Entrepreneurial Finance and Non-diversifiable Risk

Working paper 2008

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Reading Group

Oct. 14th, 2008

What is the contribution of this paper?

- The previous literature in corporate finance discusses the optimal capital structure of firm in the complete market framework. (for example Leland (1994))
- This paper discusses consumption/saving, portfolio choice, financing, investment, and endogenous default/cash-out decisions of an entrepreneur with undiversifiable idiosyncratic shock.

Why an entrepreneur?

- In the complete market, Fisher's separation theorem says that the only objective of the firm is to maximize its present value.
- In the incomplete market, the entrepreneur's risk attitude plays a crucial role for the financing and investment of the firm.

The model

- Assets in public market
  - Risk-free asset pays interest r.
  - Risky asset

$$dP_t = \mu_p P_t dt + \sigma_p P_t dB_t$$

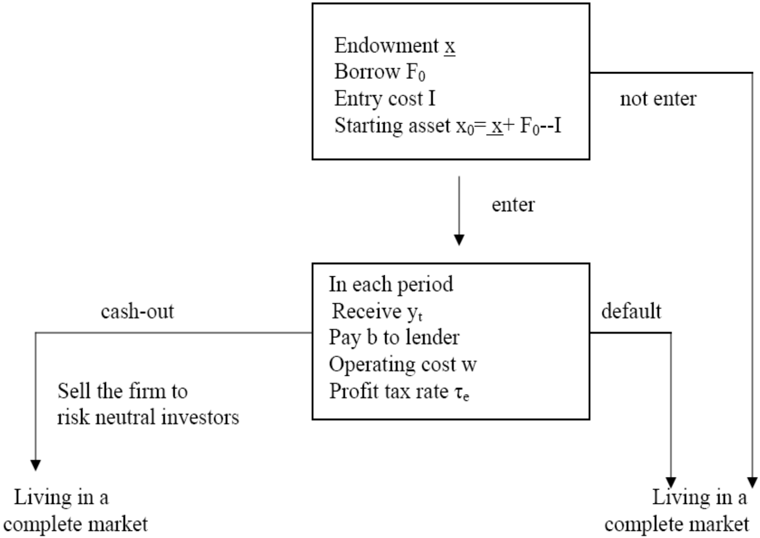
- In the private firm, the revenue process is

$$dy_t = \mu y_t dt + \omega y_t dB_t + \varepsilon y_t dZ_t$$

  where y_0 is given. Z_t is the idiosyncratic risk and is independent of market risk B_t.

- Owning a firm, the entrepreneur has to pay the debt coupon, b, and the operating cost, w. The debt has infinite maturity. The entrepreneur may default the debt and he may sell his firm.

Timing


Endowment x
Borrow F0
Entry cost I
Starting asset x0= x+ F0--I
not enter
enter
In each period
Receive yt
Pay b to lender
Operating cost w
Profit tax rate τe
cash-out
default
Sell the firm to
risk neutral investors
Living in a
complete market
Living in a
complete market

Agent's problem

- Living in a complete market

$$J^e(x) = \max_{c_t,\phi_t} E\left[\int_0^\infty e^{-\delta t}\left(-\frac{1}{\gamma}e^{-\gamma c_t}\right)dt\right]$$

$$s.t. \quad dx_t = (r(x_t - \phi_t) - c_t)dt + \phi_t(\mu_p dt + \sigma_p dB_t)$$

- Owning a firm

$$J^s(x,y) = \max_{c_t,\phi_t} E\left[\int_0^\infty e^{-\delta t}\left(-\frac{1}{\gamma}e^{-\gamma c_t}\right)dt\right]$$

$$\begin{aligned} s.t. \quad dx_t &= (r(x_t - \phi_t) + (1-\tau_e)(y_t - b - w) - c_t)dt \\ &\quad + \phi_t(\mu_p dt + \sigma_p dB_t) \end{aligned}$$

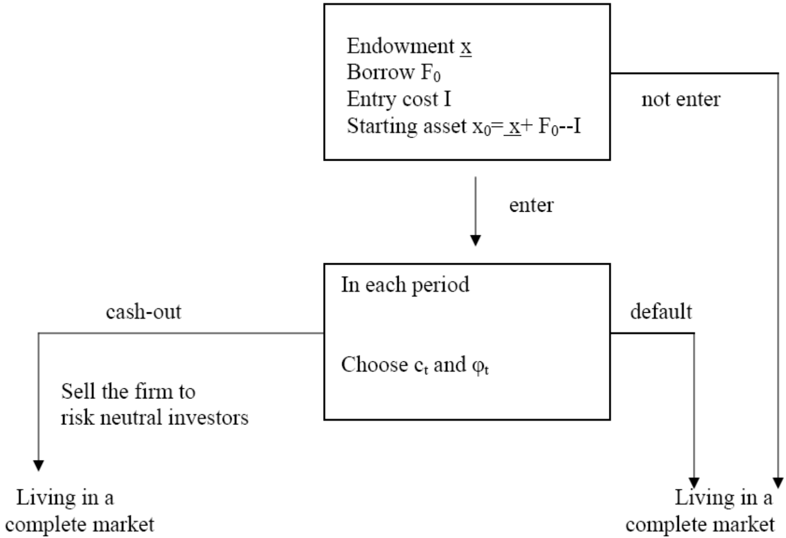
Solving the entrepreneur's utility maximization problem

