A simple model of trading and pricing risky assets under ambiguity: any lessons for policy-makers?

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Introduction

- How does the presence of ambiguity averse investors influence asset prices?

- What are the policy implications?
Model: Assets

- Risk-free bond in zero net supply, trades at period zero at unit price, and yields a return $r^f$ in period one.

- Risky asset in positive supply $\bar{z}$, trades at time zero at price $q$, and yields stochastic payoff $d$ in period one

$$d \sim \mathcal{N} (\mu_I + \mu_S, \sigma_I^2 + \sigma_S^2)$$
Model: Agents

- Two types of agents: \( m = SEU, AA \) \( (1 - \alpha) \) and \( \alpha \) respectively.

- Period-zero budget constraint

\[
\begin{align*}
  w_0^m &= qz^m + b^m
\end{align*}
\]

- End-of-period-one real wealth

\[
\begin{align*}
  w_1^m &= z^m \frac{d}{1+i} + b^m \left(1 + r^f\right)
\end{align*}
\]

- SEU investors perceive \( w_1^{SEU} \) as

\[
\begin{align*}
  w_1^{SEU} &\sim \mathcal{N} \left( z^{SEU} \frac{\mu_I + \mu_S}{1+i} + b^{SEU} \left(1 + r^f\right), \left(z^{SEU}\right)^2 \frac{\sigma_I^2 + \sigma_S^2}{(1+i)^2} \right)
\end{align*}
\]
Model: SEU Asset Demand

\[
\max_{z^{SEU}, b^{SEU}} \left[ z^{SEU} \left( \frac{\mu_I + \mu_S}{1 + i} - q \right) + b^{SEU} r^f + w_0 - \frac{1}{2} \left( z^{SEU} \right)^2 \frac{\sigma_I^2 + \sigma_S^2}{(1 + i)^2} \right]
\]

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Model: SEU Asset Demand

FOC

\[
\left( \frac{\mu_I + \mu_S}{1 + i} - q \right) - z^{SEU} \frac{\sigma_I^2 + \sigma_S^2}{(1 + i)^2} = 0
\]

FOC should be

\[
\left[ \frac{\mu_I + \mu_S}{1 + i} - q \left(1 + r^f\right) \right] - z^{SEU} \frac{\sigma_I^2 + \sigma_S^2}{(1 + i)^2} = 0
\]
Model: SEU Asset Demand

\[ z^{SEU} = \begin{cases} \frac{(1+i)[\mu_I+\mu_S-q(1+i)]}{\sigma_I^2+\sigma_S^2} > 0 & \text{if } \frac{\mu_I+\mu_S}{1+i} > q \\ 0 & \text{if } \frac{\mu_I+\mu_S}{1+i} = q \\ \frac{(1+i)[\mu_I+\mu_S-q(1+i)]}{\sigma_I^2+\sigma_S^2} < 0 & \text{if } \frac{\mu_I+\mu_S}{1+i} < q \end{cases} \]

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Model: AA Asset Demand

\[
\max_{z^{AA}, b^{AA}} \min_{(\mu_I, \sigma_I^2) \in \{\mu_1, \ldots, \mu_P\} \times \{\sigma_1^2, \ldots, \sigma_Q^2\}} E^{AA} \left(-\exp \left(-w_1^{AA}\right)\right) = \\
\min_{z^{AA}} \max_{b^{AA}} \left[ z^{AA} \left(\frac{\mu_I + \mu_S}{1 + i} - q\right) + b^{AA} r^f + w_0 - \frac{(z^{AA})^2 (\sigma_i^2 + \sigma_S^2)}{2 (1 + i)^2} \right]
\]

\[
z^{AA} = \begin{cases} 
\frac{(1+i)[\mu_{\min} + \mu_S - q(1+i)]}{\sigma_{\max}^2 + \sigma_S^2} & \text{if} \quad \frac{\mu_{\min} + \mu_S}{1+i} > q \\
0 & \text{if} \quad \frac{\mu_{\min} + \mu_S}{1+i} \leq q \leq \frac{\mu_{\max} + \mu_S}{1+i} \\
\frac{(1+i)[\mu_{\max} + \mu_S - q(1+i)]}{\sigma_{\max}^2 + \sigma_S^2} & \text{if} \quad \frac{\mu_{\max} + \mu_S}{1+i} < q
\end{cases}
\]

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If the following condition holds

\[ \mu_{\min} \leq \mu_I - \frac{\bar{z} (\sigma^2_I + \sigma^2_S)}{(1 - \alpha)(1 + i)} \]

then only SEU investors participate in the market and the equilibrium price is

\[ q^*_1 = \frac{\mu_I + \mu_S}{1 + i} - \frac{\bar{z} (\sigma^2_I + \sigma^2_S)}{(1 - \alpha)(1 + i)^2} \]
Model: Equilibrium Asset Prices - case 2

