Micro and Macro Elasticities in a Life Cycle Model with Taxes

Richard Rogerson Johanna Wallenius

Arizona State University

Working paper, December 2006

Introduction

- Policy effects on labor supply
- Tax and transfer policies affect market work
  - Prescott (2004), Rogerson (2005), Ohanian, Rafflo and Rogerson (2006)
  - time for work in continental Europe - 70% of the US
- Critique: inconsistency with low elasticity estimated with micro data, large labor supply elasticity to derive the result
  - Alesina, Glaeser and Sacerdote (2006)

Introduction

- Policy effects on labor supply
- Tax and transfer policies affect market work
  - Prescott (2004), Rogerson (2005), Ohanian, Rafflo and Rogerson (2006)
  - time for work in continental Europe - 70% of the US
- Critique: inconsistency with low elasticity estimated with micro data, large labor supply elasticity to derive the result
  - Alesina, Glaeser and Sacerdote (2006)
- Main findings
  1. macro elasticities are virtually unrelated to micro elasticities
  2. macro elasticities are large

Model

- Basic set-up
  - continuous time OLG
  - a unit mass of identical agents born at each instant
  - length of lifetime - one

Model

- Basic set-up
  - continuous time OLG
  - a unit mass of identical agents born at each instant
  - length of lifetime - one
- Preference

$$\int_0^1 U(c(a), 1 - h(a))da$$

- Production

$$Y(t) = L(t)$$

Model

- Labor services

$$l = \tilde{e}(a)g(h)$$

  - h: time devoted to market work
  - $\tilde{e}(a)$: life-cycle variation in labor productivity
    - single-peaked, twice cont. differentiable
  - $g(h)$: labor services that depends on work hours
    - continuous, increasing, $g(0) = 0$
    - strictly concave over $[\bar{h}, 1]$ but not in $[0, \bar{h}]$

