Time Consistent Taxation by a Government with Redistributive Goals

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Motivation

- In a principal-agent setting between workers and firms, the government faces a conflict between efficiency and equity.
- What are the implications of government's intervention?
- What if we allow to renegotiate the terms of the agreement?

Model: Setup

- Three types of agents: firms, workers and a government.
- There is no borrowing, lending, or saving.
- At period t, worker and firm engage in principal-agent relationship with moral hazard:
  - ◦ worker's effort $a \in A$ is unobservable
  - ◦ salary $s(\omega)$ where $\omega \in \Omega$ is a public signal with density $f(\omega|a)$
- At the end of the period t, the government can redistribute income through income taxes.

Timeline

- At the beginning of t, government announces a "minimum wage" $\underline{s}$.
- Firm j decides to be idle or offer salary contract $s_j : \Omega \rightarrow [\underline{s}, \infty)$.
- Worker chooses contract to accept and selects action $a \in A$.
- Signals ω are drawn from $f(\omega|a)$.
- Government chooses redistribution function $R : [\underline{s}, \infty) \rightarrow \mathbb{R}^+$.
- Worker consumes $c(\omega) = R(s(\omega))$.

Static Principal-Agent Problem

Given $\underline{s}$ and R,

$$\max \Phi(s, a, R)$$

$$s.t. \quad \Phi(s, a, R) \geq \Phi(s, \alpha, R) \quad \forall \alpha \in A$$

$$E(s, a) = P(a)$$

$$s(\omega) \geq \underline{s} \qquad \forall \omega \in \Omega$$

where

$$\Phi(s, a, R) \equiv \sum_{\omega \in \Omega} U(R(s(\omega)), a) f(\omega | a)$$

$$E(s, a) \equiv \sum_{\omega \in \Omega} s(\omega) f(\omega | a)$$

$$P(a) \equiv \sum_{\omega \in \Omega} p(\omega) f(\omega | a)$$

Assumptions

(A1) $A = [0, 1]$

(A2) $U : \mathbb{R}^+ \times A \rightarrow \mathbb{R}$ is continuous and $U(c, a)$ is strictly concave and increasing on c ,and decreasing on a.

(A3) $P : A \rightarrow \mathbb{R}$ is continuous, and $P(0) > 0$.

(A4) $\Omega \subset \mathbb{R}$ is finite, $f(\omega|a)$ is continuous in a, and $f(\omega|a) > 0 \quad \forall \omega \in \mathbb{R}$, $a \in A$.

(A5) $U(P(a), a) > U(P(0), 0) > U(0, i) \qquad \forall\ a \neq 0$.

Symmetric Equilibria

Let $S_t = \left\{(s^k, q_k) | \sum_k q_k = 1\right\}$ be a *contract distribution* and let $h^t = (h_0, ..., h_t)$ be a *public history* where $h_t = (\underline{s}_t, R_t, S_t)$.

DEF: A *Symmetric Strategic Profile (SSP)* σ specifies $\forall h^{t-1}$

(1) a minimum salary $\underline{s}_t$

(2) a contract s_t

(3) a contract s'_t chosen from set of contracts offered, and an action a_t

(4) a redistribution function R_t

The symmetric profile σ provides a value for the government

$\Psi(\sigma) = (1 - \delta) \sum_{t=0}^{\infty} \delta \Phi(s, a, R)$

$\Phi(s, a, R) \equiv \sum_{\omega \in \Omega} U(R(s(\omega)), a) f(\omega | a)$

Symmetric Equilibria Cont'd

DEF: A *Symmetric Equilibrium* is a SSP σ such that, after any history h^{t-1}, $H_t = (\underline{s}_t, s_t, a_t, R_t)$ prescribed by σ following h^{t-1}, satisfies:

(1) Given $\underline{s}_t$ and R_t, (s_t, a_t) solves the principal-agent problem.

(2) The government cannot gain by imposing a different minimum salary or redistribution function.

Second-Best Equilibrium

Let $\underline{s} = 0$ and $R = I$ (No government intervention)

Given action a, $s^*(\omega, a)$ solves

$$\begin{aligned} \varphi(a) \equiv \max_s \Phi(s, a, I) & \\ s.t. \quad \Phi(s, a, I) \geq \Phi(s, \alpha, I) & \quad \forall \alpha \in A \\ E(s, a) = P(a) & \\ s(\omega) \geq 0 & \quad \forall \omega \in \Omega \end{aligned}$$

Let $a^* = \arg\max \varphi(a)$

DEF: The salary schedule s *implements* action a if (s, a) satisfies constraints (IC) and (BC).

Second-Best Equilibrium Cont'd

(A6) $\varphi(a^*) > U(P(0), 0) = \varphi(0)$

Worst Punishment? No effort ($a = 0$), salaries completely flatten (R^F).

