Commitment vs Flexibility

M. Amador, I. Werning, G. Angeletos

Econometrica 2006

Reading Group – Oct.4, 2006

Motivation

- An important fraction of the population would save inadequately if choosing freely $\rightarrow$ rational for compulsory pension systems
- Are simple minimum saving rules optimal?
- What's the best policy to help agents who are subject to temptation and self-control problems?

    In deterministic intertemporal environments, full <u>commitment</u> (*freeze/remove* all future choices on savings) is the welfare maximizer (self-control is costly or inexistent).

    $\updownarrow$

    In the absence of temptation (inconsistency), and when new relevant private information is expected to arrive, full <u>flexibility</u> maximizes social welfare and is strictly preferred to commitment.

The Model in a Nutshell

- Main features
  - 2-period consumption/savings model with one consumption good
  - Individuals suffer from temptation for higher present consumption
  - Each period, i.i.d. taste shocks (only privately observed) affect relative taste for current consumption (individuals are otherwise identical)
  - No redistribution

  $\Rightarrow$ 'principal-agent' formulation: 'principal' with individual's *ex-ante* preferences
'agent' with individual's *ex-post* preferences
  - Subgame perfect equilibria
- Major insights
  - Simple minimum saving policies are optimal if the distribution of the taste shocks respects a certain condition (met by a set of common distributions).
  - Optimal flexibility decreases with the degree of temptation, and this is achieved by setting a higher minimum saving level

Basic Consumption-Savings Problem

- Budget set: $B(y) \equiv \{(c,k) \in R_+^2 : c + k \leq y\}$

- Quasi-geometric discounting (different *selves* with different preferences)

  ex-ante utility *:* $E[\theta U(c) + W(k)]$ *;* $\theta \sim F(\theta)$, $\theta \in \Theta = [\underline{\theta}, \overline{\theta}]$, $E[\theta]=1$, $f(\theta)$ cont.

  ex-post utility *:* $\theta U(c) + \beta W(k)$ *;* $0 < \beta \leq 1$, $1-\beta$ measures strength of temptation.

- Under full flexibility, the agent is constrained only by $B(y)$, and solves the simple problem $\max_{(c,k)\in B(y)} \{\theta U(c) + \beta W(k)\}$ with unique solution $(c^f(\theta), k^f(\theta))$.

- *Ex-ante* "first-best" allocation $(c^{fb}(\theta), k^{fb}(\theta))$ is defined as

$$\arg\max_{(c,k)\in B(y)} \{\theta U(c) + W(k)\}$$

Optimal Commitment without Self-Control (I)

- Choose a subset $C \subset B(y)$ that maximizes *ex-ante* utility, given that choices are to be made *ex-post:*

$$\max_{c,k} \int \left[\theta U(c(\theta)) + W(k(\theta))\right] dF(\theta)$$

$$s.t.$$

$$(IC) \qquad \theta U(c(\theta)) + \beta W(k(\theta)) \geq \theta U(c(\theta')) + \beta W(k(\theta')), \quad \forall \theta, \theta' \in \Theta$$

$$(BC) \qquad c(\theta) + k(\theta) \leq y, \quad \forall \theta \in \Theta$$

- Remark: with only two types of taste shocks, $\theta_h > \theta_l$, $\left(c^{fb}(\theta), k^{fb}(\theta)\right)$ is implementable for low enough levels of temptation (high β).

- For a continuous density $f(\theta)$, the problem is solved with the help of a change of variables from $(c(\theta), k(\theta))$ to $(u(\theta), w(\theta)) \equiv (U(c(\theta)), W(k(\theta)))$. Hence, the principal will maximize $\int_{\underline{\theta}}^{\overline{\theta}} \left(\theta u(\theta) + \beta w(\theta)\right) dF(\theta)$ subject to:
  - budget constraint,
  - incentive compatibility,
  - $u(\theta)$ non-decreasing in θ.

Optimal Commitment without Self-Control (I-II)

- Steps into solving the problem

Notice that in SPE truth telling is optimal $\Rightarrow$ (IC) is verified with equality and utility for the agent in the optimum can be expressed as:

$$V(\theta) = (\theta / \beta)u(\theta) + W(\theta)$$

By an envelope argument, $V'(\theta)$ depends only on $u(\theta)$, and $V(\theta)$ (corresponding to the left hand side of (IC)) may be expressed as

$$(\theta / \beta)u(\theta) + w(\theta) = \int_{\underline{\theta}}^{\theta} (1/\beta)u(\tilde{\theta})d\tilde{\theta} + (\underline{\theta} / \beta)u(\underline{\theta}) + w(\underline{\theta})$$

which together with monotonicity of $u(\theta)$ is necessary and sufficient for (IC).