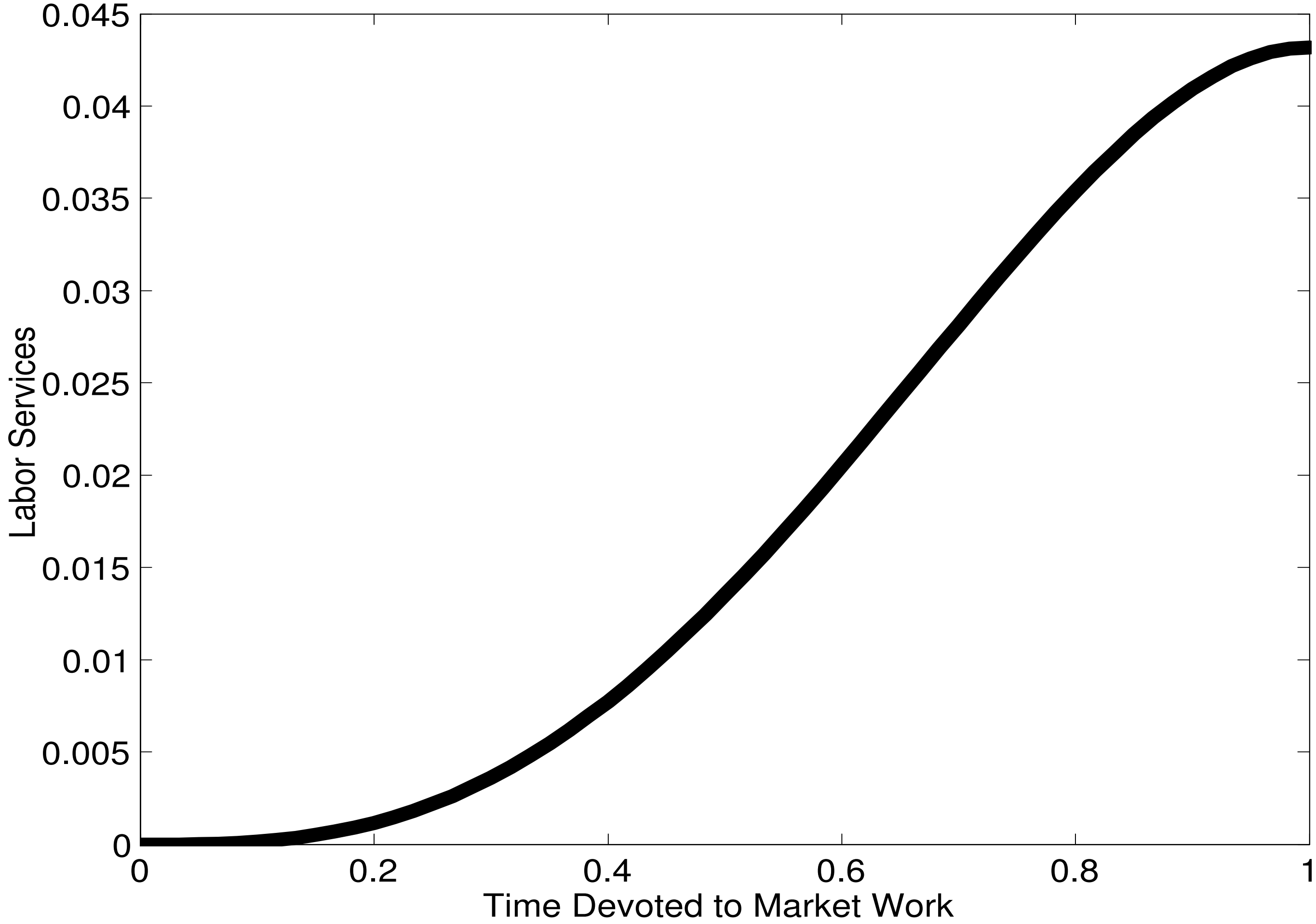
Model

- Labor services

$$l = \tilde{e}(a)g(h)$$

  - h: time devoted to market work
  - $\tilde{e}(a)$: life-cycle variation in labor productivity
    - single-peaked, twice cont. differentiable
  - $g(h)$: labor services that depends on work hours
    - continuous, increasing, $g(0) = 0$
    - strictly concave over $[\bar{h}, 1]$ but not in $[0, \bar{h}]$
- Market
  - time-zero trading for labor services and consumption at all future dates
  - consider steady state with constant price, constant interest rate
  - $w(t) = p(t)$, normalize to 1

Model: g function



Household problem

$$\max_{c(a),h(a)} \int_0^1 U(c(a), 1 - h(a))da$$

s.t.

$$\int_0^1 c(a)da = \int_0^1 \tilde{e}(a)g(h(a))da$$

Household problem

$$\max_{c(a),h(a)} \int_0^1 U(c(a), 1 - h(a))da$$

s.t.

$$\int_0^1 c(a)da = \int_0^1 \tilde{e}(a)g(h(a))da$$

Propositions 1 and 2: The optimal solution $h(a)$ satisfies

- reservation property, i.e. $h(a) > 0$ if $\tilde{e}(a) > \tilde{e}^*$ and $h(a) = 0$ if $\tilde{e}(a) < \tilde{e}^*$
- $h(a_1) > h(a_2)$ if $\tilde{e}(a_1) > \tilde{e}(a_2)$

Household problem

- Assume
$$U(c, 1-h) = u(c) - v(h)$$
- Consumption is constant at optimal
- Rewrite the problem as

$$\max_{c,h(a),A_1,A_2} u(c) - \int_{A_1}^{A_2} v(h(a))da$$

s.t.

