Asset Pricing with Heterogeneous Consumers

Constantinides and Duffie (1996)

"It is madness to make fortune
the mistress of events, because by herself
she is nothing and is ruled by prudence,"
John Dryden

Introduction

- Euler equations in a Lucas('78)-type exchange economy are unable to generate a high equity premium or a low risk-free rate.

- A number of solutions within a representative-consumer framework have been proposed:

  1. time-nonseparable preferences
  2. recursive preferences
  3. transaction costs
  4. rare-events.

- With complete markets, these economies provide full consumption insurance, the existence of which has been rejected by the data.

Introduction

- An economy with incomplete markets and heterogenous consumers is intuitively appealing, offering a broad class of pricing kernels.
- However, early attempts at modeling and calibrating such an economy (Mankiw ('86), Lucas ('91), and Telmer ('93)) have not resulted in an empirically plausible pricing kernel.
- Just one security provides "enough" risk-sharing. The set of SDF's available are still too similar to the SDF from the complete markets, representative consumer economy.

Should we give up hope?

- Exercise: To see if there is a simple model with idiosyncratic income shocks of some-level of persistence that has a chance of matching stylized facts on returns.

- Empirical plausibility in many other dimensions is left aside - as well as discussion of whether or not their results are knife edge (persistence vs. stationarity vs. mean-reverting: just persistence should be enough, but this is left out).

My intuition

To match data, we'd like a large SDF. But, by no arb, there's an upper bound on the SDF, which is a function of the agent with the worst possible $V_{c,t+1}/V_{c,t}$.

In the calibrated versions above, the idiosyncratic shocks are not persistent enough to generate large deviations in the agents' value functions.

Similar to Levine and Zame ('02), which notes that as agents become more patient, the set of possible SDFs approaches the complete markets' SDF in certain economies.

A large SDF will suffice to get a lower risk-free rate.

However, to get large equity premium, we need more - we need the large changes in the SDF to be correlated with the equity return.

The Economy

Assets

- n securities with dividend d_{jt} and ex-dividend price P_{jt}.
- $d = (d_1, ..., d_n)$ and $P = (P_1, ..., P_n)$: n-dim dividend and price processes
- $D_t = \sum_{j=1}^{n} d_{it}$: the aggregate dividend.
- Each security is in positive fixed supply, normalized to one.

- Default free discount bonds of all maturities less than or equal to fixed T.
- $B_t = (B_{t,t+T}, ..., B_{t,t+1})$: T-dim bond price process.
- $\hat{B}_t = (B_{t,t+T-1}, ..., B_{t,t+1}, 1)$.
- Bonds are in zero net supply.

Assets

They claim T is WOLG.

WRONG!

It is WOLG for the particular equilibrium they end up selecting (no trade and unit root, all consumers' idiosyncratic shocks are equally exposed to aggregate variables.) If agents wanted to trade bonds in eq. then bonds of different maturities can be used to hedge at least partially against the risk of changes in x-sectional income distribution, thus dropping it out of a pricing kernel and killing their larger risk premium.

Consumers

- Continuum of Consumers with homogeneous CRRA preferences

$$E[(1-\alpha)^{-1}\sum_{t=0}^{\infty} e^{-\rho t} C_{it}^{1-\alpha} | J_o]$$

- and budget constraint:

$$C_{it} = I_{it} + \theta_{i,t-1} \cdot (P_t + d_t) + b_{i,t-1} \cdot \hat{B}_t - \theta_{it} \cdot P_t - b_{it} \cdot B_t$$

Aggregates and Equilibrium

- Aggregate labor income: I_t
- Aggregate consumption $C_t = I_t + D_t > 0$

- Agents have common information sets including, at minimum, histories of dividends, agg and disagg labor income, and securities' and bonds' prices.

- All consumers enter period 0 with symmetric endowments of securities and zero bonds.

An equilibrium is a security and bond price process (P, B) and optimal strategies for all consumers (θ_i, b_i, C_i) given (P, B) such that markets clear.

An Equilibrium

No arbitrage implies the existence of a str. pos pricing kernel M_t such that:

$$P_{jt} = \frac{1}{M_t} E[\sum_{s=t+1}^{\infty} d_{js} M_s | J_t]$$

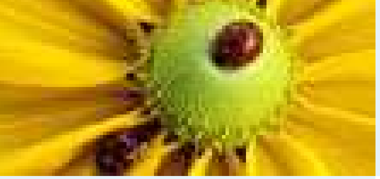
and

$$B_{t,t+s} = \frac{1}{M_t} E[M_{t+s} | J_t]$$

Assumptions:

1. Transversality: $E[M_t] \rightarrow 0$ as $t \rightarrow \infty$
2. $\frac{M_{t+1}}{M_t} \geq e^{-\rho} (\frac{C_{t+1}}{C_t})^{-\alpha}$

The latter implies: $E[R_{t+1} e^{-\rho} (\frac{C_{t+1}}{C_t})^{-\alpha} | J_t] \leq 1$ if $R_{t+1} \geq 0$. Unlike typical Euler Equality, this one is not rejected by the data.



