“Subjective Beliefs and Ex-Ante Trade”

Rigotti, Shannon, and Strzalecki (2008)

Motivation

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- Possible explanation: maybe not SEU maximizers

  → How to characterize the desire for betting in general?

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  → Identify conditions on subjective beliefs to have betting

Outline

1. Convex preferences
2. Subjective beliefs
3. Special cases
4. Ex-ante trade

Convex preferences

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- $X = \mathbb{R}_+$ set of (monetary) consequences
- $\mathcal{F} = \mathbb{R}_+^S$ set of (monetary) acts

$\rightarrow$ Act is $f = (f(1), ..., f(S))$: "pays" you money in each state

Convex preferences (cont'd)

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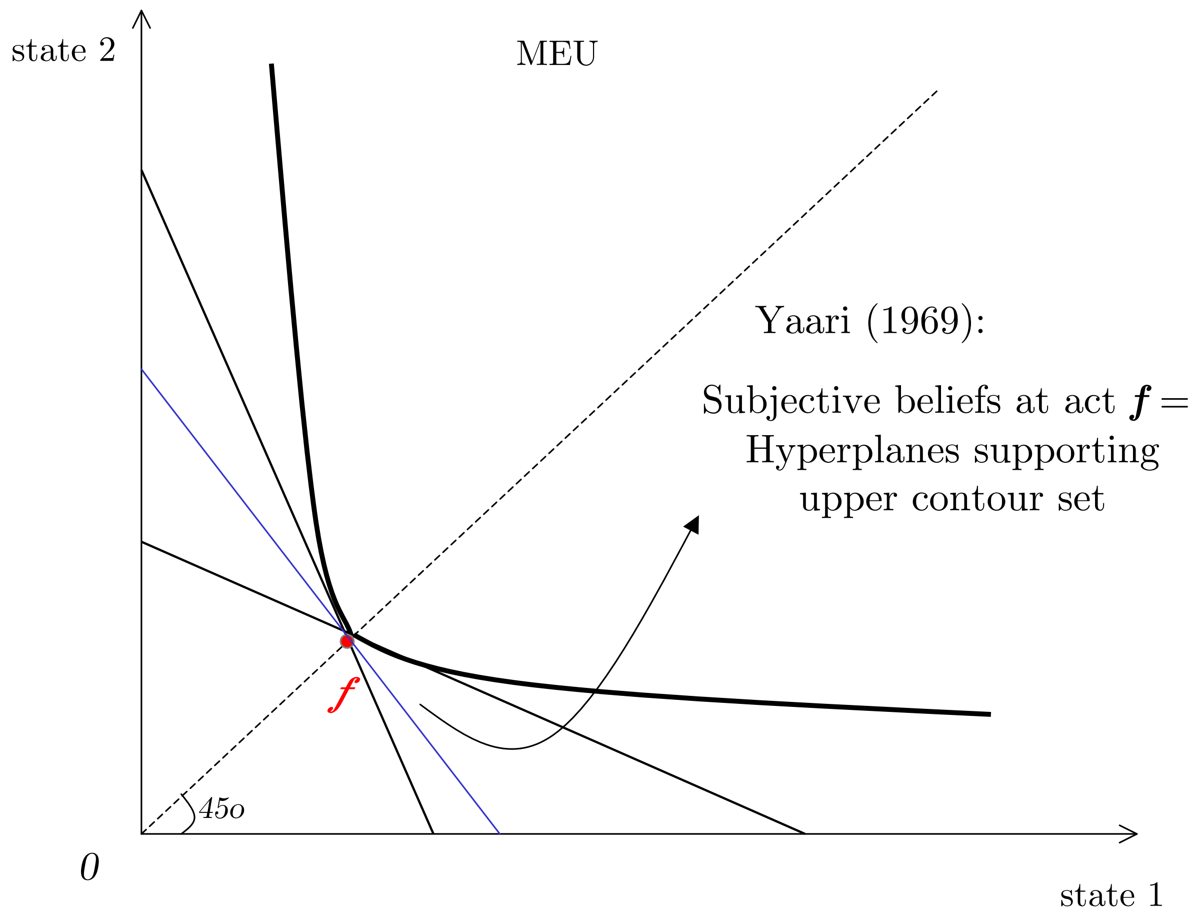
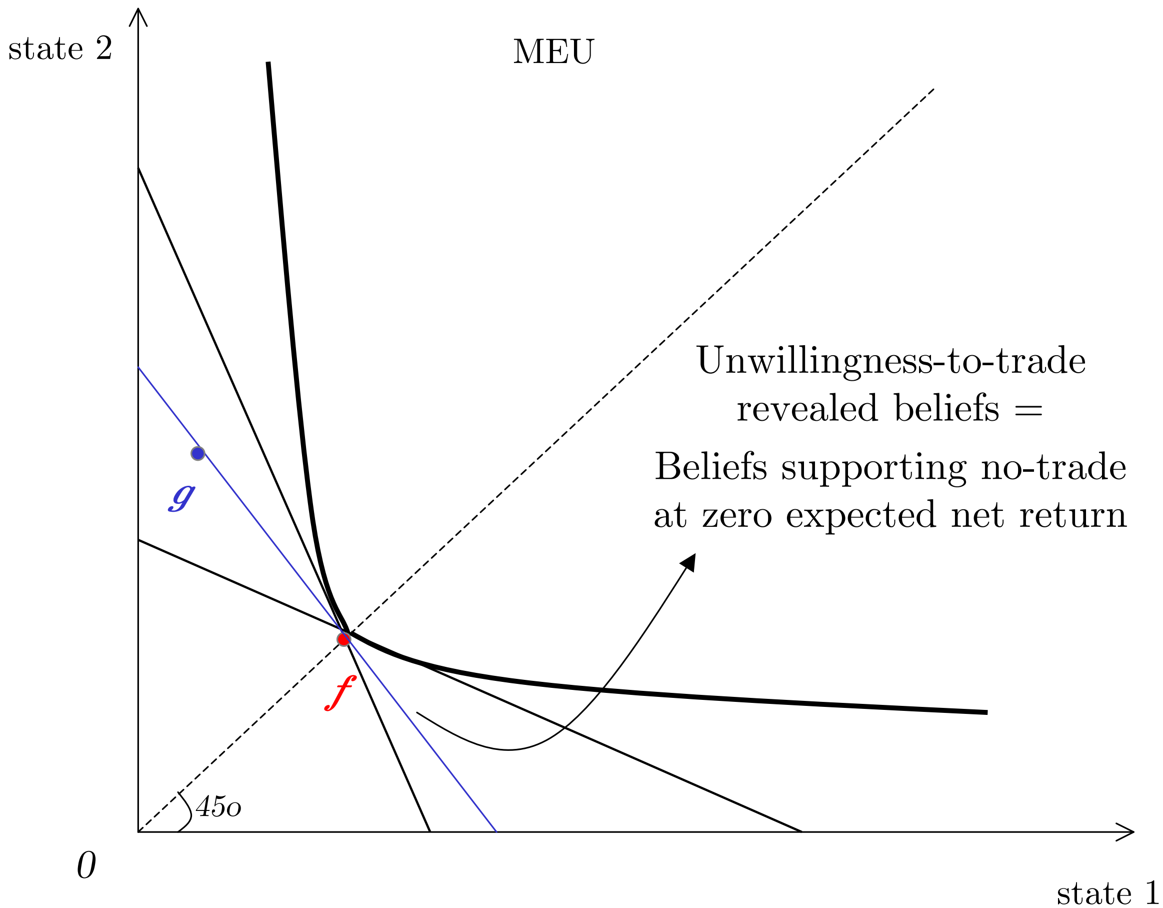
- Note: $\exists\ V \in \mathbb{R}^{\mathcal{F}}$ continuous, increasing and quasi-concave that represents $\succcurlyeq$

