The time-series properties of aggregate consumption

Implications for the costs of fluctuations

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Introduction (1)

What would be the effect on welfare of eliminating economic fluctuations?

- Lucas(1987) assumed
  1) there exists a representative consumer, whose lifetime utility function is also the social welfare function;
  2) the welfare function is time-separable and iso-elastic;
  3) log of annual per capita consumption is serially uncorrelated and normally distributed around a linear trend.
- He found that: society would be willing to sacrifice only 0.05% of consumption ($12 per year for each person) to get rid of fluctuations.
  This implies that the quantitative importance of stabilization policy is very small.

Introduction (2)

Followers: Modify the three assumptions to reexamine welfare cost.

- Heterogeneous agents and incomplete markets. (bad income shocks hurt a few households severely.)
- Use different utility functions, or use asset prices to elicit preference over risk.
- (This paper) Use alternative model for consumption dynamics. If consumption is very persistent, as in the US data, Lucas' estimates for welfare cost are severely downward-biased. (0.05% V.S. from 0.5% to 5%)
  Intuition: persistent consumption implies persistent shock. For example, if consumption follows a random walk, then any shock would be permament.

Outline

- Present some simple models of consumption that highlight the main determinants of the costs of fluctuations. (Key parameters)
- How to choose the value of these key parameters.
- Estimate the costs of fluctuations across a variety of statistical models for consumption.
- Use economic models where consumption fluctuations are an optimal response to shocks.

Measuring costs of fluctuations

The welfare cost is defined as the fraction (λ) of annual consumption that society would be willing to pay to eliminate the fluctuations. λ solves the following equation.

$$E_0[\sum_{t=0}^{\infty} e^{-\rho t}(\frac{(C_t(1+\lambda))^{1-\gamma}-1}{1-\gamma})] = \sum_{t=0}^{\infty} e^{-\rho t}(\frac{(\overline{C_t}^{1-\gamma}-1}{1-\gamma})$$

- Need to specify the stochastic process for the risky consumption path C_t in order to evaluate the expectation.
- Need to define the counterfactual "suitably modified" consumption series $\overline{C_t}$.
- Incorporate the above two in a model for consumption.
  1) Either provide a statistical model for consumption.
  2) Or assume an economic environment where society optimally chooses how much to consume.

Statistical models of consumption at first glance (1)

Define the "suitably modified" consumption series $\overline{C_t} = E[C_t] = C_0 e^{gt}$. (U.S. data shows that consumption has grown at an approximately constant rate)

Assume consumption is log-normally distributed. (Marginal distribution) i.e. $c_t \equiv \log C_t \tilde{} N(E(c_t), Var(c_t))$

We obtain a closed form solution for the costs of fluctuations: ($\rho \simeq r - \gamma g$ for iso-elastic preferences)

$$\log(1+\lambda) = \begin{cases} 0.5(1-e^{g-r}) \sum_{t=0}^{\infty} e^{(g-r)t} Var(c_t) \text{ if } \gamma = 1 \\ \frac{1}{\gamma - 1} \log[(1-e^{g-r}) \sum_{t=0}^{\infty} e^{(g-r)t} e^{0.5\gamma(\gamma-1)Var(c_t)}] \text{ else} \end{cases}$$

- We only need to compute the forecast error variance of consumption at different horizons.
- Require a model of consumption dynamics (for detrended consumption):

$$c_t = \eta c_{t-1} + \varepsilon_t, \text{ where } \varepsilon_t \tilde{} \text{ i.i.d. } N(0, \sigma^2)$$

Statistical models of consumption at first glance (2)

- LS estimates: $\eta = 0.92$; lagged consumption can account for 84% of the variability of present consumption.
- $\eta = 0$ is "Lucas consumption process"; $\eta = 1$ is "Hall consumption process".
- Approximately we have for $|\eta| \leq 1$ (exact when $\gamma = 1$)

$$\begin{aligned} \lambda &\simeq \frac{0.5\gamma\sigma^2}{r - g + 1 - \eta^2} \\ &= \frac{0.5\gamma(1-\eta^2)}{r - g + 1 - \eta^2} \times \frac{\sigma^2}{1-\eta^2} \end{aligned}$$

- Variability of shocks σ^2; persistence of shocks η.

