Joint-Search Theory: New Opportunities and New Frictions

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(based on slides provided by Gianluca Violante)
Motivation

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Bottom Line: For many households, job search is increasingly becoming a joint decision process.
Female Labor Force Participation in the US

![Graph showing labor force participation rates for all men and married white women from 1890 to 1990.]
The Rise of Dual Career Couples

**Figure 1. Percent Distribution of Marital Households by Couple’s Income Contributions: 1970–2001**

- Wife Sole-Provider
- Wife Provides Majority
- Equal Providers
- Husband Provides Majority
- Husband Sole-Provider

Joint-Search Theory
Aim of the Paper

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- Theoretical characterization of the joint job search problem of a household
- Study familiar economic environments of McCall (1970) and Burdett (1978)
- Study two cases where joint decision leads to different outcomes from single-agent search:
  1. Couple has concave utility over pooled income
  2. Couple receives job offers from multiple locations and faces a cost of living apart
Single-Search Value Functions: A Review

- Flow value for unemployed worker:

\[ rV = u(b) + \alpha \int \max \{ W(w) - V, 0 \} dF(w) \]

- Flow value for employed worker:

\[ rW(w) = u(w) \]
Baseline Joint Search Model

- Decision unit: couple, i.e., a pair of infinitely lived symmetric spouses indexed by \( i = 1, 2 \)
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- Search only during unemployment
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- No exogenous separation into unemployment
Joint-Search Value Functions

- Flow value for dual-worker couple:

\[ r_T(w_1, w_2) = u(w_1 + w_2) \]
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- Flow value for worker-searcher couple:

\[ r_\Omega(w_1) = u(w_1 + b) + \alpha \int \max [T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0] \, dF(w_2) \]
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- Flow value for dual-searcher couple:
  \[ r_U = u(2b) + 2\alpha \int \max [\Omega(w) - U, 0] dF(w) \]
Reservation Wage Functions

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... Thus, by symmetry of $T$, $\psi(.) = \phi(.)$
Risk Neutrality: Joint search = Single search w/ 2 jobs

\[ T(w_1, w_2) = W(w_1) + W(w_2) \]

\[ U = 2V \]

\[ \Omega(w_1) = V + W(w_1) \]
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Risk Aversion: HARA Utility

- HARA: Hyperbolic Absolute Risk Aversion

Risk Tolerance: $-\frac{u'(c)}{u''(c)} = \rho + \tau c$
Risk Aversion: HARA Utility

- HARA: Hyperbolic Absolute Risk Aversion

  Risk Tolerance: \[- \frac{u'(c)}{u''(c)} = \rho + \tau c\]

- Constant Absolute Risk Aversion (CARA): \(\tau = 0\)
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- Decreasing Absolute Risk Aversion (DARA): $\tau > 0$
Risk Aversion: HARA Utility

- **HARA**: Hyperbolic Absolute Risk Aversion
  
  Risk Tolerance: \(-u'(c)/u''(c) = \rho + \tau c\)

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  - Exponential: \(u(c) = -e^{-\rho c}/\rho\)

- **Decreasing Absolute Risk Aversion (DARA)**: \(\tau > 0\)
  
  - \(\rho = 0\) and \(\tau = 1/\sigma\): \(u(c) = c^{1-\sigma}/(1 - \sigma)\)
Risk Aversion: HARA Utility

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  Risk Tolerance: \(- \frac{u'(c)}{u''(c)} = \rho + \tau c\)

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- **Increasing Absolute Risk Aversion (IARA):** \(\tau < 0\)
Risk Aversion: HARA Utility

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  - Constant Absolute Risk Aversion (CARA): \(\tau = 0\)
    - Exponential: \(u(c) = -e^{-\rho c}/\rho\)
  
  - Decreasing Absolute Risk Aversion (DARA): \(\tau > 0\)
    - \(\rho = 0\) and \(\tau = 1/\sigma\): \(u(c) = c^{1-\sigma}/(1-\sigma)\)