Subjective beliefs



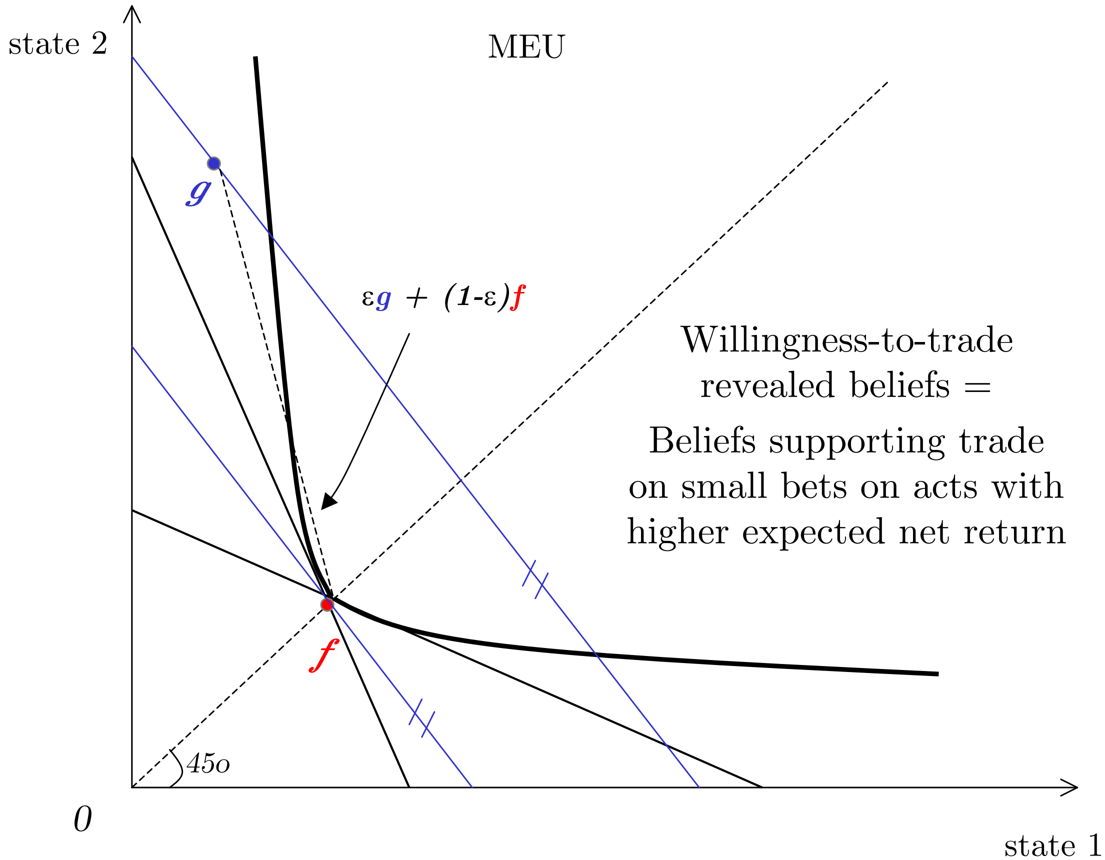
$$\pi(f) := \{p \in \Delta S : p \cdot g \geq p \cdot f \text{ for all } g \succcurlyeq f\}$$

