Limited Asset Market Participation and the Elasticity of Intertemporal Substitution

Annette Vissing-Jorgensen, JPE 2002

Roine Vestman, Professor Sargent's Reading Group, September 18 2007

The Elasticity of Intertemporal Substitution (EIS)

- Definition, certainty

$$-\frac{\frac{d\left(c_t/c_{t+1}\right)}{c_t/c_{t+1}}}{\frac{d(1+R_{ft})}{1+R_{ft}}} = \frac{d\left[\ln\left(c_{t+1}/c_t\right)\right]}{d\left[\ln(1+R_{ft})\right]}$$

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- Relevance
  - Ability of **RBC models** to fit the data (Tallarini (JME 2000))
  - Ability of **portfolio models** to fit the data (Gomes and Michaelides (JF 2005))
  - Ability of some **GE incomplete mkt models** to generate an equity premium (Guvenen (2004), Gomes and Michaelides (RFS 2007))

Research question

- Household h's Euler equation for asset i

$$E_t\left[\beta\left(\frac{C_{t+1}^h}{C_t^h}\right)^{-\gamma}(1+R_{it})\right]=1 \quad (EE_i)$$

  - $EIS=\frac{1}{\gamma}\equiv\sigma$

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- Is it important to account for market particiption?

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- What is the value of EIS among **stock and bond market participants**? Are estimates precise?
- Is it important to account for market particiption?
- In previous studies, may ignorance of participation have lead to downward biased estimates?

CEX: The Consumer Expenditure Survey

- Time period: 1982-1996
- Each household is sampled five times on a quarterly basis
- Asset ownership with respect to **four asset categories** is gathered
  1. Stocks, bonds, mutual funds, and other such securities
  2. U.S. savings bonds
  3. Savings accounts
  4. Checking accounts, brokerage accounts, and other similar accounts

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  - Semi-annual, adjusted for HH size, seasonal dummies

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  - Aggregate cross-sectionally

$$E_t\left[\beta\frac{1}{H}\sum_h\left[\left(\frac{C_{t+1}^{h,*}\varepsilon_{t+1}^{h}}{C_t^{h,*}\varepsilon_t^{h}}\right)^{-\gamma}\right](1+R_{it})\right]=1$$

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  - Let $H\longrightarrow\infty$ and invoke iid error

$$E_t\left[\beta E_h\left[\left(\frac{\varepsilon_{t+1}^{h}}{\varepsilon_t^{h}}\right)^{-\gamma}\right]E_h\left[\left(\frac{C_{t+1}^{h,*}}{C_t^{h,*}}\right)^{-\gamma}\right](1+R_{it})\right]=1$$

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- Assume **iid error, aggregate cross-sectionally, log-linearize**

$$\frac{1}{H_t^i}\sum_h \Delta \ln C_{t+1}^{h,i} =$$
$$\left(\frac{1}{\gamma}\right)^{i}\ln(1+R_{i,t})+\delta_1^i D_1+\ldots+\delta_{12}^i D_{12}+\alpha^i\frac{1}{H_t^i}\sum_h \Delta \ln(HHsize)_{t+1}^{h,i}+u_{t+1}^i$$

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- Return measure: NYSE
- Conditional moments
  - Predicting vars (lagged, logs): dividend-price ratio, NYSE return, treasury bill return, government bond horizon premium, corporate bond default premium

Remarks

- The author suggests that (EE_i) is a better representation of
  - **wealthy households**' optimization problem (participation cost and better diversification)
    - Based on reported holdings, **three layers of stock and bond holders**: Bottom, Middle, and Top
  - **single individual households**' optimization problem (no bargaining within the household)
- (EE_i) is silent on the value of EIS for non-participants. Nevertheless, estimating the EIS implied by non-participants' Euler equation may help us understand the low and insignificant estimates of previous studies

CEX: The Consumer Expenditure Survey (cont.)

Group	Mean Number of Observations per Month	Mean of $(1/H_t)\sum_{h=1}^{H_t}\Delta \ln C_{t+1}^{a}$ over the Sample	Standard Deviation of $(1/H_t)\sum_{h=1}^{H_t}\Delta \ln C_{t+1}^{h}$ over the Sample
	A. Semiannual Data, All Household Sizes		
All	217	.002	.024
Stockholders	47	.014	.041
Nonstockholders	170	−.001	.028
Bottom stockholder layer	11	.022	.088
Middle stockholder layer	11	.002	.101
Top stockholder layer	11	.014	.102
Bondholders	68	.010	.034
Nonbondholders	149	−.001	.030
Bottom bondholder layer	17	.005	.069
Middle bondholder layer	17	.012	.072
Top bondholder layer	17	.008	.073
	B. Semiannual Data, Single-Individual Households		
All	51	−.001	.045
Stockholders	10	.009	.098
Nonstockholders	41	−.003	.052
Bondholders	13	.002	.085
Nonbondholders	39	−.002	.054

Note.—The variable $(1/H_t)\sum_{h=1}^{H_t}\Delta \ln C_{t+1}$ is seasonally adjusted using seasonal dummies. The seasonally adjusted value is the sample mean of the series plus the residual from a regression on 12 dummies.

