A simple model of trading and pricing risky assets under ambiguity: any lessons for policy-makers?

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Introduction

- How does the presence of ambiguity averse investors influence asset prices?

- What are the policy implications?

Model: Assets

- Risk-free bond in zero net supply, trades at period zero at unit price, and yields a return r^f in period one.

- Risky asset in positive supply $\bar{z}$, trades at time zero at price q, and yields stochastic payoff d in period one

$$d \sim \mathcal{N}\left(\mu_I + \mu_S, \sigma_I^2 + \sigma_S^2\right)$$

Model: Agents

- Two types of agents: $m = SEU, AA$ ($(1-\alpha)$ and α respectively).

- Period-zero budget constraint

$$w_0^m = qz^m + b^m$$

- End-of-period-one real wealth

$$w_1^m = z^m \frac{d}{1+i} + b^m \left(1 + r^f\right)$$

- SEU investors perceive w_1^{SEU} as

$$w_1^{SEU} \sim \mathcal{N}\left(z^{SEU} \frac{\mu_I + \mu_S}{1+i} + b^{SEU}\left(1+r^f\right), \left(z^{SEU}\right)^2 \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2}\right)$$

Model: SEU Asset Demand

$$\max_{z^{SEU}, b^{SEU}} E^{SEU}\left(-\exp\left(-w_1^{SEU}\right)\right) =$$

$$\max_{z^{SEU}, b^{SEU}} \left[z^{SEU}\left(\frac{\mu_I + \mu_S}{1+i} - q\right) + b^{SEU} r^f + w_0 - \frac{1}{2}\left(z^{SEU}\right)^2 \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2}\right]$$

Model: SEU Asset Demand

FOC

$$\left(\frac{\mu_I + \mu_S}{1+i} - q\right) - z^{SEU}\frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2} = 0$$

FOC should be

$$\left[\frac{\mu_I + \mu_S}{1+i} - q\left(1 + r^f\right)\right] - z^{SEU}\frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2} = 0$$

Model: SEU Asset Demand

$$z^{SEU} = \begin{cases} \frac{(1+i)[\mu_I+\mu_S-q(1+i)]}{\sigma_I^2+\sigma_S^2} > 0 & \text{if} \quad \frac{\mu_I+\mu_S}{1+i} > q \\ 0 & \text{if} \quad \frac{\mu_I+\mu_S}{1+i} = q \\ \frac{(1+i)[\mu_I+\mu_S-q(1+i)]}{\sigma_I^2+\sigma_S^2} < 0 & \text{if} \quad \frac{\mu_I+\mu_S}{1+i} < q \end{cases}$$

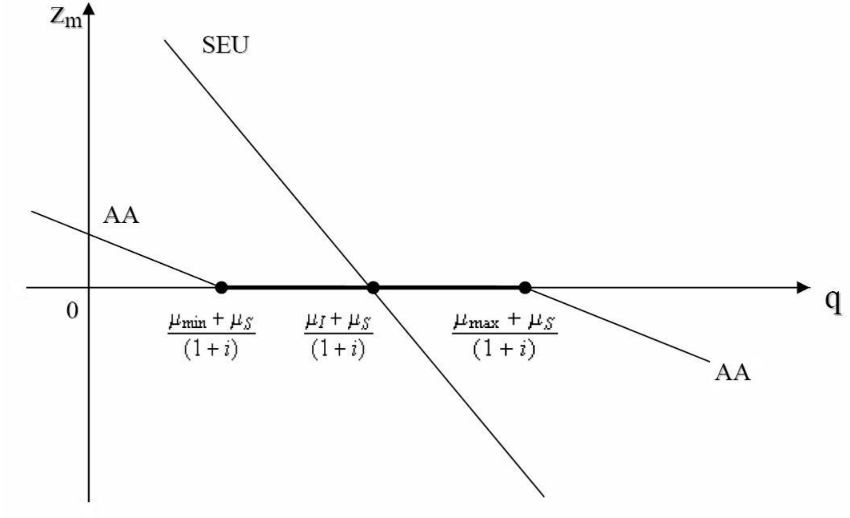
Model: AA Asset Demand

$$\max_{z^{AA},b^{AA}} \min_{(\mu_I,\sigma_I^2)\in\{\mu_1,\dots,\mu_P\}\times\{\sigma_1^2,\dots,\sigma_Q^2\}} E^{AA}\left(-\exp\left(-w_1^{AA}\right)\right) =$$

$$\max_{z^{AA},b^{AA}} \min_{(\mu_I,\sigma_I^2)\in\{\mu_1,\dots,\mu_P\}\times\{\sigma_1^2,\dots,\sigma_Q^2\}} \left[z^{AA}\left(\frac{\mu_I+\mu_S}{1+i}-q\right)+b^{AA}r^f+w_0-\frac{\left(z^{AA}\right)^2\left(\sigma_I^2+\sigma_S^2\right)}{2\left(1+i\right)^2}\right]$$

$$z^{AA}=\begin{cases} \frac{(1+i)[\mu_{\text{min}}+\mu_S-q(1+i)]}{\sigma_{\text{max}}^2+\sigma_S^2} & \text{if} \quad \frac{\mu_{\text{min}}+\mu_S}{1+i}>q \\ 0 & \text{if} \quad \frac{\mu_{\text{min}}+\mu_S}{1+i}\le q\le\frac{\mu_{\text{max}}+\mu_S}{1+i} \\ \frac{(1+i)[\mu_{\text{max}}+\mu_S-q(1+i)]}{\sigma_{\text{max}}^2+\sigma_S^2} & \text{if} \quad \frac{\mu_{\text{max}}+\mu_S}{1+i}<q \end{cases}$$