Subjective beliefs (cont'd)



$$\pi^{u}(f) := \{p \in \Delta S : f \succcurlyeq g \text{ for all } g \text{ s.t. } \mathbb{E}_p f = \mathbb{E}_p p\}$$

Subjective beliefs (cont'd)



$$P(f) = \{P \subseteq \Delta S : \mathbb{E}_p g > \mathbb{E}_p f \ \forall\ p \in P \Rightarrow \varepsilon g + (1-\varepsilon) f \succ f,\ \exists\ \varepsilon > 0\ \}$$

$$\pi^w(f) := \cap_{P \in \mathcal{P}(f)}\ P$$

Subjective beliefs (cont'd)

Proposition 1: $\succcurlyeq$ convex preference relation

$$\Rightarrow \pi(f) = \pi^u(f) = \pi^w(f), \text{ any } f \in \mathcal{F}$$

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- Other: MEU (GS (1989)), "confidence preferences" (Chateauneuf and Faro (2006)), "smooth model" (Klibanoff, Marinacci and Mukerji (2005)), "Ergin-Gul model" (2004)

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Proposition 3: Let $\succcurlyeq$ be a variational preference for which u is concave, increasing and differentiable. Then $\succcurlyeq$ is a convex preference and

$$\pi(f) = \left\{ \frac{q}{\|q\|} : q = pDU(f), \text{ some } p \in M(f) \right\}.$$

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Proof. Consequence of MMR (2006) + Lemma 1:
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- Note $\pi(x) = \{p \in \Delta S : c^{\star}(p) = 0\}$ = GS set of priors = singleton for multiplier preferences

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- Remark: finite state space (extends to infinite case)

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- allocation $(f_1, ..., f_m) \in \mathcal{F}$ is feasible if $\sum_{i=1}^m f_i = e$
- allocation $(f_1, ..., f_m) \in \mathcal{F}^m$ is Pareto optimal if there is no other allocation $(g_1, ..., g_m) \in \mathcal{F}^m$ such that $g_i \succcurlyeq_i f_i$ for all i, and $g_j \succ_j f_j$ for some j

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Proposition 7: $\succcurlyeq_i$ convex preference relation for each i. Then an interior allocation $(f_1, ..., f_m) \in \mathcal{F}^m$ is Pareto optimal iff $\cap_{i=1}^{m} \pi_i\,(f_i) \neq \emptyset$

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"$\Rightarrow$" Use second welfare theorem to find (normalized) supporting price vector p, and deduce $p \in \pi_i (f_i)$, all i

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"$\Leftarrow$" Use first welfare theorem to argue : $(f_1, ..., f_m; p)$ CE in an economy with endowments $(f_1, ..., f_m) \Rightarrow (f_1, ..., f_m)$ is Pareto optimal

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  (A6) For all $f \neq g$ and $\alpha \in (0,1)$, $f \succcurlyeq g \Rightarrow \alpha f + (1-\alpha)\, g \succ g$

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  (A7) $\forall$ $g \in \mathbb{R}^S$, $\forall$ constant acts $x, x' > 0$: $x + \lambda g \succcurlyeq x$ for some $\lambda > 0 \Rightarrow x' + \lambda' g \succcurlyeq x'$ for some $\lambda' > 0$

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Proposition 8. $\succcurlyeq$ convex preference relation satisfying (A7). Then $\pi := \pi(x) = \pi(x')$ for all constant acts $x, x' > 0$

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Proposition 9. Let each $\succcurlyeq_i$ satisfy (A1)-(A7). Then the following are equivalent:

(i) $\exists$ interior FI Pareto optimal allocation

(ii) Any Pareto optimal allocation is FI

(iii) Every FI allocation is Pareto optimal

(iv) $\cap_{i=1}^{m} \pi_i \neq \emptyset$ ($\pi_i = \pi_i(x) =$ beliefs revealed at constant acts)