Economic models of consumption in a first look

A representative consumer solves the following problem

$$\max_{\{C_t\}} E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\frac{C_t^{1-\gamma}-1}{1-\gamma}\right)\right]$$
$$s.t. \quad : \quad K_{t+1} + C_t = R_t K_t$$

where the risky R_t is log normally distributed with mean $r - 0.5\sigma^2$ and variance σ^2.
Closed form solution:

$$c_t = c_{t-1} + g - 0.5\sigma^2 + \varepsilon_t$$

where $g = (r-\rho)/\gamma + 0.5(\gamma+1)\sigma^2 - \sigma^2$, and initial condition $C_0 = (1 - e^{g-r})R_0 K_0$.

- Welfare cost is given by $\log(1+\lambda) = \frac{\gamma}{\gamma-1}\log\left(\frac{e^{r-g} - e^{-0.5(\gamma-1)\sigma^2}}{e^{r-g}-1}\right)$

Choice of parameters

- Risk aversion $\gamma \in [1, 5]$.
- $r - g$ is also the growth rate of the marginal utility of consumption. Small $r - g$ implies less discounting on future costs of a shock, given that the shock lasts for a few periods, and thus larger welfare costs.
  Calibration: $r - g = 1\%, 2\%$ and 3%.
- Epstein-Zin preferences. Role of intertemporal elasticity of substitution. Does it matter?

$$\begin{aligned} & [1 + (1 - e^{-\rho})(1-\gamma)V_t]^{\frac{1-\theta}{1-\gamma}} \\ = \quad & (1 - e^{-\rho}){C_t}^{1-\theta} + e^{-\rho}[1 + (1 - e^{-\rho})(1-\gamma)E_t V_{t+1}]^{\frac{1-\theta}{1-\gamma}} \\ s.t. \quad : \quad & K_{t+1} + C_t = R_t K_t \end{aligned}$$

  - Same welfare costs as iso-elastic preferences up to a term in order $O(\sigma^4)$, $\log(1+\lambda) = \frac{\theta}{\theta-1}\log(\frac{e^{r-g} - e^{-0.5(\theta-1)\gamma\sigma^2/\theta}}{e^{r-g}-1})$.
  - IES $1/\theta$ does not enter the (approximated) formula for costs of fluctuations(for small σ^2).

Initial Estimates of the costs of fluctuations (table 1)

Panel A: The Lucas statistical model

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
	0.04%	0.12%	0.20%
	($9)	($28)	($46)

Panel B: The AR(1) statistical model estimated by least squares

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.03%	0.10%	0.17%
	($8)	($24)	($40)
$r - g = 0.02$	0.04%	0.11%	0.18%
	($8)	($25)	($43)
$r - g = 0.01$	0.04%	0.12%	0.19%
	($9)	($27)	($45)

Panel C: The random walk economic model

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.21%	0.62%	1.03%
	($48)	($145)	($242)
$r - g = 0.02$	0.31%	0.94%	1.56%
	($73)	($219)	($365)
$r - g = 0.01$	0.63%	1.88%	3.14%
	($147)	($441)	($735)

Each cell shows the per capita costs of fluctuations as a fraction of consumption and, in brackets, in 2003 dollars. The standard deviation of shocks is 0.028, 0.011, and 0.011, for panels A to C respectively.

Statistical models of consumption, Lucas V.S. Hall

- Statistical tests cannot reject the Null hypothesis of "unit root"; but reject the Null of "trend stationary". (At 5% significance level)
- A model nests both cases

$$c_t - c_{t-1} = \overline{c} + u_t - \beta u_{t-1}$$

  - Using postwar consumption data, we get $\hat{\beta} = -0.36$(with s.d. 0.13)
  - Strongly reject Lucas case; reject Hall's model at 5% significance level.
  - Consumption growth is positively serially correlated!