Model: Demand Functions



Model: Equilibrium Asset Prices - case 1

If the following condition holds

$$\mu_{\min} \leq \mu_I - \frac{\overline{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1+i)}$$

then only SEU investors participate in the market and the equilibrium price is

$$q_1^* = \frac{\mu_I + \mu_S}{1+i} - \frac{\overline{z}\left(\sigma_I^2 + \sigma_S^2\right)}{(1-\alpha)(1+i)^2}$$

Model: Equilibrium Asset Prices - case 2

If

$$\mu_{\min} > \mu_I - \frac{\overline{z}\left(\sigma_I^2 + \sigma_S^2\right)}{\left(1-\alpha\right)\left(1+i\right)}$$

then both groups of investors participate, and the equilibrium price is

$$\begin{aligned} q_2^* \quad = \quad & \frac{\left[\alpha\mu_{\min}\left(\sigma_I^2 + \sigma_S^2\right) + \mu_S\left(\alpha\sigma_I^2 + \left(1-\alpha\right)\sigma_{\max}^2 + \sigma_S^2\right) + \left(1-\alpha\right)\mu_I\left(\sigma_{\max}^2 + \sigma_S^2\right)\right]}{\left(1+i\right)\left[\sigma_S^2 + \alpha\sigma_I^2 + \left(1-\alpha\right)\sigma_{\max}^2\right]} \\ & - \frac{\overline{z}\left(\sigma_I^2 + \sigma_S^2\right)\left(\sigma_{\max}^2 + \sigma_S^2\right)}{\left(1+i\right)^2\left[\sigma_S^2 + \alpha\sigma_I^2 + \left(1-\alpha\right)\sigma_{\max}^2\right]} \end{aligned}$$

Risk Premia Under Ambiguity

- Define the real risk premium (under $r^f = 0$) as

$$E_{(j)}\left(1+\widetilde{R}\right) \equiv \frac{E\left(d\right)}{\left(1+i\right)q^*}$$

- In the case of equilibrium with *SEU* only

$$E_{(1)}\left(1+\widetilde{R}\right) = \frac{E^{SEU}\left(d\right)}{\left(1+i\right)q_1^*}$$

- In the equilibrium with participation of both types

$$E_{(2)}\left(1+\widetilde{R}\right) = \frac{\alpha E^{AA}\left(d\right)+\left(1-\alpha\right)E^{SEU}\left(d\right)}{\left(1+i\right)q_2^*}$$

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- In turns out that $E_{(2)}\left(1+\widetilde{R}\right) > E_{(1)}\left(1+\widetilde{R}\right)$

Affecting the diffusion of AA behaviours (α)

- Both types of investors participate initially,
  $\alpha \downarrow \Rightarrow q_2^* \uparrow \Rightarrow q_2^*$ may enter into the region of nonparticipation by AA investors.
- If initially only SEU investors participate,
  $\alpha \uparrow \Rightarrow q_1^* \downarrow \Rightarrow q_1^*$ may enter into a region where AA investors are willing to participate.

Changing the amount of ambiguity-relevant uncertainty $\left(\mu_{\min} \text{ and } \sigma^2_{\max}\right)$

- If initially only SEU investors participate in the market, then variation in $\sigma^2_{\max}$ has no effect on the market.

- If initially both groups of investors are participating, $\sigma^2_{\max} \downarrow \Rightarrow z^{AA} \uparrow \Rightarrow q_2^* \uparrow \Rightarrow q_2^*$ may enter into the region of nonparticipation by AA investors.

Generalizations?

- Ambiguity about the distribution of the systematic component of asset payoff (μ_S, σ_S^2)?
- Welfare implications?

If only SEU investors participate

$$q_1^* = \frac{\mu_I+\mu_S}{1+i} - \frac{\overline{z}\left(\sigma_I^2+\sigma_S^2\right)}{(1-\alpha)(1+i)^2}$$
$$z^{AA} = 0$$
$$z^{SEU} = \frac{1}{(1-\alpha)}\overline{z}$$
$$b^{AA} = w_0^{AA}$$
$$b^{SEU} = w_0^{SEU} - q_1^* z^{SEU} = w_0^{SEU} - \left(\frac{\mu_I+\mu_S}{1+i} - \frac{\overline{z}\left(\sigma_I^2+\sigma_S^2\right)}{(1-\alpha)(1+i)^2}\right)\frac{1}{(1-\alpha)}\overline{z}$$

$$\alpha b^{AA} + (1-\alpha)\, b^{SEU} = 0 \Rightarrow$$
$$\alpha w_0^{AA} + (1-\alpha)\left[w_0^{SEU} - \left(\frac{\mu_I+\mu_S}{1+i} - \frac{\overline{z}\left(\sigma_I^2+\sigma_S^2\right)}{(1-\alpha)(1+i)^2}\right)\frac{1}{(1-\alpha)}\overline{z}\right] = 0$$
$$\alpha w_0^{AA} + (1-\alpha)\, w_0^{SEU} = \overline{z}\left(\frac{\mu_I+\mu_S}{1+i} - \frac{\overline{z}\left(\sigma_I^2+\sigma_S^2\right)}{(1-\alpha)(1+i)^2}\right)$$