STOCKHOLDERS

POINT ESTIMATES

	INSTRUMENT SET 1		INSTRUMENT SET 2			INSTRUMENT SET 3		
	$\hat{\sigma}$	Wald Test Equals σ	$\hat{\sigma}$	Overidentification Test	Wald Test Equals σ	$\hat{\sigma}$	Overidentification Test	Wald Test Equals σ
	A. Euler Equation for Stocks 1. All Household Sizes							
All	.098 (.071)		.066 (.062)	.086		.068 (.059)	.314	
Stockholders	**.299** **(.146)**		.281 (.114)	.260		.200 (.091)	.018	
Nonstockholders	**.057** **(.079)**		.017 (.070)	.048		.049 (.070)	.568	
Bottom layer	.046 (.186)		−.054 (.163)	.570		.052 (.158)	.828	
Middle layer	.175 (.274)		.350 (.207)	.547		.173 (.261)	.027	
Top layer	.486 (.325)		.417 (.235)	.203		.292 (.188)	.027	
Nonstockholders vs. stockholders		3.255 (.071)			4.340 (.037)			3.296 (.069)
Nonstockholders vs. top layer		2.146 (.143)			2.378 (.123)			2.941 (.086)

Stockholders

Wald test on participants / non-participants

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STOCKHOLDERS

TEST OF OVERIDENTIFYING RESTRICTIONS

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BOND HOLDERS

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Nonstockholders	.077		−.115	.071		.160	.250	
	(.143)		(.137)			(.123)		
Nonstockholders vs. stockholders		1.733			1.958			3.866
		(.188)			(.162)			(.049)
	B. Euler Equation for Treasury Bills 1. All Household Sizes							
All	.372		.362	.097		.264	.143	
	(.232)		(.225)			(.220)		
Bondholders	.932		.839	.215		.783	.147	
	(.368)		(.360)			(.353)		
Nonbondholders	.105		.087	.024		.005	.382	
	(.270)		(.257)			(.244)		
Bottom layer	.986		.798	.492		.726	.244	
	(.662)		(.612)			(.580)		
Middle layer	.287		.336	.476		.277	.627	
	(.550)		(.544)			(.523)		
Top layer	1.648		1.672	.356		1.530	.067	
	(.515)		(.505)			(.505)		
Nonbondholders vs. bondholders		4.029			3.192			4.127
		(.045)			(.074)			(.042)
Nonbondholders vs. top layer		8.055			8.905			7.871
		(.005)			(.003)			(.005)

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Conclusions and discussion

- Estimates of EIS are positive (a leap forward?)
- Participation matters
- Possible extensions
  - In terms of data: Better HH wealth data would give more accurate participation classification and potentially better return data
  - The work of Chen, Favilukis and Ludvigson on Epstein-Zin preferences seems to be the next step within the Euler equation framwork
  - Joint estimation of γ and σ - easier with a richer model?

Stockholders

Joint estimation

	Instrument Set 1			Instrument Set 2			Instrument Set 3		
	$\hat{\sigma}$	Overidentification Test	Wald Test Equals σ	$\hat{\sigma}$	Overidentification Test	Wald Test Equals σ	$\hat{\sigma}$	Overidentification Test	Wald Test Equals σ
	A. All Household Sizes								
All	.080	.025		.046	.036		.026	.016	
	(.077)			(.058)			(.045)		
Stockholders	.442	.022		.404	.071		.261	.011	
	(.144)			(.114)			(.079)		
Nonstockholders	.013	.036		.003	.041		.003	.028	
	(.057)			(.022)			(.033)		
Bottom layer	.006	.141		.003	.372		.007	.078	
	(.054)			(.021)			(.026)		
Middle layer	.068	.175		.242	.218		.048	.041	
	(.173)			(.203)			(.040)		
Top layer	.700	.020		.422	.190		.324	.004	
	(.354)			(.260)			(.179)		
Nonstockholders vs. stockholders			9.228 (.002)			12.886 (.0003)			13.677 (.0002)
Nonstockholders vs. top layer of stockholders			3.890 (.049)			2.127 (.145)			4.211 (.040)