Liquidity and Risk Management

American Economic Review Papers and Proceedings, 2007

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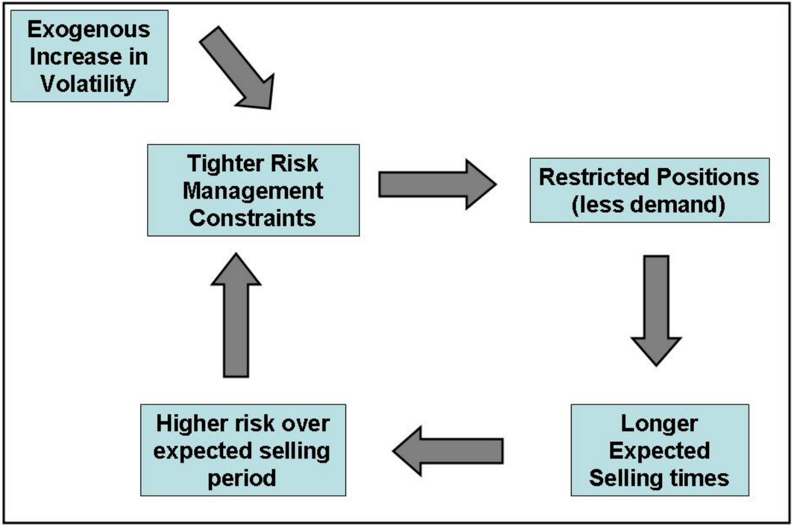
Reading Group
October 23, 2008

Motivation

- Risk Management
  - Risk Management Constraints widely used to allocate capital within financial institutions
  - Aggregate Effects: Methodological Herding
- Liquidity
  - Liquidity affects length of time required to sell an asset
  - Risk management constraints measure risk over expected selling period
- This Paper:
  - Models the interaction between (shared) risk management practices and market liquidity
  - Main Finding: A feedback effect can arise: tighter risk management leads to less liquidity in the market, this illiquidity further tightens risk management, can lead to sudden drops in liquidity and prices

Feedback Effect

Risk Management Affects Liquidity and Liquidity Affects Risk Management



Assets

Two Securities:

Liquid security with risk free return r

Illiquid risky security

- risky dividend-rate process, X
- X(t) follows a Levy process w/finite variance and zero drift, e.g. Brownian motion

$$dX(t) = \sigma_x dB(t)$$

Agents

Continuum of Agents

- Risk neutral, infinitely lived
- Maximize lifetime consumption flow
- Intrinsic Agent Types:
  - Type $i \in \{h, l\}$
  - Type process follows a Markov chain, independent across agents
  - i switches from l to h with intensity λ_u
  - i switches from h to l with intensity λ_d

Risk Management

An agent of type i holding θ_t shares of the asset incurs a holding cost of $\delta > 0$ if he violates his *risk management constraint*

$$Var_t(\theta_t[P(X_{t+\tau}) - P(X_t)]) \leq (\sigma^i)^2$$

σ^i: risk bearing capacity

$\sigma^h = \bar{\sigma} > 0$ and $\sigma^l = 0$

τ: holding period

Requirements on Risk Measure:

- risk measure increases with the size of the position in the risky security
- risk measure increases with the length of the time period over which the risk is assessed

Risk Management

Two types of risk measurement:

- Simple risk management: variance of the position computed over a fixed horizon τ (VaR)
- Liquidity adjusted risk management: variance of the position computed over the time required to sell the asset (LVaR)

Liquidity

Illiquidity due to Search and Bargaining Costs (DGP (2005,2007))

Meeting Rates:

- meeting times: successive event times of a Poisson process with intensity λ
- successful meeting rate for one agent initiating meetings with a relevant subpopulation (potential buyers) of size μ_b: $\lambda\mu_b$
- successful meeting rate between all agents in a subpopulation of size μ_s (potential sellers) initiating meetings with a subpopulation (potential buyers) of size μ_b: $\mu_s\lambda\mu_b$
- Total meeting rate between potential sellers and buyers when both sides initiate is $2\mu_s\lambda\mu_b$

Negotiation:

- Nash bargaining between buyer and seller, seller has (exogenous) bargaining power $q \in (0, 1)$

Distribution of Types

Restrict our attention to equilibria in which:

- Agents hold either 0 or $\bar{\theta}$ units of the asset
- $\bar{\theta}$ is the largest position that satisfies the risk management constraint of the high risk bearing type

$$\text{Set of Agent Types } T = \{ho, hn, lo, ln\}$$

stationary Distribution over types

$$\mu_{ho} + \mu_{hn} + \mu_{lo} + \mu_{ln} = 1$$

and

$$\Theta = \bar{\theta}(\mu_{ho} + \mu_{lo})$$

The Investor's Problem

$$U(W_t, \sigma(t), X(t), t) = \sup_{C,\theta} E_t \int_0^\infty e^{-rs} dC_{t+s}$$

s.t.

$$dW_t = rW_t dt - dC_t + \theta_t (X(t) - \delta 1_{(\sigma^\theta(t)=lo)}) dt - P_t d\theta_t$$

$P(t)$ depends on $X(t)$ *and* on distribution of types (liquidity)

Equilibrium

Equilibrium:

- steady state distribution μ over types
- allocation: $\bar{\theta}$
- price: $P(X, \mu)$
- Value Function: $V(W, \mu, X, P)$

s.t.

