Collateral Constraints and Noisy Fluctuations
by Jennifer La’O - Job Market Paper 2010

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January 2010
Motivation

- Little variation of output explained by changes in productivity. 
- "Demand Shocks": shock orthogonal to technology shock that causes positive co-movement between output, employment and consumption.
**Aim of the paper:** Provide a theory of how financial frictions may imply movements in both the business cycle and asset prices not driven by fundamentals.

Candidates:

*Noise + Collateral constraints*
The Model

- 2 stages $t = 1, 2$
- 2 types of agents:
  - Continuum of workers grouped in a continuum of households or "families"
  - Continuum of competitive representative firms each of whom lives in an island. $i \in I$
- 2 goods traded in global markets:
  - output (produced by firms): price $P$
  - Land or Housing: price $Q$
- Local labor markets, one market in each island: wage $w_i$
Households

- Preferences of the household $k$

$$\mathcal{U}_k = C_k^{1-\gamma} H_k^\gamma - N_k$$

- $C_k$ is the household’s consumption of the good produced by firms
- $H_k$ is the composite of housing $h_{jk}$ purchased by each member $i$ of the household $k$

$$H_k = \left[\int_j h_{jk} \frac{v-1}{v} dj\right]^{\frac{\nu}{\nu-1}}$$

- $N_k$ is the composite of the labor $n_{jk}$ supplied by workers $j$ in household $k$

$$N_k = \int_j n_{jk} dj$$

- Budget constraint

$$PC_k + Q \int_j h_{jk} dj \leq \int_i \pi_i di + \int_j w_j n_{jk} dj$$
Islands and Firms

- Continuum of islands \( i \in I \)
- Islands determine boundaries for labor market and information
- 1 representative competitive firm in each island
- Production function in island \( i \)

\[ y_i = A_i n_i^\theta \]

where \( A_i \) is the productivity level on island \( i \) and \( n_i \) is the firm’s employment level.

- Each firm is endowed with \( L_i \) units of land.
- Profits

\[ \pi_i = P y_i - w_i n_i + Q L_i \]
Collateral Constraints

- Firms cannot commit to repay workers after they produced
- Firms use land holdings as collateral to pay workers

$$w_i n_i \leq \chi QL_i$$

where $\chi > 0$ is the fraction of land that can be collateralizable.
Shocks

Each island economy is subject to 2 shocks

1. **Technology shock:**
   \[ a_i \equiv \log A_i = \bar{a} + \zeta_{a,i} \]
   
   - \( \bar{a} \sim N(0, \sigma_{A,0}^2) \) is an aggregate shock to productivity
   - \( \zeta_{a,i} \overset{iid}{\sim} N(0, \sigma_{A,x}^2) \) is a purely idiosyncratic productivity shock

2. **Labor Endowment shock:**
   \[ l_i \equiv \log L_i = \bar{l} + \zeta_{l,i} \]
   
   - \( \bar{l} \sim N(0, \sigma_{L,0}^2) \) is an aggregate shock to land endowment
   - \( \zeta_{l,i} \overset{iid}{\sim} N(0, \sigma_{L,x}^2) \) is a purely idiosyncratic land shock
**Stage 1**

Nature draws $\tilde{a}$ and $\tilde{l}$

Workers are sent to islands

Workers and firms in island $i$ learn $a_i$, $l_i$ and $z$

Local labor market $w_i$ and $y_i$

Global trade in claims to land - $Q$

Workers return home
Timing

Stage 2

- All information becomes common knowledge
- Commodity market opens
- Consumption and housing
Information Structure

Stage 1:

- Aggregate productivity and land supply are not observed.
- Workers and firms know perfectly their local productivity and their local land endowment.
- Public signal of aggregate productivity

\[ z = \bar{a} + \varepsilon, \varepsilon \sim N(0, \sigma^2_{A,z}) \]

\( \varepsilon \) = noise, correlated errors in expectations.
- Price of land in the global market \( Q \) is observed by everyone.
- All variances and distributions are common knowledge

Stage 2:

- Everything is common knowledge
• Let $\omega = (a_i, l_i, z)$ be all exogenous information available in island $i$.

• An island of type $\omega = (a_i, l_i, z)$ has productivity $a_i$ and land endowment $l_i$ when the public signal is $z$.

• Aggregate state: $\Omega = (\bar{a}, \bar{l}, z)$

• The distribution of types $\omega$ among islands depends on $\Omega$.