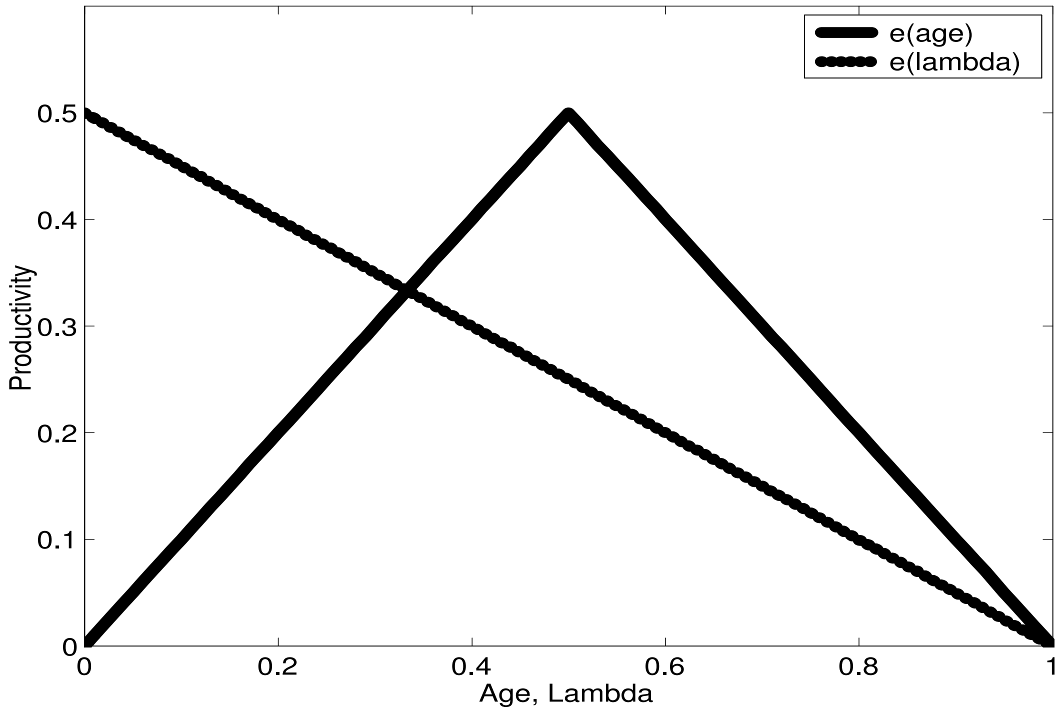
$$c = \int_{A_1}^{A_2} \tilde{e}(a)g(h(a))da$$

Household problem

- Change of variables: define $e(\lambda)$ s.t. $\lambda = \int_0^1 I(\tilde{e}(a) > e(\lambda))da$
- $e(\lambda)$ is the productivity s.t. the individual has a higher productivity for the fraction λ of their life

Household problem

- Change of variables: define $e(\lambda)$ s.t. $\lambda = \int_0^1 I(\tilde{e}(a) > e(\lambda))da$
- $e(\lambda)$ is the productivity s.t. the individual has a higher productivity for the fraction λ of their life



The $\tilde{e}(a)$ and $e(\lambda)$ Functions

Household problem

$$\max_{c,h(\lambda),\lambda^*} u(c) - \int_0^{\lambda^*} v(h(\lambda))d\lambda \qquad \textbf{s.t.} \qquad c = \int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda$$

Household problem

$$\max_{c,h(\lambda),\lambda^*} u(c) - \int_0^{\lambda^*} v(h(\lambda))d\lambda \qquad \textbf{s.t.} \qquad c = \int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda$$

- FOC wrt λ^*: extensive margin

$$\frac{v(h(\lambda^*))}{u'(\int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda)} = e(\lambda^*)g(h(\lambda^*))$$

- FOC wrt $h(\lambda)$: intensive margin

$$\frac{v'(h(\lambda))}{u'(\int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda)} = e(\lambda)g'(h(\lambda))$$

Household problem

Optimal composition of work curve

$$\frac{v'(h(\lambda))}{e(\lambda)g'(h(\lambda))} = \frac{v(h(\lambda^*))}{e(\lambda^*)g(h(\lambda^*))}$$

Household problem

Optimal composition of work curve

$$\frac{v'(h(0))}{e(0)g'(h(0))} = \frac{v(h(\lambda^*))}{e(\lambda^*)g(h(\lambda^*))}$$

- $h(0)$ increasing in λ^*
  Marginal disutility equated at both margins. More work hours $\Rightarrow$ higher MD $\Rightarrow$ raise MD on the other margin

Household problem

Optimal composition of work curve

$$\frac{v'(h(0))}{e(0)g'(h(0))} = \frac{v(h(\lambda^*))}{e(\lambda^*)g(h(\lambda^*))}$$

