Collateral Constraints and Noisy Fluctuations

by Jennifer La'O - Job Market Paper 2010

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Motivation

- Little variation of output explained by changes in productivity Blanchard and Quah (1989), Shapiro and Watson (1988), Cochrane (1994)
- "Demand Shocks": shock orthogonal to technology shock that causes positive co-movement between output, employment and consumption.

Aim of the paper: Provide a theory of how financial frictions may imply movements in both the business cycle and asset prices not driven by fundamentals.

Candidates:

Noise + Collateral constraints

The Model

- 2 stages $t = 1, 2$
- 2 types of agents:
  - Continuum of workers grouped in a continuum of households or "families"
  - Continuum of competitive representative firms each of whom lives in an island. $i \in I$
- 2 goods traded in global markets:
  - output (produced by firms): price P
  - Land or Housing: price Q
- Local labor markets, one market in each island: wage w_i

Households

- Preferences of the household k

$$\mathcal{U}_k = C_k^{1-\gamma} H_k^{\gamma} - N_k$$

  - C_k is the household's consumption of the good produced by firms
  - H_k is the composite of housing h_{jk} purchased by each member i of the household k

$$H_k = \left[\int_J h_{jk}^{\frac{\nu-1}{\nu}} dj \right]^{\frac{\nu}{\nu-1}}$$

  - N_k is the composite of the labor n_{jk} supplied by workers j in household k

$$N_k = \int_J n_{jk} dj$$

- Budget constraint

$$PC_k + Q \int_J h_{jk} dj \leq \int_I \pi_i di + \int_J w_j n_{jk} dj$$

Islands and Firms

- Continuum of islands $i \in I$
- Islands determine boundaries for labor market and information
- 1 representative competitive firm in each island
- Production function in island i

$$y_i = A_i n_i^{\theta}$$

  where A_i is the productivity level on island i and n_i is the firm's employment level.
- Each firm is endowed with L_i units of land.
- Profits

$$\pi_i = Py_i - w_i n_i + QL_i$$

Collateral Constraints

- Firms cannot commit to repay workers after they produced
- Firms use land holdings as collateral to pay workers

$$w_i n_i \leq \chi Q L_i$$

where $\chi > 0$ is the fraction of land that can be collateralizable.

Shocks

Each island economy is subject to 2 shocks

1. **Technology shock:**

$$a_i \equiv \log A_i = \bar{a} + \xi_{a,i}$$

- $\bar{a} \sim N\left(0, \sigma^2_{A,0}\right)$ is an aggregate shock to productivity
- $\xi_{a,i} \overset{iid}{\sim} N\left(0, \sigma^2_{A,x}\right)$ is a purely idiosyncratic productivity shock

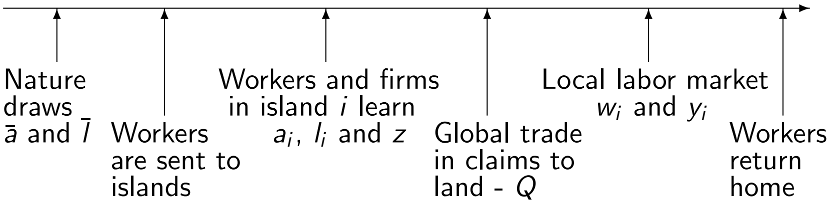
2. **Labor Endowment shock:**

$$l_i \equiv \log L_i = \bar{l} + \xi_{l,i}$$

- $\bar{l} \sim N\left(0, \sigma^2_{L,0}\right)$ is an aggregate shock to land endowment
- $\xi_{l,i} \overset{iid}{\sim} N\left(0, \sigma^2_{L,x}\right)$ is a purely idiosyncratic land shock

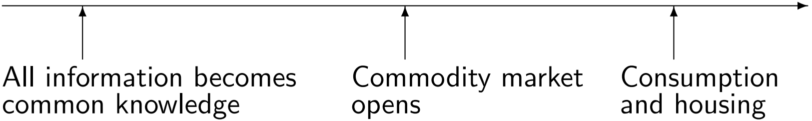
Timing

Stage 1



Timing

Stage 2



Information Structure

Stage 1:

- Aggregate productivity and land supply are not observed.
- Workers and firms know perfectly their local productivity and their local land endowment.
- Public signal of aggregate productivity

$$z = \overline{a} + \varepsilon,\ \varepsilon \sim N\left(0, \sigma^2_{A,z}\right)$$

$\varepsilon =$ noise, correlated errors in expectations.

