Raouf Boucekkine
Fernando del Rio
Omar Licandro

Endogenous vs Exogenously Driven Fluctuations in Vintage Capital Models

published in Journal of Economic Theory (1999)

presented by
Matthias Kredler

Sargent Reading Group
10 October 2006

Main question of the paper

- Recurrence of business fluctuations: "A boom is necessarily followed by a recession and vice versa"
- Thus they want to study a "fundamental source of endogenous fluctuations, the replacement of old equipment".
- Also, want to study how this interacts with exogenously driven fluctuations.

Two sectors

- Produced good c: Used in consumption and investment
- Non-produced good m (in the spirit of Caballero and Hammour, 1996):
  - Consumed by households
  - Used as intermediate input in production
  - Economy is endowed with $\bar{m}$ of it every period
  - Its price is an exogenously given deterministic sequence

Technology

- The technology for the produced good c is Leontieff. If we throw in
  - $e^{\gamma\tau}$ unit of capital of vintage τ
  - one unit of labor
  - one unit of m

  we get $e^{\gamma t}$ units of c. "Labor-augmenting technical progress is continuously embodied in new capital goods".
- Capital of the newest vintage t can be obtainted one-to-one from the consumption good.
- Capital cannot be turned into c again.

Aggregate output

- Denote by $T(t)$ the age of the oldest vintage alive at t. Then aggregate output at t is

$$y(t) = \int_{t-T(t)}^{t} e^{\gamma\tau} h(\tau) d\tau,$$

where $h(\tau)$ is employment associated with vintage τ.

- Employment is

$$l(t) = \int_{t-T(t)}^{t} h(\tau) d\tau$$

- Unemployment is $u(t) = 1 - l(t)$.

Preferences

- There is a continuum of identical agents with measure one. Preferences are linear:

$$\int_0^\infty e^{-\rho t}\Big[c(t) + p(t)e^{\gamma t}m(t)\Big]dt$$

  $e^{\gamma t}p(t)$ is marginal utility of the non-produced good at t. So the price of non-produced goods must be $p(t)e^{\gamma t}$ in equilibrium.

- No disutility of labor, labor supply normalized to 1.

Time-τ cash flow of vintage-t firm

- Revenue (one unit of labor): $e^{\gamma t}$
- Labor costs: $e^{\gamma\tau}\tilde{w}(\tau)$,
  where $\tilde{w}(\tau)$ is "the worker's shadow wage at τ".
- Intermediate-input costs: $e^{\gamma\tau}p(\tau)$

(all quantities in units of time-τ consumption good)

Optimal scrapping time $J(t)$

- Want to maximize
$$\int_t^{t+j} \underbrace{e^{-\rho(\tau-t)}}_{\text{discounting}} \Big[\underbrace{e^{\gamma t} - e^{\gamma\tau}\tilde{w}(\tau) - e^{\gamma\tau}p(\tau)}_{\text{cashflow at } \tau} \Big] d\tau$$
- The optimal scrapping time $J(t)$ for capital of vintage t fulfills
$$p[t + J(t)] + \tilde{w}[t + J(t)] = e^{-\gamma J(t)}$$
- The job is destroyed when it becomes profitable to re-allocate labor and non-produced resources to the latest technology.
- The relationship to the age of the oldest capital in use $T(t)$ is
$$J(t) = T[t + J(t)]$$

Timing for labor market (my interpretation)

- Firm sets up capital stock of vintage t at time t.
- Worker decides if to join firm or not.
- Nash bargaining over the appropriable surplus.
- From t until $J(t)$, production occurs and both parties recieve the agreed-upon shares.

Appropriable surplus and bargaining

- Appropriable surplus (this is *after* having invested in the capital stock) of one job in vintage τ is

$$\pi(t) = \int_t^{t+J(t)} e^{-\rho(\tau - t)} \Big[e^{\gamma t} - e^{\gamma \tau} \tilde{w}(\tau) - e^{\gamma \tau} p(\tau) \Big] d\tau$$

- Nash bargaining: β goes to the worker, and $1 - \beta$ to the firm.

Resulting equilibrium conditions

- Zero profits:

$$(1-\beta)\pi(t) = \underbrace{e^{\gamma t}}_{\text{investment at } t}$$

- Unemployed are indifferent between accepting job or not:

$$e^{\gamma t}\tilde{w}(t) = \underbrace{\frac{h(t)}{u(t)}\beta\pi(t)}_{\text{expected utility flow for unemployed}}$$

Market clearing

- Non-produced good:

$$\bar{m} = \underbrace{m(t)}_{\text{consumed}} + \underbrace{l(t)}_{\text{used as intermediate input}}$$

- Produced good:

$$y(t) = c(t) + \underbrace{e^{\gamma t} h(t)}_{\text{investment}}$$

Collect equilibrium conditions

- For a given path of $p(t)$ and given initial conditions $h(t) \geq 0$, $\forall t < 0$, an equilibrium is a path for $T(t)$, $J(t)$, $h(t)$ and $u(t)$ such that

$$
\begin{aligned}
u(t) &= 1 - \int_{t-T(t)}^{t} h(\tau) d\tau \\
\frac{h(t)}{u(t)} \frac{\beta}{1-\beta} &= e^{-\gamma T(t)} - p(t) \\
\frac{1}{1-\beta} &= \int_{t}^{t+J(t)} e^{-\rho(\tau - t)} \left[1 - e^{-\gamma(t - \tau + T(\tau))} \right] d\tau \\
J(t) &= T(t + J(t))
\end{aligned}
$$

- Also, we need $0 \leq u(t) \leq 1$ and $p(t) < e^{-\gamma T(t)}$.

Results (I)

- All results in the following apply to the case

$$p(t) = p_0 + p_1 \sin(p_2 t),$$

  with period $\Omega = 2\pi/p_2$.

- Optimal scrapping is constant:

$$T(t) = J(t) = T^0$$

  (the *endogenous period*).

Results (II)

- The planner sets unemployment to zero: The exogenous cycle is not important, the endogenous period dominates.
- For the decentralized economy, there can be positive unemployment.

Differential difference equation for decentralized economy

- Differential-difference equation (DDE) for jobs in vintage t:

$$h'(t) = k_1(t)h(t) + k_2(t)h(t - T_0),$$

where $k_1(\cdot)$ and $k_2(\cdot)$ are functions of period Ω.

- Can be solved via successive resolutions of orinary differential equations on intervals of length T^0.
- It is not clear at the outset which periodicity (T^0 or Ω) will dominate in the long run.

Limit behavior

- If T^0/Ω is a rational number, the authors show that the exogenous cycle Ω dominates in the end.
- Mathematically spoken: The solution paths for job creation converge to a limit cycle with period Ω.
- But: Short-run fluctuations are governed mainly by replacement echoes (see numerical examples).

Figures describing dynamics

- No exogenous cycle: See page 15 in the paper
- Introduce exogenous cycle: The numerical examples show that replacement echoes dominate in the short run; see page 20 in the paper

The authors' main conclusions

- Replacement echoes dominate in short run: period T^0.
- Exogenous cycle dominates in the long run: period Ω.
- The transition displays very interesting characteristics: asymmetries and highly irregular patterns.