- Optimality: Given prices and stationary distribution, allocations are optimal and V solves investor's problem
- Market Clearing: $\Theta = \bar{\theta}(\mu_{lo} + \mu_{ho})$
- $\bar{\theta}$=maximum value of VaR constraint given prices and stationary distribution

Characterizing the Equilibrium

Key insight: only encounter that provides gains from trade is one in which low-type owners sell to high-type non owners,bargaining theory tells us that if two such parties meet, trade occurs immediately

$\Rightarrow$ Asset allocations (in steady state equilibrium) can be determined without reference to prices

This "separability" allows us to solve the model as follows:

- Solve for Steady State distribution, given $\bar{\theta}$
- Solve for Prices (and Value Function), taking distribution over types as given
- Risk Management Constraint (and thus $\bar{\theta}$) depends on prices (and steady state distribution)
- Fixed Point: $\bar{\theta} \Rightarrow$ Prices $\Rightarrow \bar{\theta}$

Equilibrium

Steady State Distribution

Given $\bar{\theta}$, μ_{lo} must remain constant

$$0 = -2\lambda\mu_{hn}(t)\mu_{lo}(t) - \lambda_u\mu_{lo}(t) + \lambda_d\mu_{ho}(t)$$

3 more such equations for $\mu_{ho}, \mu_{lo}, \mu_{ln}$

$$\mu_{ho} + \mu_{hn} + \mu_{lo} + \mu_{ln} = 1,$$

$$\Theta = \bar{\theta}(\mu_{ho} + \mu_{lo})$$

DGP (2005) show that a unique stable steady-state mass distribution exists as long as $\bar{\theta} \geq \Theta$

Equilibrium

Value Function

$$V_\varsigma(X(t), W(t)) = W_t + 1_{\varsigma \in \{ho, lo\}} \frac{\bar{\theta} X(t)}{r} + \bar{\theta}\nu_\varsigma$$

ν_ς depends on price,distribution over types, and δ

Bargaining:
Seller's reservation price: $V_{ln} + P\bar{\theta} > V_{lo} \Rightarrow P\bar{\theta} > V_{lo} - V_{ln}$
Reservation Price of buyer: $V_{ho} - P\bar{\theta} > V_{hn} \Rightarrow P\bar{\theta} < V_{ho} - V_{hn}$
Nash Bargaining with a seller bargaining power of $q \in (0, 1)$ yields

$$P\bar{\theta} = (V_{lo} - V_{ln})(1 - q) + (V_{ho} - V_{hn})q$$

Equilibrium

Fixed Point: $V(P) \Rightarrow P(V)$

4 HJB equations, zero change in steady state utility for agent of each type, e.g. type lo:

$$0 = -r\nu_{lo} + \lambda_u(\nu_{ho} - \nu_{lo}) + 2\lambda\mu_{hn}(P - \nu_{lo} + \nu_{ln}) - \delta$$

bargaining price relationship

$$P\bar{\theta} = (V_{lo} - V_{ln})(1-q) + (V_{lo} - V_{ln})q$$

Price expression given by:

$$P(X_t) = \frac{X_t}{r} - \frac{\delta}{r}\frac{r(1-q) + \lambda_d + 2\lambda\mu_{lo}(1-q)}{r + \lambda_d + 2\lambda\mu_{lo}(1-q) + \lambda_u + 2\lambda\mu_{hn}q}$$

Equilibrium

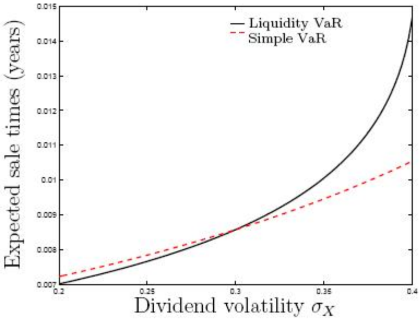
$$Var_t(\theta_t[P(X_{t+\tau}) - P(X_t)]) \leq (\sigma^i)^2$$

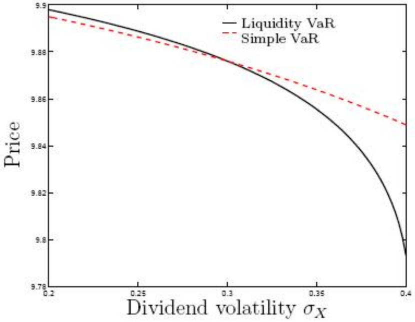
$$\bar{\theta} \Rightarrow \text{Price} \Rightarrow \bar{\theta}$$

- Simple risk management, τ is exogenous: $\bar{\theta} = \frac{r\bar{\sigma}}{\sigma_x}\frac{1}{\tau}$
- Liquidity Adjusted Risk Management, expected selling time - endogenously determined: $\bar{\theta} = \frac{r\bar{\sigma}}{\sigma_x}\sqrt{(2\lambda\mu_{hn})}$

Results

Comparative Statics: Comparison of Steady States





Results

Comparative Statics: Comparison of Steady States

Proposition 2 *Suppose that $\bar{\sigma}$ is large enough for existence of an equilibrium. Consider a stable equilibrium with liquidity-adjusted risk management and let $\tau = \frac{1}{2\lambda\mu_{hn}}$, which means that the equilibrium allocations and price are the same with simple risk management.*

Consider any combination of the conditions (a) higher dividend volatility σ_X, (b) lower risk limit $\bar{\sigma}$, (c) lower meeting intensity λ, (d) lower switching intensity λ_u to the high risk-bearing state, and (e) higher switching intensity λ_d to the low risk-bearing state. Then, (i) the equilibrium position $\bar{\theta}$ decreases, (ii) expected search times for selling increase, and (iii) prices decrease. All three effects are larger with liquidity-adjusted risk management.

The Big Picture

Model generates the abrupt changes in prices and selling times that characterize crises

Why is this Important?

- Policy Implications - aggregate risk cannot be diversified away
- Pricing Implications - liquidity is priced, binding risk management constraints constitute an additional risk factor in the economy