- Price of land in the global market Q is observed by everyone.
- All variances and distributions are common knowledge

Stage 2 :

- Everything is common knowledge

- Let $\omega = (a_i, l_i, z)$ be all exogenous information available in island i.
- An island of type $\omega = (a_i, l_i, z)$ has productivity a_i and land endowment l_i when the public signal is z.
- Aggregate state: $\Omega = (\bar{a}, \bar{l}, z)$
- The distribution of types ω among islands depends on Ω Abuse of notation: Distribution of ω given Ω is $\Omega(\omega)$

Household's Problem

Representative household.

$$\max_{h(\omega,Q),n(\omega,Q)} E\left(\begin{array}{c} C(\Omega)^{1-\gamma}\left(\left[\int_J h(\omega,Q)^{\frac{\nu-1}{\nu}}\,d\Omega(\omega)\right]^{\frac{\nu}{\nu-1}}\right)^{\gamma} \\ -\int_J n(\omega,Q)\,d\Omega(\omega) \end{array}\right)$$

where

$$\begin{aligned} PC(\Omega) &= \int \pi(\omega,Q)\,d\Omega(\omega)+\int_J w(\omega,Q)\,n(\omega,Q)\,d\Omega(\omega) \\ &\quad -Q\int_J h(\omega,Q)\,d\Omega(\omega) \end{aligned}$$

Firm's Problem

$$\max_{y(\omega,Q),n(\omega,Q)} E\left[\lambda\left(\Omega\right)\left(y\left(\omega,Q\right)-w\left(\omega,Q\right)n\left(\omega,Q\right)\right)|\omega,Q\right]$$

s.t.

$$\begin{aligned} y\left(\omega,Q\right) &= A\left(\omega\right)n\left(\omega,Q\right)^{\theta} \\ w\left(\omega,Q\right)n\left(\omega,Q\right) &\leq \chi QL\left(\omega\right) \end{aligned}$$

where where $\lambda\left(\Omega\right)$ is the household's marginal value of wealth when aggregate state is Ω.

Allocation rule and pricing functions

An **allocation rule** is :

- an employment strategy $n(\omega, Q)$,
- a housing demand strategy $h(\omega, Q)$
- a production strategy $y(\omega, Q)$
- and a consumption strategy $C(\Omega)$.

The **pricing functions** in this economy are

- a wage function $w(\omega, Q)$
- a land price $Q(\Omega)$
- and a commodity price $P(\Omega)$ normalized to 1

Equilibrium

Definition

An equilibrium consists of an allocation rule and pricing functions such that:

- given the production choices in stage 1, $C(\Omega)$ and $P(\Omega)$ are a Walrasian equilibrium for the complete information exchange economy in stage 2
- for a given realization of the land price Q, $n(\omega, Q)$, $y(\omega, Q)$ and $w(\omega, Q)$ are a Bayesian-Nash equilibrium for the incomplete-information game played in each island in stage 1
- Q and $h(\omega, Q)$ are a Noisy Rational Expectations equilibrium in the land market in stage 1.