  - Increasing Absolute Risk Aversion (IARA): \(\tau < 0\)
    - \(\tau = -1\): \(u(c) = -(\rho - c)^2\)
CARA case

- Let $w^*$ be reservation wage in single-search
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- Result 2:
  $$
  \phi (w_1) = \begin{cases} 
  w_1 & \text{if } w_1 < w^* \\
  w^* & \text{if } w_1 \geq w^* 
  \end{cases} \quad \text{(quit)}
  $$
  $$
  \phi (w_1) = \begin{cases} 
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CARA case

- Let $w^*$ be reservation wage in single-search

- Result 1: $w^{**} < w^*$
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- Because of CARA, the reservation wage of the unemployed spouse in the no-quit range is independent of the wage of the employed spouse, $w_1$
CARA

Joint-Search Theory
CARA: Breadwinner Dynamics

\[ w_2 \]

\[ w^* \]

\[ w^{**} \]

\[ w^{**} = w^* = \hat{w} \]

1: search
2: work

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2: search
General Characterization for HARA

- $w^{**} < w^*$
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- For $w_1 < \hat{w}$
  \[ \phi(w_1) = w_1 \] (i.e., 45$^0$ line)
General Characterization for HARA

- $w^{**} < w^*$

- For $w_1 < \hat{w}$
  \[ \phi (w_1) = w_1 \quad \text{(i.e., 45° line)} \]

- For $w_1 \geq \hat{w}$:
  \[
  \phi' (w_1) \begin{cases} 
  > 0 & \text{if DARA} \\
  = 0 & \text{if CARA} \\
  < 0 & \text{if IARA}
  \end{cases}
  \]
General Characterization for HARA

- $w^{**} < w^*$

- For $w_1 < \hat{w}$
  \[
  \phi(w_1) = w_1 \quad \text{(i.e., 45° line)}
  \]

- For $w_1 \geq \hat{w}$:
  \[
  \phi'(w_1) \begin{cases} 
  > 0 & \text{if DARA} \\
  = 0 & \text{if CARA} \\
  < 0 & \text{if IARA}
  \end{cases}
  \]

- Breadwinner Cycle always exists. However, the nature of the region changes depending on DARA, CARA, or IARA.
DARA

Joint-Search Theory
CARA
Joint-Search: HARA prefs

IARA

Joint-Search Theory
Two Extensions

- Symmetric on-the-job search \((\alpha_e = \alpha_u)\)
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- Symmetric on-the-job search ($\alpha_e = \alpha_u$)
  - The search strategies of the jointly searching couple are identical to those of the single-agent

- Exogenous separation
  - In the CARA and DARA cases, breadwinner cycle exists, and $\phi(w)$ is strictly increasing
Two-location Model

- Risk-neutrality
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- Inside location \((i)\) and outside location \((o)\)
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- No cost of migration across locations
Value functions

- Dual-worker and Separate Dual-worker couple:

\[
\begin{align*}
    r_T(w_1, w_2) &= w_1 + w_2 \\
    r_S(w_1, w_2) &= w_1 + w_2 - \kappa
\end{align*}
\]
Value functions

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\[ r_T(w_1, w_2) = w_1 + w_2 \]
\[ r_S(w_1, w_2) = w_1 + w_2 - \kappa \]

- Worker-searcher couple:

\[ r_{\Omega}(w_1) = w_1 + b + \alpha_i \int \max [T(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0] dF(w_2) \]
\[ + \alpha_o \int \max [S(w_1, w_2) - \Omega(w_1), \Omega(w_2) - \Omega(w_1), 0] dF(w_2) \]
Value functions

- **Dual-worker and Separate Dual-worker couple:**
  \[ rT(w_1, w_2) = w_1 + w_2 \]
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- **Worker-searcher couple:**
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- **Dual-searcher couple:**
  \[ rU = 2b + 2(\alpha_i + \alpha_o) \int \max [\Omega (w) - U, 0] \, dF(w) \]
Inside (Left) and Outside (Right) Offers