Need a model with richer dynamics

- About the AR(1) in table 1?
  - For very persistent series like consumption, the LS estimate of η is downward-biased. For example, if the true model is a random walk, then the LS estimate of η will be below 1 with a probability of 68%.
- Local-to-unity Models: Model η as lying within a circle of radius c/n around 1 (n is sample size). As sample size $\uparrow$, consumption becomes closer to a R.W. After some time "n", the forecast error variance is indistinguishable from that of a R.W. For log utility, the welfare cost $\hat{L} = 0.5(1 - e^{g-r})\left[\sum_{t=0}^{n} e^{(g-r)t}\hat{v}(c_t) + \sum_{t=n+1}^{\infty} e^{(g-r)t}[\hat{v}(c_n) + \hat{v}(c_1)(t-n)]\right]$ where $\hat{v}(c_t)$ is the LS estimator of the forecast error variance t steps ahead. For a R.W., $Var(c_t) = \sigma^2 t$, so $Var(c_t) = Var(c_n) + Var(c_1)(t-n)$ $\hat{L} \longrightarrow \log(1+\lambda)$; For AR(1) $\frac{\hat{L}}{n} \Rightarrow 0.5\sigma^2 e^{g-r}\frac{e^{g-r+2U}-1}{g-r+2U}$, where U is R.V.

Median-unbiased estimates and 90% confidence intervals

Panel A: Costs in percentages of annual per capita consumption

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.21% (0.19 ; 0.21)	0.63% (0.57 ; 0.64)	1.05% (0.95 ; 1.07)
$r - g = 0.02$	0.31% (0.29 ; 0.32)	0.95% (0.87 ; 0.96)	1.58% (1.46 ; 1.61)
$r - g = 0.01$	0.63% (0.60 ; 0.64)	1.90% (1.81 ; 1.92)	3.19% (3.03 ; 3.22)

Panel B: Costs in annual per capita 2003 dollars

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	\$49 (44 ; 50)	\$147 (133 ; 150)	\$246 (223 ; 256)
$r - g = 0.02$	\$74 (68 ; 75)	\$222 (204 ; 225)	\$371 (341 ; 377)
$r - g = 0.01$	\$148 (140 ; 149)	\$446 (423 ; 450)	\$747 (709 ; 755)

Each cell shows the median unbiased estimate and, in parenthesis, the 90% confidence interval. The Ng and Perron (2001) modified BIC picked the autoregression's order.

Dickey-Fuller regression:

$$\Delta c_t = k_0 + k_1 t + q c_{t-1} + \sum_{j=1}^{m} \phi_j \Delta c_{t-j} + u_t$$

Initial Estimates of the costs of fluctuations (table 1)

Panel A: The Lucas statistical model

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
	0.04%	0.12%	0.20%
	($9)	($28)	($46)

Panel B: The AR(1) statistical model estimated by least squares

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.03%	0.10%	0.17%
	($8)	($24)	($40)
$r - g = 0.02$	0.04%	0.11%	0.18%
	($8)	($25)	($43)
$r - g = 0.01$	0.04%	0.12%	0.19%
	($9)	($27)	($45)

Panel C: The random walk economic model

	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.21%	0.62%	1.03%
	($48)	($145)	($242)
$r - g = 0.02$	0.31%	0.94%	1.56%
	($73)	($219)	($365)
$r - g = 0.01$	0.63%	1.88%	3.14%
	($147)	($441)	($735)

Each cell shows the per capita costs of fluctuations as a fraction of consumption and, in brackets, in 2003 dollars. The standard deviation of shocks is 0.028, 0.011, and 0.011, for panels A to C respectively.

ARMA models

Test does not reject the null that log difference of C is stationary!