One main then insert this condition into the objective function and budget constraint…

Optimal Commitment without Self-Control (II)

- The principal problem, then, reduces to

$$\max_{u,\underline{w}\in\Phi}\left\{\frac{\underline{\theta}}{\beta}u(\underline{\theta})+\underline{w}+\frac{1}{\beta}\int_{\underline{\theta}}^{\bar{\theta}}(1-G(\theta))u(\theta)d\theta\right\} \qquad (PP)$$

$$s.t.\quad W(y-C(u(\theta)))+\frac{\theta}{\beta}u(\theta)-\frac{\underline{\theta}}{\beta}u(\underline{\theta})-\frac{1}{\beta}\int_{\underline{\theta}}^{\theta}u(\tilde{\theta})d\tilde{\theta}\geq 0,\qquad \forall\theta\in\Theta$$

with

$$\Phi=\{u,\underline{w}\mid \underline{w}\in W(R_+),u:\Theta\to U(R_+),u_\theta(\theta)\geq 0\}$$

$$G(\theta)=F(\theta)+\theta(1-\beta)f(\theta)$$

$\Longrightarrow$ ***Bunching*** is always a necessary feature of the optimum:

Prop.: *An optimal allocation* $(u^*,\underline{w}^*)$ *satisfies* $u^*(\theta)=u^*(\theta_p)$ *for* $\theta\geq\theta_p$*; where* θ_p *is the lowest value in* Θ *such that*

$$\int_{\hat{\theta}}^{\bar{\theta}}\left(1-G(\tilde{\theta})\right)d\tilde{\theta}\leq 0$$

for $\hat{\theta}\geq\theta_p$ *. In the optimum, the budget constraint holds with equality at* $\theta_{p.}$ $\rightarrow$ ***no money burning***

Optimal Commitment without Self-Control (III)

$\Rightarrow$ ***Minimum-Savings Policies*** completely describe the optimum if

$G(\theta) = F(\theta) + \theta(1-\beta) f(\theta)$ is increasing for all $\theta \leq \theta_p$

$\Leftrightarrow$ (Ass. A): $\theta \dfrac{f'(\theta)}{f(\theta)} \geq -\dfrac{2-\beta}{1-\beta}$ $\rightarrow$ lower bound for the elasticity of *f*

Assumption A is satisfied by any differentiable non-decreasing density functions, exponential distribution, log normal, and Pareto and Gamma distributions (under some parameters).

- It follows that $(u^*, \underline{w}^*)$ consists in a simple Minimum-Savings Policy:

$$k \geq k^f(\theta_p) \Leftrightarrow \begin{cases} \left(U\left(c^f(\theta)\right)\right) & for \quad \theta \leq \theta_p \\ \left(U\left(c^f(\theta_p)\right)\right) & for \quad \theta > \theta_p \end{cases} \quad ; \quad \underline{w}^* = W\left(k^f(\underline{\theta})\right)$$

<u>if and only if</u> Assumption A holds.

Optimal Commitment without Self-Control (IV)

- Furthermore,

  - Prop.: *If Assumption A doesn't hold, no minimum-savings rule is optimal*

    Expl: Departing from a minimum-savings allocation; if $G(\theta)$ is decreasing for some open interval (θ_a, θ_b), with $\theta_b \leq \theta_p$, it is welfare improving to remove consumption choices between $c^f(\theta_a)$ and $c^f(\theta_a)$.

    Types in (θ_a, θ_b) will move to one of the extremes, and the welfare gain depends on the number of them going to $c^f(\theta_a)$ or $c^f(\theta_a)$. Assumption A affects precisely this relative quantities.

  - Prop.: *The bunching point θ_p increases with β.*

    The minimum-savings level $k_{min} = y - C(u(\theta_p))$ decreases with β.

Optimal Commitment with Self-Control (I)

- Individuals may partially resist temptation, although it is costly to do so.

