Consumption and risk sharing over the life cycle

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Outline of the presentation

- Facts based on data
- Estimation of an income process
- Model
- Main results
- Reflections

Three facts

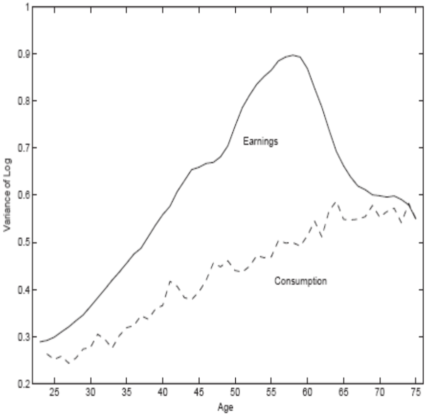


Fig. 1. The graphs represent the cross-sectional variance of the logarithm of earnings and consumption. The basic data unit is the household. Consumption data are from the CEX and are taken directly from Deaton and Paxson (1994). Earnings data are taken from the PSID. The variances are net of 'cohort effects:' dispersion which is unique to a group of households with heads born in the same year. This is accomplished, as in Deaton and Paxson (1994), via a cohort and age dummy-variable regression. The graphs are the coefficients on the age dummies, scaled so as to mimic the overall level of dispersion in the data. Further details are in Appendix A.

- Age-dependent inequality in earnings and consumption increases between ages 23 and 60
- The increase in consumption is less than the increase in earnings
- The increase in both is approximately linear

The data suggests that in terms of risk sharing we need a model in between complete markets and autarky

- Complete markets $\Rightarrow$ consumption inequality is constant across age
- Autarky $\Rightarrow$ consumption inequality mimics earnings inequality
- Remark: if limited risk sharing *does not* play a key role, an explanation coherent with complete markets could be
  - Predetermined heterogeneity in skills
  - Differences in wage growth across skill levels
  - Limited borrowing against future wage growth
  - One would need to rule out this explanation by looking at the inequality within each skill group (inequality would be constant over age within a skill group)

Income process

- Let log earnings of individual i in cohort c of age h follow

  $$y^c_{ih} = g(x^c_{ih}, \theta) + u_{ih}$$

- Ultimately, we want a "cohort-free" time-series process for u_{ih}

  $$u_{ih} = \alpha_i + \varepsilon_{ih} + z_{ih};\ \alpha_i \sim N(0, \sigma^2_\alpha), \varepsilon_{ih} \sim N(0, \sigma^2_\varepsilon)$$
  $$z_{ih} = \rho z_{i,h-1} + \eta_{ih};\ \eta_{ih} \sim N(0, \sigma^2_\eta)$$

- Want to summarize all h-dependent variability with $(\rho, \sigma^2_\alpha, \sigma^2_\varepsilon, \sigma^2_\eta) \mid z_{i0} = 0$
- To get $var(u_{ih})$, construct the following moment restrictions

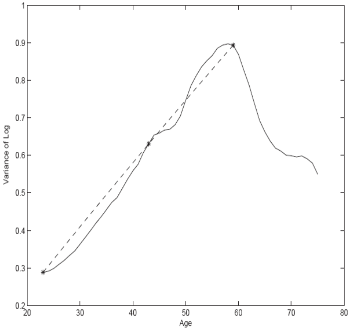
  $$\begin{cases} E[(y^c_{ih} - \overline{y^c_h})^2|h] - \bar{a} - b_h = 0 & H \text{ such moments} \\ E[(y^c_{ih} - \overline{y^c_h})^2|c] - \bar{b} - a_c = 0 & C \text{ such moments} \\ E[(y^c_{ih} - \overline{y^c_h})^2|h = 42] - m = 0 & \text{scaling} \\ b_h + (m - b_{42}) - b^*_h = 0 & \text{moments} \end{cases}$$

- We get cross-sect. variances to match with the parameters

  $$b^*_h = var(u_{ih}) = \sigma^2_\alpha + \sigma^2_\varepsilon + \sigma^2_\eta \sum_{j=0}^{h-1} \rho^{2j}$$

Income process - graphical GMM to get parameters

- Pop. moments $b_h^* = var(u_{ih}) = \underbrace{\sigma_\alpha^2 + \sigma_\varepsilon^2}_{\text{"intercept"}} + \underbrace{\sigma_\eta^2 \sum_{j=0}^{h-1} \rho^{2j}}_{\text{"slope"}}$



- "Intercept": $\sigma_\alpha^2 + \sigma_e^2 = 0.27$
- "Slope components": $\sigma_\eta^2 = 0.017$; $\rho = 0.9989$
- Formally, $(\rho, \sigma_\alpha^2, \sigma_\varepsilon^2, \sigma_\eta^2)$ is estim. jointly with b_h^* by GMM