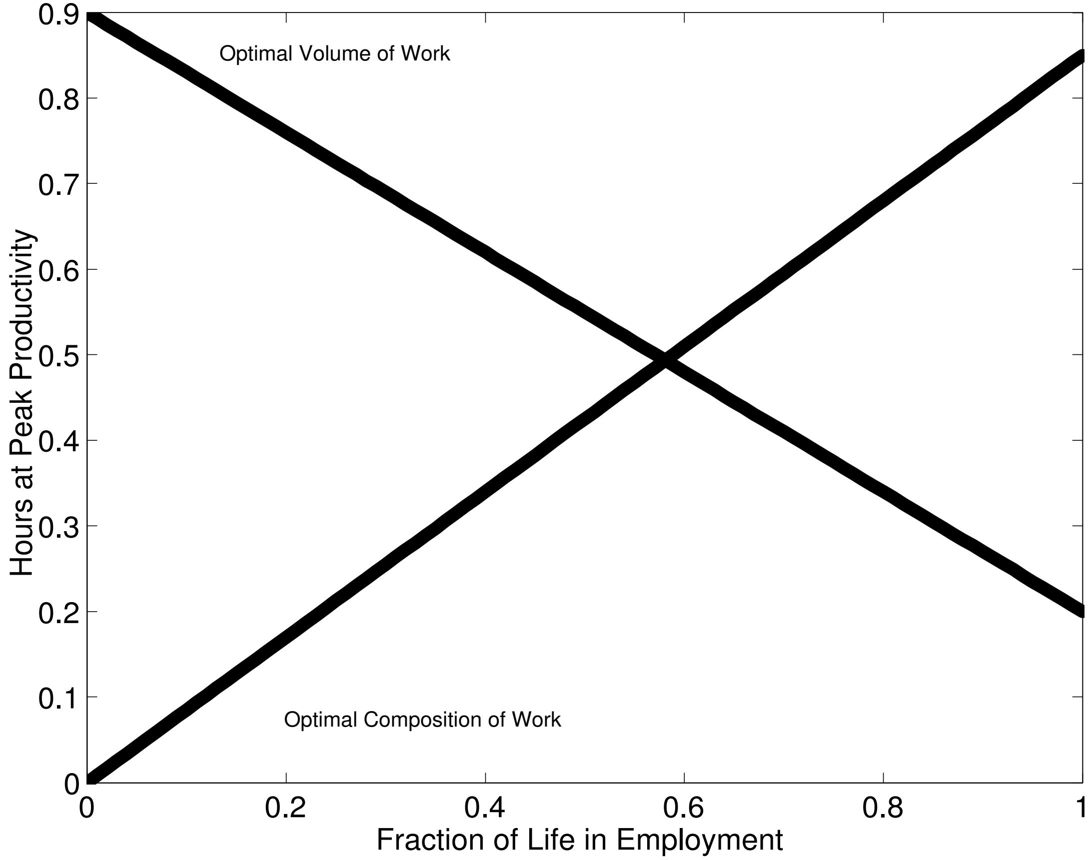
- $h(0)$ increasing in λ^*
  Marginal disutility equated at both margins. More work hours $\Rightarrow$ higher MD $\Rightarrow$ raise MD on the other margin

Optimal volume of work curve

$$\frac{e(0)g'(h(0))}{v'(h(0))} = \frac{1}{u'(\int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda)}$$

- $h(0)$ decreasing in λ^*
  MRS equated with MP of work. More work hours $\Rightarrow$ more consumption $\Rightarrow$ higher MRS $\Rightarrow$ reduce the other margin

$h(0)$ and λ^* in equilibrium



Equilibrium Determination of $h(0)$ and λ^*

Tax policies

$$\max_{c,h(\lambda),\lambda^*} u(c) - \int_0^{\lambda^*} v(h(\lambda))d\lambda \quad \textbf{s.t.} \quad c = (1-\tau)\int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda + T$$

Tax policies

$$\max_{c,h(\lambda),\lambda^*} u(c) - \int_0^{\lambda^*} v(h(\lambda))d\lambda \quad \text{s.t.} \quad c = (1-\tau)\int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda + T$$

Optimal composition of work curve

$$\frac{v'(h(0))}{e(0)g'(h(0))} = \frac{v(h(\lambda^*))}{e(\lambda^*)g(h(\lambda^*))}$$

Optimal volume of work curve

$$(1-\tau)\frac{e(0)g'(h(0))}{v'(h(0))} = \frac{1}{u'(\int_0^{\lambda^*} e(\lambda)g(h(\lambda))d\lambda)}$$

Tax policies: quantitative results

- Functional forms

$$u(c) = \log(c), \quad v(h) = \alpha \frac{h^{1+\gamma}}{1+\gamma}$$

$$g(h) = (h - \bar{h}) \quad \text{for} \quad h \geq \bar{h}, \qquad 0 \qquad \text{otherwise}$$

$$e(\lambda) = e_0 - (e_0 - e_1)\lambda$$

- Calibrate α, $\bar{h}$ and e_1 to match
  - λ^*=0.67 – work approx. 40 yrs out of adult life of 60 yrs
  - $h(0)$=0.45 – work peak of 45 hrs out of 100 discretionary hrs per week
  - hourly wages at peak twice as large as the lowest