No trade Equilibrium

Consumer i's labor income:

$$I_{it} = \delta_{it} C_t - D_t$$

where:

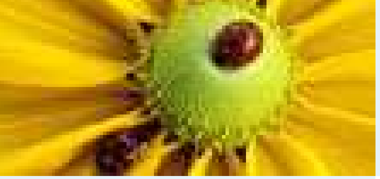
$$\log(\delta_{it}) = \sum_{s=1}^{t} (\eta_{is} y_s - \frac{y_s^2}{2})$$

$$y_t = \sqrt{\frac{2}{\alpha^2 + \alpha}} [\log(\frac{M_t}{M_{t-1}}) + \rho + \alpha \log(\frac{C_t}{C_{t-1}})]^{1/2}$$

where η_{it} are IID SN.

Thus the distribution of innovations to $\log \delta_{it}$ are **symmetric** across agents at t and independent across time.

Note: law of large numbers across agents implies goods market clearing.



Comment

If we had a mean reverting process instead, with the speed of mean-reversion time-varying:

$$\delta_{it} = y_s \delta_{i,t-1} + \eta_{it},$$

then agents will trade bonds, possibly eliminating increased risk premia.

Note: η not independent of aggregates. The part of idiosyncratic risk that is correlated with aggregate risk asymmetrically across agents will be traded away if possible.

Intuition

With no trade, MRS of consumer i:

$$e^{-\rho}(\frac{C_{i,t+1}}{C_{it}})^{-\alpha} = e^{-\rho}(\frac{I_{i,t+1} + D_{i,t+1}}{I_{it} + D_{it}})^{-\alpha} = e^{-\rho}(\frac{C_{t+1}}{C_t})^{-\alpha} \exp[-\alpha(\eta_{i,t+1} y_{t+1} - \frac{y_{t+1}^2}{2})]$$

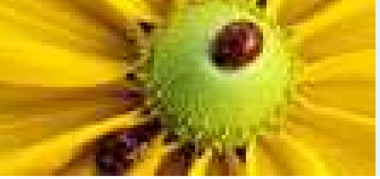
So Consumer i's valuation of security j at time t is:

$$\hat{P}_{jt}(i) = E[(P_{j,t+1} + d_{j,t+1}) e^{-\rho}(\frac{C_{t+1}}{C_t})^{-\alpha} \exp[-\alpha(\eta_{i,t+1} y_{t+1} - \frac{y_{t+1}^2}{2})] | J_t]$$

$$= E[(P_{j,t+1} + d_{j,t+1}) e^{-\rho}(\frac{C_{t+1}}{C_t})^{-\alpha} Z_{it} | J_t]$$

where:

$$Z_{it} = E[\exp[-\alpha(\eta_{i,t+1} y_{t+1} - \frac{y_{t+1}^2}{2})] | J_t \cup \{y_{t+1}\}]$$



Intuition

Independence of $\eta_{i,t+1}$ from y_{t+1} and J_t and law of iterated expectations imply:

$$Z_{it} = \exp(\frac{\alpha(\alpha+1)}{2} y_{t+1}^2) = e^{\rho}(\frac{C_{t+1}}{C_t})^{\alpha}(\frac{M_{t+1}}{M_t})$$

so

$$\hat{P}_{jt}(i) = E[(P_{j,t+1} + d_{j,t+1})\frac{M_{t+1}}{M_t}|J_t] = P_{jt}$$

thus we have a no-trade eq (informally).

Economic Interpretation

In the no-trade eq:

$$E[R_{j,t+1}e^{-\rho}(\frac{C_{t+1}}{C_t})^{-\alpha}\exp(\frac{\alpha(\alpha+1)}{2}y_{t+1}^2)|J_t] = 1$$

and note that:

$$\log(\frac{C_{i,t+1}/C_{t+1}}{C_{i,t}/C_t}) = \log(\frac{\delta_{i,t+1}}{\delta_{it}})$$

$$= \eta_{i,t+1}y_{t+1} - \frac{y_{t+1}^2}{2} \sim N(-\frac{y_{t+1}^2}{2}, y_{t+1}^2)$$

so y_{t+1}^2 is the x-sectional variation.

Wouldn’t want to use Krusell and Smith here.

Special Case

If

$$y_{t+1}^2 = a + b \log(\frac{C_{t+1}}{C_t}).$$

then:

$$E[R_{j,t+1} e^{-\hat{\rho}} (\frac{C_{t+1}}{C_t})^{-\hat{\alpha}}] = 1$$

where

$$\hat{\rho} = \rho - \frac{\alpha(\alpha+1)}{2} a$$

$$\hat{\alpha} = \alpha - \frac{\alpha(\alpha+1)}{2} b$$

If $b < 0$ then we get Mankiw's ('86) counter-cyclical increase in x-sectional variance and thus counter-cyclical risk premium. Nice, but recall that Hansen and Jagannathan ('91) reject EE with that parametric form.

IICAPM

In general, the excess return of security j is:

$$E_t[R_{j,t+1}] - R_{F,t+1} = -\frac{cov_t(R_{j,t+1}, H_{t+1})}{E_t[H_{t+1}]},$$

where

$$H_{t+1} = (\frac{C_{t+1}}{C_t})^{-\alpha} \exp(\frac{\alpha(\alpha+1)}{2} y_{t+1}^2)$$

Even if aggregate consumption growth is constant, assets which positively co-vary with x-sectional variance will require a positive excess return.

Conclusions

Idiosyncratic income risk in incomplete markets can solve equity premium puzzle. But we need several things:

1. Idiosyncratic shocks dependent on aggregate shocks.
2. Dependence must have some kind of symmetric (non-diversifiable) component.
3. (2) should be persistent to get quantitatively large effects.