The model

- Preferences

  $$E \sum_{h=1}^{H} \beta^h \phi_h U(c_h);$$
  ϕ_h: uncond prob of surviving up to age h
  $$U(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

- Endowment: log *hours* worked

  $$\log(n_h) = k_h + \alpha + z_h + \varepsilon_h$$
  $$z_h = \rho z_{h-1} + \eta_h;\ \eta_h \sim N(0, \sigma_\eta^2),\ z_0 = 0,\ E[n] = 1$$

- Production (repr. firm)

  $$Y = ZK^\theta N^{1-\theta}$$

- Law of motion for K

  $$K' = Y - C + (1-\delta)K$$

- Average labor endowment during an agent's working life: $\overline{n}$

The problem of the agent

Let V_h denote the value function of an h year old agent

$$
\begin{aligned}
& V_h(\alpha, z_h, \varepsilon_h, a_h, \overline{n}_h) = \\
& = \max_{a'_{h+1}} \left\{ U(c_h) + \widehat{\beta}\xi_{h+1} E\left[V'_{h+1}(\alpha, z'_{h+1}, \varepsilon'_{h+1}, a'_{h+1}, \overline{n}'_{h+1}) \right] \right\}
\end{aligned}
$$

s.t.

$$
\left.\begin{aligned}
& c_h + (1+g)a'_{h+1} \leq a_h R/\xi_h + n_h(1-\tau)W \\
& a'_{h+1} \geq \underline{a}(\alpha, z, h) \\
& \overline{n}'_{h+1} = \overline{n}_h + n_h/I
\end{aligned}\right\} \text{before retirement}
$$

$$
\left.\begin{aligned}
& c_h + a'_{h+1} \leq a_h R/\xi_h + B(\overline{n}_h)W \\
& \overline{n}'_{h+1} = \overline{n}_h
\end{aligned}\right\} \text{after retirement}
$$

$\widehat{\beta} \equiv \beta(1+g)^{1-\gamma}$

ξ_h cond. survivor prob.

I number of years before retirement

Unique stationary equilibrium

A stationary equilibrium is defined as prices R, W, a set of functions $\left\{V_h, a'_{h+1}\right\}_{h=1}^{H}$, aggregate capital stock K and labor supply N and a cross-sectional distr of agents μ across ages, idiosyncratic shocks, asset holdings, and past earnings such that

a) prices W and R are given by the firm's marginal productivities
b) individual optimiz. problems are satisfied
c) $W \int_S B(\overline{n}) d\mu = WN(1-\tau)$
d) the distr μ is stationary given individuals' decisions
e) aggr. quantities are determined by $K = \int_S a_h d\mu$; $N = \int_S n_h d\mu$

Calibration

- Aggregate wealth / income = 3.1 to match bottom 99%: $\beta = 0.962$
- Set $\rho = 1$ for convinience (and adjust σ_η^2 slightly)
- Mortality rate for females
- Pop. growth: 1%
- Growth in per capita wages $g = 1.5\%$
- Capital share $\theta = 0.4$
- $\delta = 0.109$
- $\gamma = 2$

Result in benchmark economy (no social security)

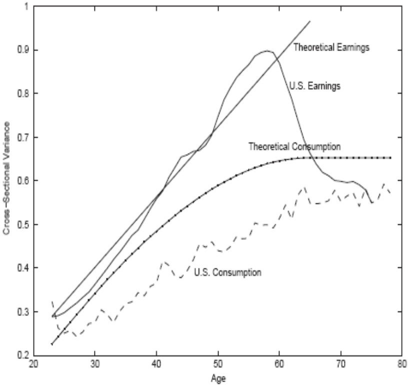
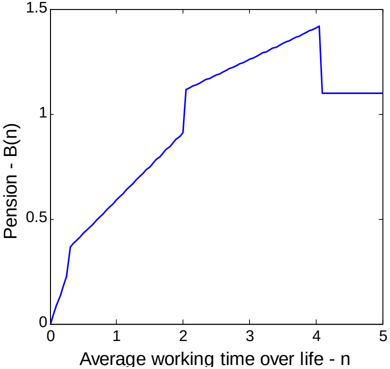


Fig. 4. This graph compares the population moments from our benchmark model with those of the data. The solid lines (without dots) represent the theoretical and empirical cross-sectional variance of log earnings. The dashed line represents the empirical cross-sectional variance of consumption and the solid-dotted line represents the theoretical cross-sectional variance of consumption from the benchmark economy.