- Government: $R^F = \arg\max_R \min_{(s,a)\in I} \Psi(\sigma)$
- Workers and Firms: $(P(0), 0) = \arg\max_{(s,a)\in I} \min_R \Phi(s, a, R)$
- Static NE.

(s^*, a^*) is sustainable iff $\nexists$ profitable deviation for the government:

$$\varphi(a^*) \geq (1-\delta)U(P(a^*), a^*) + \delta\varphi(0)$$

$$\Leftrightarrow \delta \geq \underline{\delta} \equiv \frac{U(P(a^*), a^*) - \varphi(a^*)}{U(P(a^*), a^*) - \varphi(0)}$$

What if $\delta < \underline{\delta}$?

Distortionary Effects of Government Intervention

Consider the family of linear redistribution functions with slope $m \in (0, \infty)$

$$L(s, a, m) \equiv ms + (1 - m)P(a) \qquad \text{if } s \geq \underline{s}_m \equiv \left(\frac{1 - m}{m}\right) P(a)$$

(A7) P is concave

(A8) $\forall a \in A$, $K(\alpha, a, 1)$ is convex and increasing in α.

DEF: For any given a, the triple $(\underline{s}, s, R) = \sum(a)$ is defined as

$$m \equiv \mu(a) \qquad R \equiv L_m \qquad s(\omega) \equiv L_m^{-1}(s^*(\omega, a))$$

Distortionary Effects of Government Intervention Cont'd

LEMMA: Let a be an implementable action, $m = \mu(a) \equiv \frac{1}{P'(a)} \frac{\partial K}{\partial \alpha}(a, a, 1)$. If the government imposes linear $R \equiv L(\cdot, a, m)$, and $\underline{s} = \underline{s}_m$:

(i) Firm: implements a with $s(\omega) \equiv L_m^{-1}(s^*(\omega, a))$

(ii) Workers: $c(\omega) = R(s(\omega)) = s^*(\omega, a)$, with expected utility $\varphi(a)$.

Hence, when $\delta < \underline{\delta}$, the best equilibrium is $\sigma(\underline{s}, s, \bar{a}, R)$ where $\bar{a}$ solves

$$\max_a \varphi(a)$$
$$s.t. \quad \varphi(a) \geq (1 - \delta) U(P(a), a) + \delta \varphi(0)$$

Strategic Dynnamic Programming

DEF: Let $W \in \mathbb{R}$ be bounded. Define $\underline{W} = \inf(W)$ and $\overline{W} = sup(W)$.

A tuple $(\underline{s}, s, a, R, w)$ is *admissible w.r.t.* W if

(i) Given $(\underline{s}, R)$, (s, a) solves the principal-agent problem.

(ii) $\sum_{\omega \in \Omega} R(s(\omega)) f(\omega | a) = E(s, a)$

(iii) $w \in W$ and $(1 - \delta)\Phi(s, a, R) + \delta w \geq (1 - \delta)U(P(a), a) + \delta \underline{W}$

DEF: The value set generated by W is

$$B(W) \equiv \{(1 - \delta)\Phi(s, a, R) + \delta w | (\underline{s}, s, a, R, w) \text{ is admissible w.r.t. } W\}$$

Strategic Dynamic Programming Cont'd

DEF: A bounded $W \subset \mathbb{R}$ is *self-generating* if $W \subset B(W)$

THM: (Factorization) $V = B(V)$

THM: (Self-generation) If W is self-generating, $B(W) \subset V$.

THM: (Compactness) If W is self-generating, so is $cl(W)$

LEMMA: Let W be a closed self-generating set. Then:

(i) $\exists$ equilibrium γ with value $\underline{W}$, all of those continuation values are in W.

(ii) $\exists$ action $a : \sigma(\underline{s}, s, a, R, \gamma)$, where $(\underline{s}, s, R) \equiv \sum(a)$, is an equilibrium with value $\varphi(a) = sup\ B(W)$

Renegotiation

Once the government has deviated, the severest punishment is too harsh.

DEF: $\underline{r} \equiv sup \{\underline{X} | X \text{ is self-generating}\}$

DEF: W is *renegotiation-proof set* if it is self-generating and $\underline{W} = \underline{r}$

DEF: $\bar{r} = \max \{W | W \text{ is renegotiation-proof}\}$

THM: Suppose $(\underline{s}, s, a, R, r)$ satisfies (i) and (ii) of admissibility, and

(iii) $\Phi(s, a, R) \geq (1-\delta)U(P(a), a) + \delta r$ and $r \geq \varphi(0)$.

Then, $[r, \varphi(a)]$ is self-generating.

COR: $[\underline{r}, \bar{r}]$ is the largest renegotiation-proof set.