- Entrepreneur's value function satisfies the HJB

$$
\begin{aligned}
\delta J^s(x,y) \quad = \quad & \max_{c,\phi}\{-\frac{1}{\gamma}e^{-\gamma c} \\
& +[rx+\phi(\mu_p-r)-c+(1-\tau_e)(y-b-w)]J^s_x(x,y) \\
& +\mu y J^s_y(x,y)+\frac{(\sigma_p\phi)^2}{2}J^s_{xx}(x,y) \\
& +\frac{(\sigma y)^2}{2}J^s_{yy}(x,y)+\phi\sigma_p\omega y J^s_{xy}(x,y)\}
\end{aligned}
$$

- Boundary conditions
  - Default
  - Cash-out

Timing


Endowment x
Borrow F0
Entry cost I
Starting asset x0= x+ F0--I
not enter
enter
In each period
Choose ct and φt
cash-out
default
Sell the firm to
risk neutral investors
Living in a
complete market
Living in a
complete market

Optimal default

- After the entrepreneur defaults at time T_d, he exits and lives in a complete market.

$$J^s(x_{T_d}, y_{T_d}) = J^e(x_{T_d})$$

- The entrepreneur optimally chooses the default time. Smooth pasting condition

$$\left.\frac{\partial J^s(x,y)}{\partial x}\right|_{T_d} = \left.\frac{\partial J^e(x)}{\partial x}\right|_{T_d}$$

and

$$\left.\frac{\partial J^s(x,y)}{\partial y}\right|_{T_d} = \left.\frac{\partial J^e(x)}{\partial y}\right|_{T_d}$$

- The smooth pasting conditions give us $y_d(x)$. Generally, $y_d(x)$ depends on x. Thanks to the exponential utility, $y_d(x)$ is a constant.

Optimal cash-out

- At the cash-out time T_u, the entrepreneur
  - sells firm to well diversified investors and collects firm value $V^*(y)$,
  - pays the fixed cost K,
  - pays retire debt at par F_0,
  - and pays capital gains taxes at the rate of τ_g.
- After cash-out the entrepreneur's asset

$$x_{T_u} = x_{T_u -} + V^*(y_{T_u}) - F_0 - K - \tau_g[V^*(y_{T_u}) - K - I]$$

Optimal cash-out

- Boundary condition

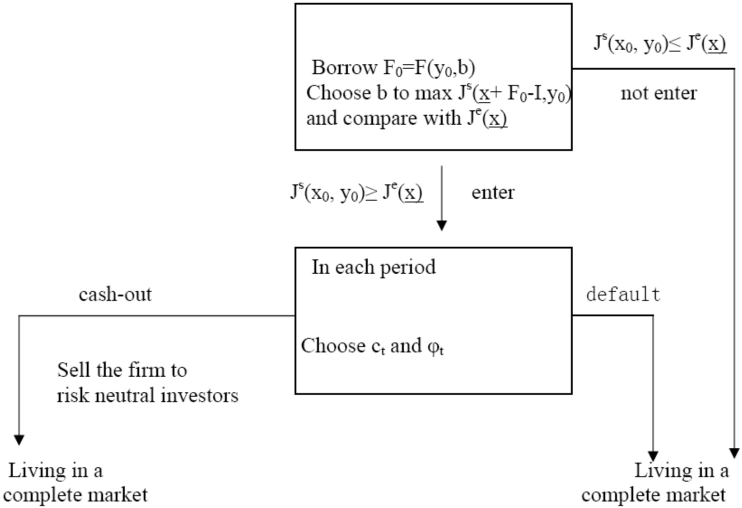
$$J^s(x,y) = J^e(x + V^*(y) - F_0 - K - \tau_g[V^*(y) - K - I])$$

- Smooth pasting condition

$$\frac{\partial J^s(x,y)}{\partial x} = \frac{\partial J^e(x + V^*(y) - F_0 - K - \tau_g[V^*(y) - K - I])}{\partial x}$$

$$\frac{\partial J^s(x,y)}{\partial y} = \frac{\partial J^e(x + V^*(y) - F_0 - K - \tau_g[V^*(y) - K - I])}{\partial y}$$

Timing


Borrow $F_0=F(y_0,b)$
Choose b to max $J^s(\underline{x}+F_0-I,y_0)$
and compare with $J^e(\underline{x})$
$J^s(x_0, y_0) \leq J^e(\underline{x})$
not enter
$J^s(x_0, y_0) \geq J^e(\underline{x})$
enter
In each period
Choose c_t and φ_t
cash-out
Sell the firm to
risk neutral investors
default
Living in a
complete market
Living in a
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Numerical example