Tax policies: quantitative results

- Estimate the micro Frisch labor supply elasticity

$$\log(h_t) = b_0 + b_1 \log(w_t^h) + \varepsilon_t$$

γ	0.5	1	2	10
Frisch elast. $\hat{b}_1$	1.29	0.59	0.28	0.05

- Policy experiment
  - calibrate the model with $\tau = 30\%$
  - increase τ from 30% to 50%

Tax policies: quantitative results

γ	Frisch elst.	agg. hours	λ^*	$h(0)$
0.5	1.29	.777	.857	.856
1	0.59	.784	.788	.918
2	0.28	.788	.808	.956
10	0.05	.790	.794	.991

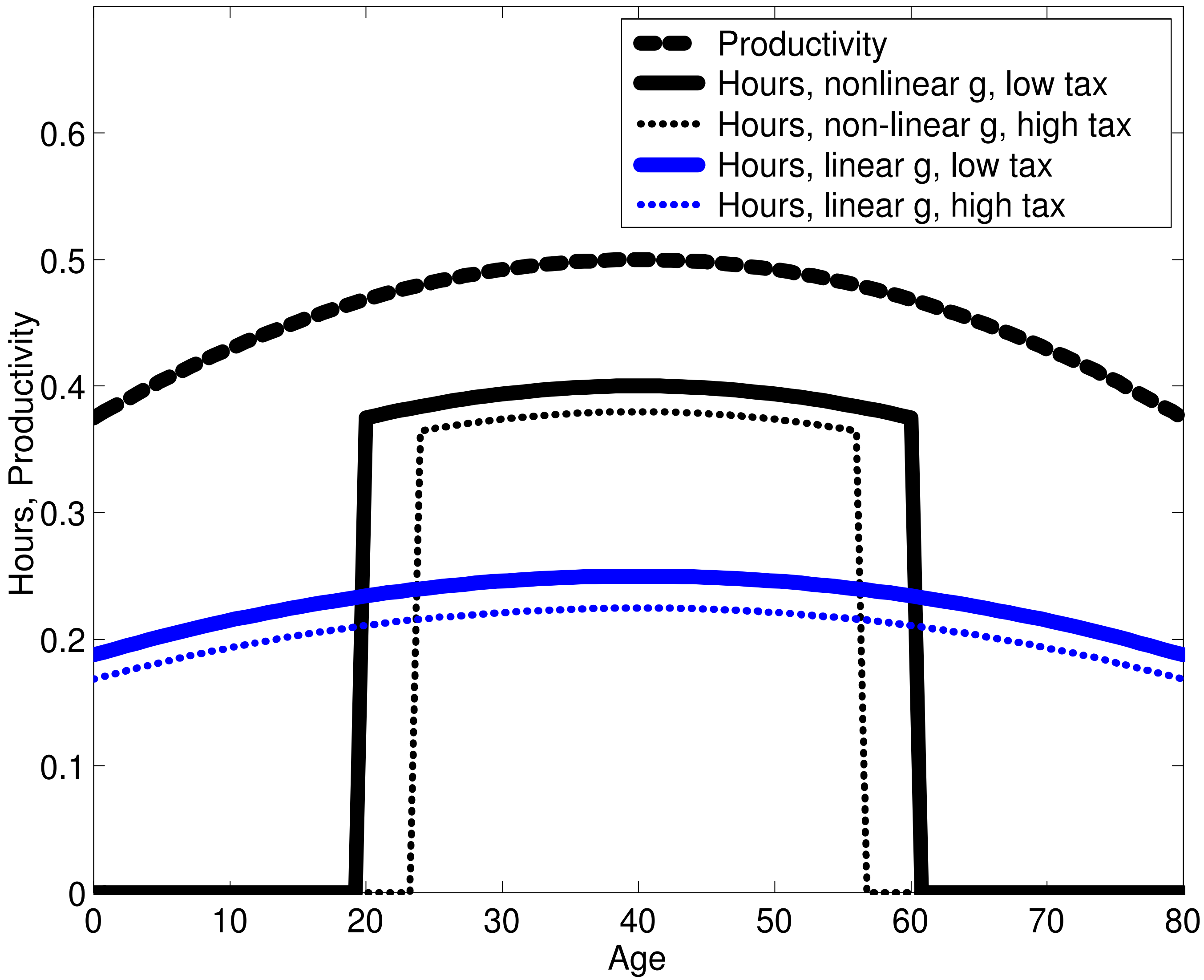
1. tax policy has large effect on aggregate hours worked
2. difference in micro elasticities has little effect on macro elasticity
3. γ has large decomposition effect - changes in working life vs changes in hours while employed