Abuse of notation: Distribution of $\omega$ given $\Omega$ is $\Omega(\omega)$
Household’s Problem

Representative household.

$$\max_{h(\omega,Q), n(\omega,Q)} \mathbb{E} \left( C(\Omega)^{1-\gamma} \left( \left[ \int J h(\omega, Q) \frac{v-1}{v} d\Omega(\omega) \right]^{\frac{v}{v-1}} - \int J n(\omega, Q) d\Omega(\omega) \right)^\gamma \right)$$

where

$$PC(\Omega) = \int \pi(\omega, Q) d\Omega(\omega) + \int J w(\omega, Q) n(\omega, Q) d\Omega(\omega) - Q \int J h(\omega, Q) d\Omega(\omega)$$
Firm’s Problem

\[
\max_{y(\omega,Q), n(\omega,Q)} E \left[ \lambda(\Omega) (y(\omega,Q) - w(\omega,Q)n(\omega,Q)) \right] |\omega, Q]
\]

s.t.

\[
y(\omega,Q) = A(\omega)n(\omega,Q)^\theta
\]
\[
w(\omega,Q)n(\omega,Q) \leq \chi Q L(\omega)
\]

where \( \lambda(\Omega) \) is the household’s marginal value of wealth when aggregate state is \( \Omega \).
Allocation rule and pricing functions

An allocation rule is:

- an employment strategy \( n(\omega, Q) \),
- a housing demand strategy \( h(\omega, Q) \),
- a production strategy \( y(\omega, Q) \),
- and a consumption strategy \( C(\Omega) \).

The pricing functions in this economy are

- a wage function \( w(\omega, Q) \),
- a land price \( Q(\Omega) \),
- and a commodity price \( P(\Omega) \) normalized to 1.
Equilibrium

Definition
An equilibrium consists of an allocation rule and pricing functions such that:

- given the production choices in stage 1, \( C(\Omega) \) and \( P(\Omega) \) are a Walrasian equilibrium for the complete information exchange economy in stage 2
- for a given realization of the land price \( Q, n(\omega, Q), y(\omega, Q) \) and \( w(\omega, Q) \) are a Bayesian-Nash equilibrium for the incomplete-information game played in each island in stage 1
- \( Q \) and \( h(\omega, Q) \) are a Noisy Rational Expectations equilibrium in the land market in stage 1.
Output

Given asset price $Q$, output is given by

$$y(\omega, Q) = \min \{ y^u(\omega, Q) \cdot y^c(\omega, Q) \}$$

where

$$y^u(\omega, Q) = A(\omega)^{\frac{1}{1-\theta}} \left[ \frac{\theta}{w(\omega, Q)} \right]^{\frac{\theta}{1-\theta}}$$

$$y^c(\omega, Q) = A(\omega) \left[ \chi Q \frac{L(\omega)}{w(\omega, Q)} \right]^\theta$$

and

$$w(\omega, Q) = E \left[ (1 - \gamma) \left( \frac{H(\Omega)}{Y(\Omega)} \right)^\gamma \mid \omega, Q \right]^{-1}$$
Lemma

In equilibrium, the land price is given by

$$\log Q(\Omega) = \left( \bar{E} \log Y(\Omega) - \bar{E} \log L(\Omega) \right) - \frac{1}{V} \left( \log L(\Omega) - \bar{E} \log L(\Omega) \right)$$

where $\bar{E}$ denotes the average expectation in the population.
**Mechanism**

There is a two way feedback between the financial and real sides of the economy generated by collateral constraints.

- The production of a constrained firm is increasing in the asset price.
- The asset price is increasing in the average expectation of aggregate output.

**Remark:** Amplification? The price of land affects expectations by aggregating information when information is dispersed.
Positive Co-Movement
Contribution of Noise

![Graph 1: Sensitivity of Output to Productivity](image1)

![Graph 2: Sensitivity of Output to Noise](image2)
Variance of Output and Asset Prices
Substitutability and Complementarity

If workers are optimistic about aggregate income (high $\varepsilon$):

- Negative income effect on labor, reduces employment and output. $\implies$ Source of **strategic substitutability**.
- Drives the price of land up, relaxing the collateral constraint and increasing the production of constraint firms. $\implies$ Source of **strategic complementarity**.

The second effect is only present in the constraint economy. Interaction between dispersed information and collateral constraints introduces strategic complementarity.