If

\[ \mu_{\text{min}} > \mu_l - \frac{z (\sigma_l^2 + \sigma_S^2)}{(1 - \alpha) (1 + i)} \]

then both groups of investors participate, and the equilibrium price is

\[
q^*_2 = \frac{\left[ \alpha \mu_{\text{min}} (\sigma_l^2 + \sigma_S^2) + \mu_S (\alpha \sigma_l^2 + (1 - \alpha) \sigma_{\text{max}}^2 + \sigma_S^2) + (1 - \alpha) \mu_l (\sigma_{\text{max}}^2 + \sigma_S^2) \right]}{(1 + i) \left[ \sigma_S^2 + \alpha \sigma_l^2 + (1 - \alpha) \sigma_{\text{max}}^2 \right]} \]

\[ - \frac{\bar{z} (\sigma_l^2 + \sigma_S^2) (\sigma_{\text{max}}^2 + \sigma_S^2)}{(1 + i)^2 \left[ \sigma_S^2 + \alpha \sigma_l^2 + (1 - \alpha) \sigma_{\text{max}}^2 \right]} \]
Risk Premia Under Ambiguity

- Define the real risk premium (under \( r^f = 0 \)) as
  
  \[
  E(j) \left( 1 + \tilde{R} \right) \equiv \frac{E(d)}{(1 + i) q^*}
  \]

- In the case of equilibrium with SEU only
  
  \[
  E(1) \left( 1 + \tilde{R} \right) = \frac{E^{SEU}(d)}{(1 + i) q_1^*}
  \]

- In the equilibrium with participation of both types
  
  \[
  E(2) \left( 1 + \tilde{R} \right) = \frac{\alpha E^{AA}(d) + (1 - \alpha) E^{SEU}(d)}{(1 + i) q_2^*}
  \]
**Risk Premia Under Ambiguity**

- Define the real risk premium (under $r^f = 0$) as
  \[
  E(j) \left(1 + \tilde{R}\right) \equiv \frac{E(d)}{(1 + i)q^*}
  \]

- In the case of equilibrium with SEU only
  \[
  E(1) \left(1 + \tilde{R}\right) = \frac{E^{SEU}(d)}{(1 + i)q^*_1}
  \]

- In the equilibrium with participation of both types
  \[
  E(2) \left(1 + \tilde{R}\right) = \frac{\alpha E^{AA}(d) + (1 - \alpha) E^{SEU}(d)}{(1 + i)q^*_2}
  \]

- In turns out that $E(2) \left(1 + \tilde{R}\right) > E(1) \left(1 + \tilde{R}\right)$

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Affecting the diffusion of AA behaviours ($\alpha$)

- Both types of investors participate initially,
  $\alpha \downarrow \Rightarrow q_2^* \uparrow \Rightarrow q_2^*$ may enter into the region of nonparticipation by AA investors.

- If initially only SEU investors participate,
  $\alpha \uparrow \Rightarrow q_1^* \downarrow \Rightarrow q_1^*$ may enter into a region where AA investors are willing to participate.
Changing the amount of ambiguity-relevant uncertainty \( (\mu_{\min} \text{ and } \sigma^2_{\max}) \)

- If initially only SEU investors participate in the market, then variation in \( \sigma^2_{\max} \) has no effect on the market.

- If initially both groups of investors are participating, \( \sigma^2_{\max} \downarrow \Rightarrow z^{AA} \uparrow \Rightarrow q^*_2 \uparrow \Rightarrow q^*_2 \) may enter into the region of nonparticipation by AA investors.
Generalizations?

- Ambiguity about the distribution of the systematic component of asset payoff ($\mu_S, \sigma^2_S$)?
- Welfare implications?
If only SEU investors participate

\[ q_1^* = \frac{\mu_I + \mu_S}{1+i} - \frac{\bar{z}(\sigma_I^2 + \sigma_S^2)}{(1-\alpha)(1+i)^2} \]

\[ z^{AA} = 0 \]

\[ z^{SEU} = \frac{1}{(1-\alpha)} \bar{z} \]

\[ b^{AA} = w_0^{AA} \]

\[ b^{SEU} = w_0^{SEU} - q_1^* z^{SEU} = w_0^{SEU} - \left( \frac{\mu_I + \mu_S}{1+i} - \frac{\bar{z}(\sigma_I^2 + \sigma_S^2)}{(1-\alpha)(1+i)^2} \right) \frac{1}{(1-\alpha)} \bar{z} \]

\[ \alpha b^{AA} + (1-\alpha) b^{SEU} = 0 \Rightarrow \]

\[ \alpha w_0^{AA} + (1-\alpha) \left[ w_0^{SEU} - \left( \frac{\mu_I + \mu_S}{1+i} - \frac{\bar{z}(\sigma_I^2 + \sigma_S^2)}{(1-\alpha)(1+i)^2} \right) \frac{1}{(1-\alpha)} \bar{z} \right] = 0 \]

\[ \alpha w_0^{AA} + (1-\alpha) w_0^{SEU} = \bar{z} \left( \frac{\mu_I + \mu_S}{1+i} - \frac{\bar{z}(\sigma_I^2 + \sigma_S^2)}{(1-\alpha)(1+i)^2} \right) \]

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