-searcher couple:

$$rU = u(2b) + 2\alpha \int \max\left[\Omega(w) - U, 0\right] dF(w)$$

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... Thus, by symmetry of T, $\psi(.) = \phi(.)$

Risk Neutrality: Joint search = Single search w/ 2 jobs

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$$U = 2V$$

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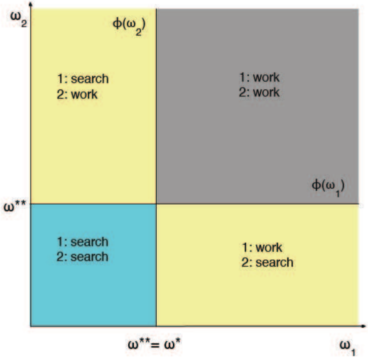

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- HARA: Hyperbolic Absolute Risk Aversion

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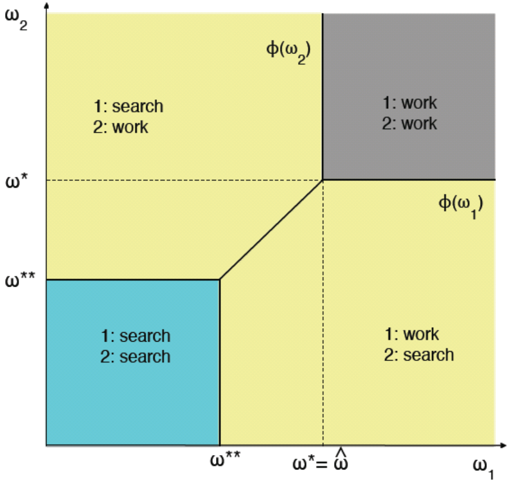
- Because of CARA, the reservation wage of the unemployed spouse in the no-quit range is independent of the wage of the employed spouse, w_1

CARA


ω2
φ(ω2)
1: search
2: work
1: work
2: work
ω*
φ(ω1)
ω**
1: search
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1: work
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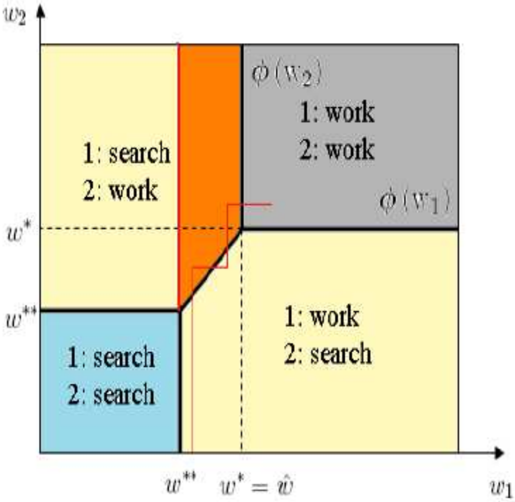
ω**
ω*= ω̂
ω1

CARA: Breadwinner Dynamics



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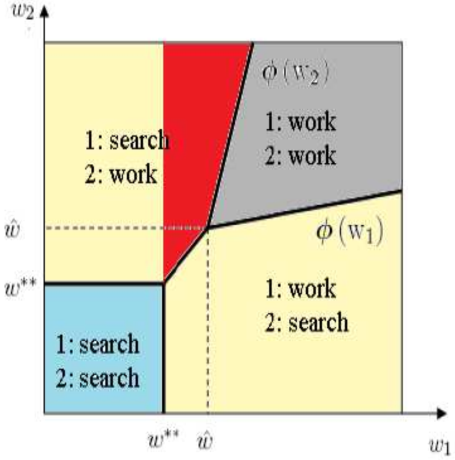
- Breadwinner Cycle always exists. However, the nature of the region changes depending on DARA, CARA, or IARA.

DARA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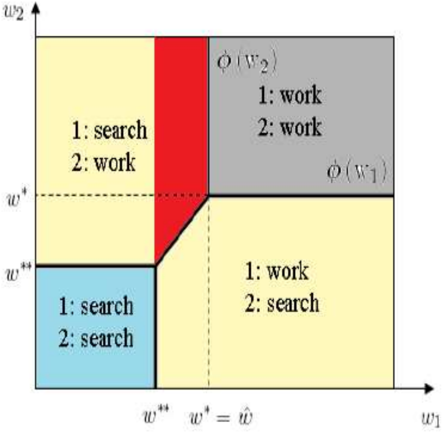
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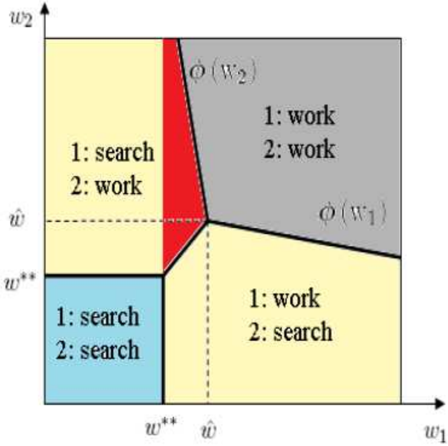
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  - In the CARA and DARA cases, breadwinner cycle exists, and $\phi(w)$ is strictly increasing

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Value functions

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$$rU = 2b+2(\alpha_i+\alpha_o)\int \max\left[\Omega(w)-U,0\right] dF(w)$$

Inside (Left) and Outside (Right) Offers