Output

Given asset price Q, output is given by

$$y(\omega, Q) = \min\left\{y^u(\omega, Q) . y^c(\omega, Q)\right\}$$

where

$$\begin{aligned} y^u(\omega, Q) &= A(\omega)^{\frac{1}{1-\theta}}\left[\frac{\theta}{w(\omega, Q)}\right]^{\frac{\theta}{1-\theta}} \\ y^c(\omega, Q) &= A(\omega)\left[\chi Q \frac{L(\omega)}{w(\omega, Q)}\right]^{\theta} \end{aligned}$$

and

$$w(\omega, Q) = E\left[(1-\gamma)\left(\frac{H(\Omega)}{Y(\Omega)}\right)^{\gamma} \middle| \omega, Q\right]^{-1}$$

Lemma

In equilibrium, the land price is given by

$$\log Q(\Omega) = \left(\overline{E}\log Y(\Omega) - \overline{E}\log L(\Omega)\right) - \frac{1}{v}\left(\log L(\Omega) - \overline{E}\log L(\Omega)\right)$$

where $\overline{E}$ denotes the average expectation in the population.

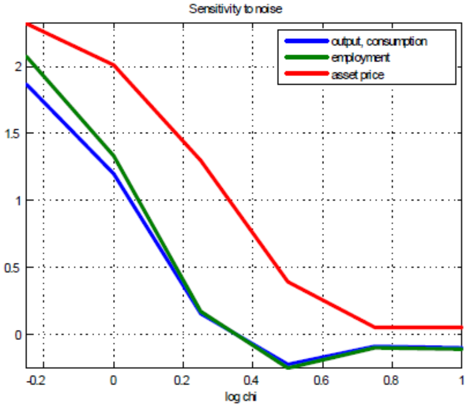
Mechanism

There is a two way feedback between the financial and real sides of the economy generated by collateral constraints.

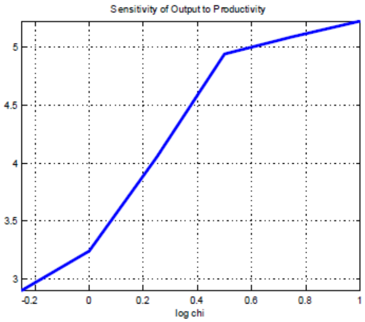
- The production of a constrained firm is increasing in the asset price.
- The asset price is increasing in the average expectation of aggregate output.

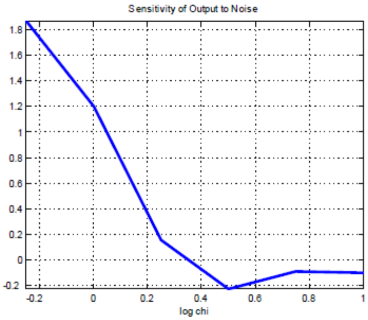
Remark: Amplification? The price of land affects expectations by aggregating information when information is dispersed.

Positive Co-Movement

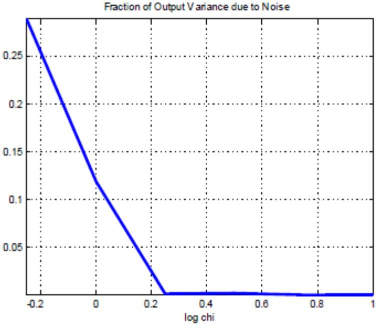


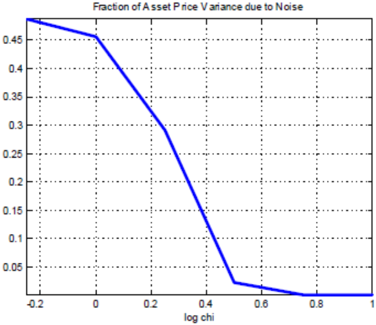
Contribution of Noise





Variance of Output and Asset Prices





Substitutability and Complementarity

If workers are optimistic about aggregate income (high ε):

- Negative income effect on labor, reduces employment and output.$\Longrightarrow$Source of **strategic substitutability.**
- Drives the price of land up, relaxing the collateral constraint and increasing the production of constraint firms.$\Longrightarrow$ Source of **strategic complementarity.**

The second effect is only present in the constraint economy. Interaction between dispersed information and collateral constraints introduces strategic complementarity.