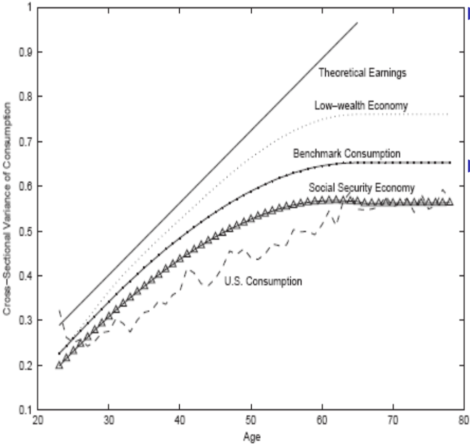
- Consumption inequality grows too much in the model
- Insurance opportunities not accounted for represents 20% of the variance in income shocks (decrease $(\sigma^2_\eta, \sigma^2_\varepsilon)$ with 20% to match data with model)

Introduction of social security: B(n)

- Reminder - budget constr. after retirement:
  $c_h + a'_{h+1} \leq a_h R/\xi_h + B(\overline{n}_h)W$
- A representation of Old Age Insurance of the U.S. social security system:



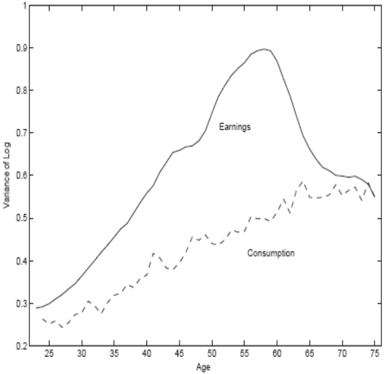
The effects of social security and the level of aggr. wealth

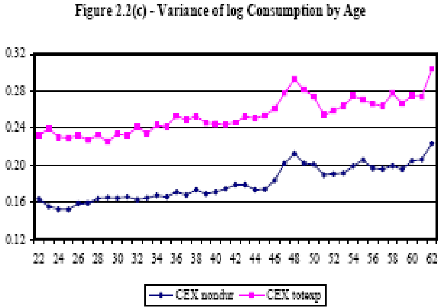


- Social security provides insurance across cohorts as well as within cohorts (if $B(\overline{n_h})$ is concave)
- Aggr. wealth determines the relative position of the earnings and consumption profile
  - Low wealth (low β,aggr wealth/income $= 1.5$) $\Rightarrow$ more inequality

Reflection 1: Are they overestimating consumption ineq.?

Deaton & Paxton (1994) data and (age,cohort) dummies vs. updated data and (age,time) dummies





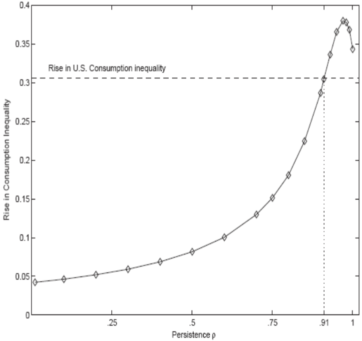
- Heathcote, Storesletten and Violante (2005), Kaplan (2006)
  - More correct to control for time than cohort effects
- Increase in variance over life-cycle: 0.35 vs. 0.07

Reflection 2: Initial conditions (fixed effects) vs. life-cycle shocks

Conflict with micro evidence?

- First, let $\sigma_\alpha^2 > 0$ but $\sigma_\varepsilon^2 = \sigma_\eta^2 = 0$
  - Need to increase $\{c\}$ by 27.4% in benchmark for agent to be indifferent
- Second, let $\alpha^i = 0 \; \forall i$ and let $\sigma_\varepsilon^2, \sigma_\eta^2$ as in benchmark
  - Need to increase $\{c\}$ by 20.2% in benchmark for agent to be indifferent under veil of ignorance
- Keane & Wolpin (1997):
  - *"According to our estimates, unobserved endowment heterogeneity, as measured at age 16, accounts for 90 percent of the variance in life-time utility."* (p. 515)

Reflection 3: High persistence is key with this type of income process - replace with heterogeneity?



Recent alternative income process (Guvenen, 2006):

$$y_t^i = g(\theta^0, X_t^i) + f(\theta^i, X_t^i) + z_t^i + \varepsilon_t^i$$
$$z_t^i = \rho z_{t-1}^i + \eta_t^i$$

-Restricted income profiles (this paper): $f(\theta^i, X_t^i) = \alpha^i$

-Heterogenous income profiles: $f(\theta^i, X_t^i) = \alpha^i + \beta^i \cdot age_i$

-ρ lower