Panel A: Estimated ARMA (2,2) model			
$(1-0.66L-0.32L^2)\Delta c_t = (1+1.03L+0.56L^2)u_t, \quad \sigma_u = 0.011$			
	$\gamma=1$	$\gamma=3$	$\gamma=5$
$r-g=0.03$	0.31% ($72)	0.94% ($219)	1.60% ($375)
$r-g=0.02$	0.47% ($109)	1.43% ($334)	2.47% ($579)
$r-g=0.01$	0.94% ($220)	2.93% ($687)	5.33% ($1248)
Panel B: Estimated ARMA (1,0) model			
$(1-0.34L)\Delta c_t = u_t, \quad \sigma_u = 0.010$			
	$\gamma=1$	$\gamma=3$	$\gamma=5$
$r-g=0.03$	0.40% ($94)	1.23% ($288)	2.13% ($498)
$r-g=0.02$	0.61% ($144)	1.89% ($442)	3.33% ($780)
$r-g=0.01$	1.25% ($292)	3.94% ($923)	7.40% ($1734)
Panel C: Estimated ARMA (0,1) model			
$\Delta c_t = (1+0.36L)u_t, \quad \sigma_u = 0.011$			
	$\gamma=1$	$\gamma=3$	$\gamma=5$
$r-g=0.03$	0.34% ($79)	1.02% ($240)	1.76% ($412)
$r-g=0.02$	0.51% ($120)	1.56% ($366)	2.73% ($638)
$r-g=0.01$	1.03% ($242)	3.23% ($752)	5.92% ($1388)

Economic models of consumption

Neoclassical stochastic growth model

$$\max_{\{C_t\}} E\left[\sum_{t=0}^{\infty} e^{-\rho t}\left(\frac{C_t^{1-\gamma}-1}{1-\gamma}\right)\right]$$
$$s.t. \quad : \quad K_{t+1}+C_t = A_t^{1-\alpha}K_t^{\alpha}+(1-\delta)K_t$$

- Key parameters: capital share α, process for A_t.
  - Physical + human capital $\alpha = 0.75$; physical capital $\alpha = 0.36$.
  - $a_t = \mu + \tau(1-\phi)t + \phi a_{t-1} + w_t$, with $w_t \tilde{} N(0, \varpi)$. $\phi = 0.9$ or 1 (Prescott 1986)

Costs of fluctuation in the stochastic growth model

Panel A: Stationary productivity and strongly diminishing returns			
	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.07% ($16)	0.08% ($19)	0.09% ($22)
$r - g = 0.02$	0.07% ($17)	0.09% ($21)	0,10% ($24)
$r - g = 0.01$	0.08% ($19)	0.10% ($24)	0.12% ($27)
Panel B: Stationary productivity and mildly diminishing returns			
	$\gamma = 1$	$\gamma = 3$	$\gamma = 5$
$r - g = 0.03$	0.17% ($40)	0.15% ($36)	0.13% ($31)
$r - g = 0.02$	0.18% ($43)	0.18% ($42)	0.16% ($38)
$r - g = 0.01$	0.21% ($48)	0.23% ($55)	0.23% ($55)
Panel C: Non-stationary productivity			
	$\alpha = 0.36$	$\alpha = 0.75$	
$r - g = 0.03$	0.55% ($129)	0.26% ($60)	
$r - g = 0.02$	0.85% ($199)	0.39% ($92)	
$r - g = 0.01$	1.76% ($412)	0.81% ($190)	

Conclusion

- Persistence of shocks is a key determinant of costs of fluctuations.
  - As persistence increases, costs of fluctuation rise substantially.
- If an economist was able to come up with a policy that...made a country grow 1% faster forever, his work would have a more importance on society's welfare than probably any other economist has ever had. Until this happens though, lowering inflation, reducing taxes on capital income, and dampening consumption fluctuations, are aims that are within the grasp of our knowledge. If better stabilization policy can bring society a gain of $200 billions, this is a large enough impact on well-being to motivate the work of a modest economist.