DSGE Models in a Data-Rich Environment

Jean Boivin and Marc Giannoni, 2006

Motivation

- Conventional estimation practice for DSGE models.
  - Assume economic variable is properly measured by a single index.
  - All relevant informatin for estimation is summarized by a small number of data series.
- Possible solutions.
  - Measurement error. (Sargent, 1989)
  - Dynamic index model — a large panel data set contains useful information for estimation of the state of the economy. (Quah and Sargent, 1993)

An Empirical example

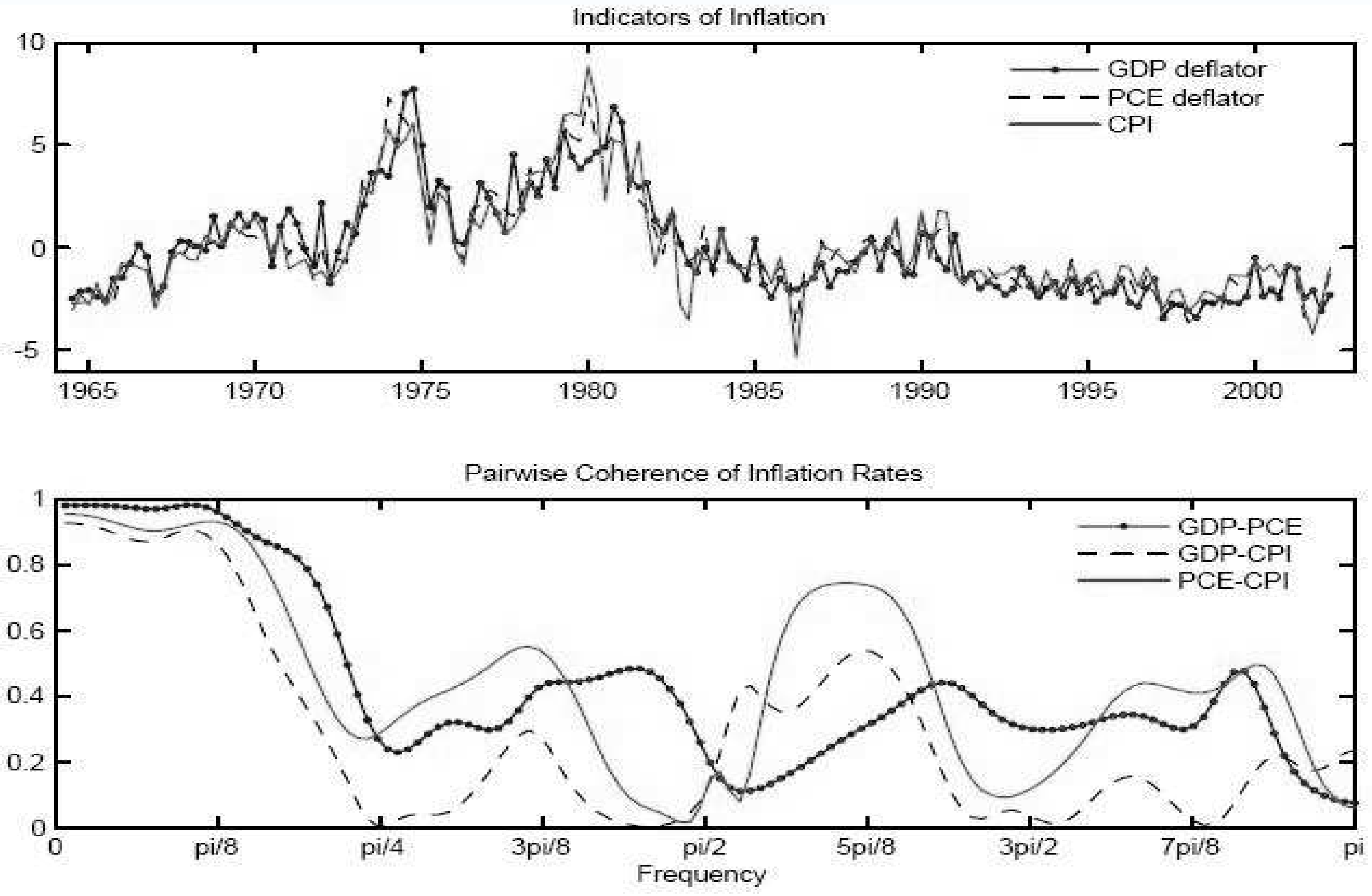

Figure 2: Indicators of quarterly inflation rates (de-meaned) and pairwise coherence.