  Ex-ante, their utility of a set of allocations *C* is given by:

$$P(C) = E[\max_{(c,k)\in C}\left(\theta U(c) + W(k) + \varphi\left(\theta U(c) + \beta W(k)\right)\right) - \\ - \varphi \max_{(c,k)\in C}\left(\theta U(c) + \beta W(k)\right)]$$

  $\varphi > 0$: cost (lack) of self-control

  θ, β and φ are sochastic, drawn from a continuous joint distribution with bounded rectangular support $\left[\underline{\theta}, \overline{\theta}\right] \times \left[\underline{\beta}, \overline{\beta}\right] \times \left[\underline{\varphi}, \overline{\varphi}\right]$, $\underline{\beta} > 0$.

- The problem now is now to maximize $P(C)$, subject to $C \subset B(y)$.

- Commitment devices are valuable because they reduce over-consumption (as before) and reduce the cost of self control.

Optimal Commitment with Self-Control (II)

- Like before, maximization is done over utilities instead of consumption bundles (change of variable), and $P(C)$ is rewritten as

$$E\left\{(1+\beta\varphi)\max_{(\upsilon,\omega)\in C}\left[\underbrace{\left(\theta/\hat{\beta}\right)}_{\hat{z}}\upsilon+\omega\right]-\varphi\beta\max_{(\upsilon,\omega)\in C}\left[\underbrace{\left(\theta/\beta\right)}_{z}\upsilon+\omega\right]\right\};\quad \hat{\beta}=\frac{1+\beta\varphi}{1+\varphi}$$

$$\hat{z},z\in\hat{\Theta}\equiv\left[\underline{x},\overline{x}\right]\equiv\left[\underline{\theta}\left(1+\underline{\varphi}\right)/\left(1+\overline{\beta}\,\underline{\varphi}\right),\overline{\theta}/\underline{\beta}\right]$$

- Thus, the principal problem in this case is to find an allocation that maximizes

$$E\left[(1+\beta\varphi)\left[\hat{z}u(\hat{z})+w(\hat{z})\right]-\varphi\beta\left[zu(z)+w(z)\right]\right]$$

$$s.t.$$

$$(IC)\qquad \left(u(x),w(x)\right)\in\arg\max_{(\upsilon,\omega)\in C}\left[x\upsilon+\omega\right],\qquad \forall x\in\hat{\Theta}$$

$$(BC)\qquad C\left(u(x)\right)+K\left(w(x)\right)\leq y,\qquad \forall x\in\hat{\Theta}$$

Optimal Commitment with Self-Control (III)

Steps to analyse the problem mirror those for the model without self-control.
We again obtain that:

- ***Bunching*** a necessary feature of the optimum:

  Prop.: *An optimal allocation* $(u^*, \underline{w}^*)$ *satisfies* $u^*(x) = u^*(x_p)$ *for* $x \geq x_p$ *; where* x_p *is the lowest valur in* Θ *such that*

  $$\int_{\hat{x}}^{\overline{x}} \left(1 - \hat{G}(x)\right) dx \leq 0$$

  for $\hat{x} \geq x_p$ *. The budget constraint holds with equality at* $x_{p.}$ $\rightarrow$ ***no money burning***

- ***Minimum-Savings policies*** fully characterizes the optimum iff $\hat{G}(x)$ is non-decreasing for all $x \leq x_{p.}$

- When temptation and self control are non-stochastic, the bunching point x_p increases and the minimum savings decreases with β and the opposite happens with an increase in φ.

Other applications/interpretations of the result

- Optimal Fiscal Constitutions

  If a government holds private information on its bias towards higher public consumption, relative to the society as a whole → Maximum-Spending Limits

- Optimal Paternalism

  E.g., when individual utility of leisure is private information → Minimum-Schooling Hours

- Externalities

  Individual utility affected (in heterogeneous fashion) by total consumption in the economy. A planner would like to remove some consumption choices that do not take into account eventual externalities over social welfare →Minimum/Maximum consumption levels.

- Under some utility specifications (exponential), taste shocks maybe directly reinterpreted as income shocks.

Further interesting questions

- Pay-as-You-Go vs Fully Funded systems

  Krusell, Krurusçu, Smith (2005) find in a capital accumulation model that is optimal to subsidize investment (financing it with income taxes).

- Allowing for transfers across types (which mechanism? consumption taxes?)

- What's does empirics say about distribution of θ in the different proposed applications?