

# Long-Run Labor Supply and the Elasticity of Intertemporal Substitution for Consumption

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We suspect that these could maintain simultaneously:

- A) Consumption and labor are additively separable in an additively time-separable utility function.
- B) The elasticity of intertemporal substitution for consumption is relatively low-well below 1.
- C) Long-run labor supply is not totally inelastic. Income and substitution effects are not both zero. But they cancel.

# Given A) separable utility function

- And B) empirical estimates of the elasticity of intertemporal substitution found quite low values. Hall (1988)

$$\Delta \ln(C_t) = s(r_t - \rho) + \varepsilon_t + \theta \varepsilon_{t-1}$$

Hall (1988) gets point estimates of EIS,  $s$ , equal to 0.1 or 0.2 that are not significantly different from zero.  $s=0.2$  gives us

$$U(C, N) = -C^{-4} - v(N)$$

where  $v(N)$  is a convex function of labor  $N$ .

- The implied real consumption wage is

$$\frac{W}{P_C} = -\frac{U_N(C, N)}{U_C(C, N)} = C^5 v'(N)$$

Per capita consumption  $C$  has roughly doubled in the 35 years since 1960. The average work hours  $N$  has stayed fairly constant. Thus, this functional form implies, counterfactually, that the real consumption wage should have increased by a factor of

$$2^5 = 32$$

over that time period!

# Theory

- Make the equality of income and substitution effects on labor supply a maintained assumption when estimating the elasticity of intertemporal substitution in consumption.
- This assumption implies a real wage proportional to consumption times some function of the quantity of labor

$$\frac{W}{P_C} = -\frac{U_N(C, N)}{U_C(C, N)} = Cv'(N)$$

- The period utility function must be of the form

$$U(C, N) = \Phi(\ln(C) - v(N))$$

for some monotonically increasing function  $\Phi$  .

- The reasonable additional assumption of a constant elasticity of substitution in consumption when the quantity of labor is held constant narrows the utility function down to the King-Plosser-Rebelo form

$$U(C, N) = \frac{C^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N)}$$

- We write  $s = \frac{1}{\gamma}$  where s now represents the

labor-held-constant elasticity of intertemporal substitution in consumption.

- The intertemporal Euler equation

$$U_C(C_{t-1}, N_{t-1}) = E_{t-1} e^{(r_t - \rho)} U(C_t, N_t)$$

- Using the K-P-R utility function, log-linearize the Euler equation

$$\Delta c = s(r_t - \rho) + \tau(1 - s)\Delta n + \varepsilon_t + h.o.t.$$

$c = \ln(C)$ ,  $n = \ln(N)$ , and

$$N^* v'(N^*) = \left( \frac{WN}{P_C C} \right)^* = \tau$$

where  $N^*$  is the trend level of labor.

- One more rearrangement shows that this is a very simple IV estimation:

$$\Delta c - \tau \Delta n = \text{const} + s[r_t - \tau \Delta n] + \varepsilon_t$$

# Evidence: Data

- Quarterly, seasonally-adjusted, aggregate U.S. data from 1949:1-1999:2
- The real interest rate is computed as the after-tax nominal rate on three-month U.S. Treasury bills minus inflation.
- Two kinds of data for inflation: one is the ex post inflation, the other is the ex ante expectation from survey data.



**Results:**  $\Delta c - \tau \Delta n = \text{const} + s[r_t - \tau \Delta n] + \varepsilon_t + \theta \varepsilon_{t-1}$

- The results are reported for three sample periods: 1982-1999, 1949-1982 and 1949-1999.
- For the period of 1982-1999, the estimated values of EIS (Elasticity of Intertemporal Substitution) is significantly greater than zero unlike Hall (1988) and most subsequent work in this area.
- $\tau$  equals labor income divide by nominal consumption expenditure. For the data,  $\tau = 0.77$ . In the regression, they use  $\tau = 0.8$ .

- Instruments

$\Delta c(-2)$ ,  $\Delta n(-2)$ ,  $r(-2)$ ,  $\Delta y(-2)$ , and  $c(-2)-y(-2)$ .

- The estimated  $s$  is not sensitive to the instrument set used. All the results say that  $s$  is about two-third or one-half, depending on the inflation data.
- For  $s=0.5$ , the corresponding utility function is

$$-\frac{e^{v(N)}}{C}$$

- For  $s=0.67$ , the corresponding utility function is

$$-\frac{2e^{\frac{1}{2}v(N)}}{C^{\frac{1}{2}}}$$

- Adding disposable income to the regression.

$$\Delta c - \tau \Delta n = \text{cons} \tan t + s[r_t - \tau \Delta n] + \beta \Delta y + \varepsilon_t + \theta \varepsilon_{t-1}$$

- The disposable income variable is insignificant and the estimate of  $s$  is significant. This contrast with Campbell and Mankiw's (1989) rule-of-thumb hypothesis and shows that excess sensitivity does not exist.
- Statistically, we can not reject the restriction of K-P-R functional form.

# Early sample and entire sample

- For the earlier sample: 1949-1982, as well as the entire sample 1949-1999, the model is significantly less well.
- The estimates of the intertemporal elasticity of substitution are much smaller and insignificantly different from zero, akin to Hall (1988).
- The restriction of the K-P-R functional form are rejected in both cases.

# Compared with Hall (1988)

- The models are different

Hall (1988)  $\Delta \ln(C_t) = s(r_t - \rho) + \varepsilon_t + \theta \varepsilon_{t-1}$

The model of this paper

$$\Delta c - \tau \Delta n = \text{const} + s[r_t - \tau \Delta n] + \varepsilon_t + \theta \varepsilon_{t-1}$$

- The results are different

Low EIS in Hall (1988)

For the time period of 1982-1999, EIS is greater than zero in this paper.

# What if the cancellation is not exact?

- If the elasticity of real wage with respect to consumption,  $\xi$  is not unity, the model becomes

$$\Delta c - \tau \Delta n = \text{constant} + s[r_t - \xi \tau \Delta n] + \varepsilon_t + \theta \varepsilon_{t-1}$$

- They believe that  $\xi$  is close enough to 1 in order to match the long-run labor supply elasticity.

# Conclusion

- We need depart from the assumption of additive separability between consumption and labor in order to explain the fact that a permanent increase in the real wage has very little effect on long-run labor supply.
- Combining separable utility assumption and K-P-R utility functional form gives us the estimate of EIS about 0.5-0.75. The omitted variable of labor can account for Campell and Mankiw's (1989) finding about excess sensitivity.

# Further investigation

- K-P-R implies the complementarity between consumption and labor. This means the household should plan to have their consumption drop at retirement. And the drop is quite larger than data.
- The implication for monetary policy. Is complementarity a solution to the channel for monetary expansion to cause an increase in consumption?