Main Contribution

- Take a structure-factor approach to study the problem of measurement error and state estimation.
  - Structure restrictions imposed by DSGE models.
  - Advantage: factor will have its economic meanings.

Usefulness of Cross-sectional information

- State equation:

$$f_t = \rho f_{t-1} + \eta_t, |\rho| < 1$$

  where η_t is called "structural shock".
- Measurement equation:

$$x_{1t} = f_t + e_{1t}$$

  where x_{1t} is a noisy indicator of f_t, and e_{1t} is the measurement error.
- when $\rho \neq 0$, use Kalman filter.
- when $\rho = 0$, cannot identify η_t and e_{1t} separately.
- Additional indicators:

$$x_{it} = f_t + e_{it}, i = 2, 3, ..., n_x$$

- f_t can be understood as a "common factor", which can be identified through cross-section.
- Beyond "identification": efficient estimation of states/factors, as well as measurement of economic variables whose indicators are available.

The Model

- Linear (or linearized) rational expectations model.

$$
\begin{aligned}
A \cdot E_t \begin{bmatrix} z_{t+1} \\ Z_{t+1} \end{bmatrix} &= B \begin{bmatrix} z_t \\ Z_t \end{bmatrix} + C s_t \\
s_t &= M s_{t-1} + \varepsilon_t
\end{aligned}
$$

- Information set: $I_t = \{z_k, Z_{k+1}, s_k, \varepsilon_k, k \leq t; A, B, C, Q\}$
- Numerical solution

$$
\begin{aligned}
z_t &= D S_t \\
S_t &= G S_{t-1} + H \varepsilon_t
\end{aligned}
$$

where $S_t = \begin{bmatrix} Z_t \\ s_t \end{bmatrix}$

An Example

- A log-linearized RBC model

$$\begin{bmatrix} \hat{l}_t \\ \hat{c}_t \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix}$$

$$\begin{bmatrix} \hat{k}_t \\ a_t \end{bmatrix} = \begin{bmatrix} g & h\rho \\ 0 & \rho \end{bmatrix} \begin{bmatrix} \hat{k}_{t-1} \\ a_{t-1} \end{bmatrix} + \begin{bmatrix} h \\ 1 \end{bmatrix} v_t$$

Here $z_t = \left[\hat{l}_t, \hat{c}_t\right]', S_t = \left[\hat{k}_t, a_t\right]'$

The Model, continued

- Economic variable whose indicators are available.

$$F_t \equiv F\begin{bmatrix} z_t \\ S_t \end{bmatrix} = \Phi S_t$$

- Observation equation.
  - Part 1: Structure-factor representation.

$$X_{F,t} = \Lambda_F F_t + e_{F,t}$$

    where Λ_F is highly structured: 1)at most 1 nonzero element for each row — each indicator corresponds to only one economic variable. 2) column allows many nonzero terms — economic variable is imperfectly measured by many indicators.
  - Part 2: Nonstructure-factor representation.

$$X_{S,t} = \Lambda_S S_t + e_{S,t}$$

    where Λ_S is not restricted.
  - Combine the above two: $X_t = \Lambda S_t + e_t$ by noticing that $F_t = \Phi S_t$

State Space Representation

- $$\begin{aligned} S_t &= GS_{t-1} + H\varepsilon_t \\ X_t &= \Lambda S_t + e_t \end{aligned}$$
- Cross-sectional information is useful for estimation of states.
- Dynamic structure is used to uncover structure shocks.

Estimation

- Bayesian approach.
- Alternative: EM method using Kalman filter.
- data: post-1982.

Open Issues

- Including pre-1982 data.
  - Modelling of the "great moderation".
  - Drifting coefficients and volatilities.
- Optimal choice of indicators.