Tax policies: quantitative results

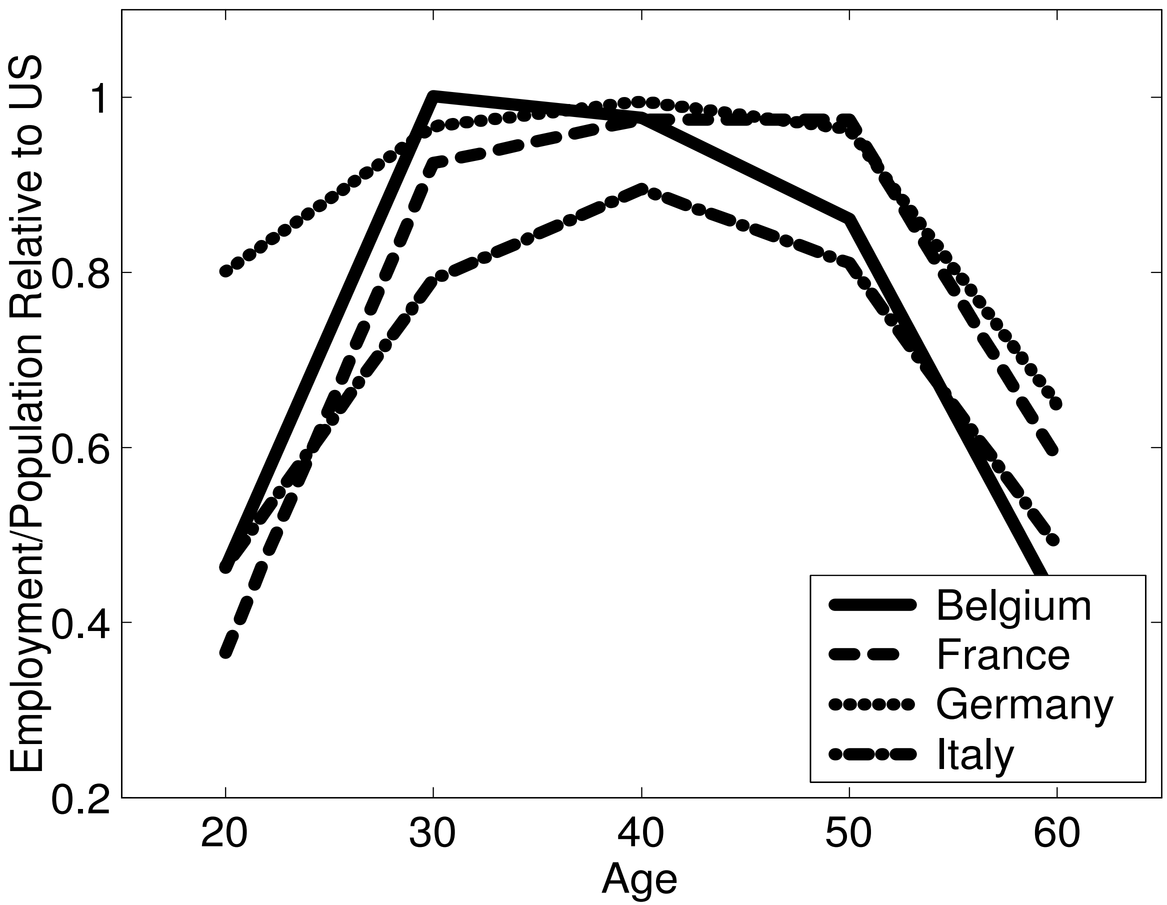
Key to the result

- nonlinear mapping from work hours to aggregate labor services
- life-cycle profile for hours worked and a choice of no work over life
- OLG profile is not enough

Linearity of $g(h)$



US vs Europe



Life Cycle Employment Profiles Relative to the US

Remarks

- can the model explain the time series of the difference in US vs Europe?
  - maybe, if cross-sectional or life-cycle distribution of productivity has shifted differently
- missing GE effects
  - high tax $\Rightarrow$ less work $\Rightarrow$ higher wage (?)