Capital Taxation and Ownership when Markets are Incomplete

Emmanuel Farhi
2005

April 22, 2008

Introduction

- When markets are incomplete, hedging of the government budget is limited in scope.
- Capital taxation provides a state contingent source of revenues.
- What is the role of capital taxation as a hedging instrument under incomplete markets?
- What about capital ownership?
- Under incomplete markets, what price kernel should the government use to price non-traded assets?

Model Setup

- Two types of agents: government and continuum of households.
- Uncertainty: $s_t \in S = \{1, 2, ..., S\}$ with transition matrix $P(s'|s)$ and initial distribution π_0
- Stochastic government expenditure $g_t = g(s_t)$
- Productivity shock in production function $F(k_{t-1}, l_t, s_t)$
- Government can trade a risk-free bond b^g, levy fully state contingent linear taxes on labor τ_s^l and set taxes on capital one period in advance τ^k.

Competitive Equilibrium and Ramsey Problem

- Household's Problem:

$$\max E_{-1} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t)$$

$$\begin{aligned} \text{s.t. } c_t + b_t^g &\leq (1-\tau_s^l) w_t l_t + (1-\tau^k) k F_k(k_{t-1}, l_t^d, s_t) \\ &+ k_{t-1} - k_t + (1+r_t) b_{t-1}^g + T_t \end{aligned}$$

- Government:

$$\begin{aligned} (1+r_t) b_{t-1}^g + g_t &\leq \tau_s^l w_t l_t + \tau^k k F_k(k_{t-1}, l_t) + b_t^g \\ \underline{M}(k_t, u_c(c_t, l_t)) &\leq u_c(c_t, l_t) b_t^g \leq \overline{M}(k_t, u_c(c_t, l_t)) \end{aligned}$$

Competitive equilibrium and Ramsey Problem Cont'd

- Given $b^g_{-1}, r_0, k_0, \tau^k_0$ and $\{s_t\}$, **a competitive equilibrium** is HH allocations $\{c_t, l_t, l^d_t, k_t, b^g_t\}_{t\geq 0}$, government policies $\{\tau^l_t, \tau^k_t, b^g_t\}_{t\geq 0}$, and prices $\{r_t, w_t\}_{t\geq 0}$ such that:

  (i) Given $b^g_{-1}, r_0, k_0, \tau^k_0$ and government policies, $\{c_t, l_t, l^d_t, k_t, b^g_t\}_{t\geq 0}$ solves household's problem.

  (ii) Government b.c and debt constraints are satisfied $\forall t \geq 0$.

  (iii) Labor market clears: $l_t = l^d_t$

  (iv) $c_t + g_t + k_t \leq F(k_{t-1}, l_t) + k_{t-1} \qquad \forall t \geq 0, \forall s^t \in S^t$

- A **Ramsey outcome** is a competitve equilibrium that maximizes HH's lifetime utility.

Competitive equilibrium conditions

From FONC:

$$\begin{aligned}
1 &= (1+r_t)E_t\left\{\beta\frac{u_{c,t+1}}{u_c}\right\} \\
1 &= E_t\left\{\beta\frac{u_{c,t+1}}{u_c}\left[1+(1-\tau_t^k)F_{k,t+1}\right]\right\} \\
\tau_t^l &= 1+\frac{u_{l,t}}{u_{c,t}w_t} \\
w_t &= F_l(k_{t-1}, l_t^d, s_t)
\end{aligned}$$

Recursive Formulation of Ramsey Problem

Let $\theta \equiv u_c(c_-, l_-, s_-)$

$$V(k, b, \theta, s_-) = \max E\left\{u(c_s, l_s) + \beta V(k'_s, b'_s, u_c(c_s, l_s), s)|s_-\right\}$$

$$s.t.(1+r)E\left\{\beta u_c(c_s, l_s)|s_-\right\} = \theta$$

$$E\left\{\beta u_c(c_s, l_s)\left[1 + (1-\tau^k)F_k(k, l_s, s)\right]|s_-\right\} = \theta$$

$$\tau^l_s = 1 + \frac{u_l(c_s, l_s)}{u_c(c_s, l_s)F_l(k, l_s, s)} \qquad \forall s \in S$$

$$(1+r)b + g_s \leq \tau^l_s l_s F_l(k, l_s, s) + \tau^k k F_k(k, l_s, s) + b'_s \qquad \forall s \in S$$

$$c_s + g_s + k'_s \leq F(k, l_s, s) + k \qquad \forall s \in S$$

$$\underline{M}(k'_s, u_c(c_s, l_s), s) \leq u_c(c_s, l_s)b'_s \leq \overline{M}(k'_s, u_c(c_s, l_s), s) \qquad \forall s \in S$$

Quasi-linear case

Assume $u(c, l, s) = c + H(l)$, where H is continuous, $H_l < 0, H_{ll} < 0, H'(0) = \infty$

Eliminate government's ability to manipulate intertemporal prices

Let $\widetilde{b} = b\theta$

$$\widetilde{V}(\widetilde{b}, s_-) = \max E \left\{ F_s + k(1 - \frac{1}{\beta}) - g_s + \beta \widetilde{V}(\widetilde{b}'_s, s) | s_- \right\}$$

$$E \left\{ \beta \left[1 + (1 - \tau^k) F_{k,s} \right] | s_- \right\} = 1 \qquad (\mu)$$

$$\widetilde{b} \frac{1}{\beta} + g_s \leq l_s F_{l,s} + l_s H_{l,s} + \tau^k k F_{k,s} + \widetilde{b}'_s \quad \forall s \in S \qquad (v_s)$$

$$\underline{M}_s \leq \widetilde{b}'_s \leq \overline{M}_s \qquad \forall s \in S \quad (v_{1,s}, v_{2,s})$$

Ramsey Outcome

$$v_{s_-} = E\left\{v_s | s_-\right\} + v_{2,s_-} - v_{1,s_-}$$

Off debt limits, $\left\{v_{s_-}\right\}$ is a martingale.

$$\beta \widetilde{V}_b(\widetilde{b}'_s, s) = -v_s + v_{2,s} - v_{1,s}$$

Hence, debt inherits a near random walk component.

Optimal capital tax:

$$\tau^k = \underbrace{\frac{\frac{E\left\{-(1-\tau^k)kF_{kk,s}|s_-\right\}}{E\left\{F_{k,s}|s_-\right\}}}{1+\frac{E\left\{F_{k,s}v_s|s_-\right\}}{E\left\{F_{k,s|s_-}\right\}}}}_{Term\ 1} \left[\underbrace{\frac{Cov\left\{kF_{k,s}, v_s|s_-\right\}}{E\left\{kF_{k,s}\,|s_-|\right\}}}_{Term\ 2} - \underbrace{\frac{Cov\left\{kF_{kk,s}, v_s|s_-\right\}}{E\left\{kF_{kk,s}\,|s_-|\right\}}}_{Term\ 3}\right]$$

Lemma: If F is Cobb-Douglas, $\tau^k_0 = 0 \quad \forall t \geq 1$

Long-Run Behavior with Natural Borrowing Limits

- Multiplier $v_{1,t} = 0$:

$$v_{s_-} = E\left\{v_s | s_-\right\} + v_{2,s_-}$$

- Hence, $\{v_s\}$ is a non-negative supermartingale
- Supermartingale Conv. Thm: v_s converges a.s. to non-negative r.v.

Lemma: If the Markov process $\{s_t\}$ is ergodic, the value function $\widetilde{V}$ is continuously differentiable and concave in $\widetilde{b}$, policy functions are continuous, then:

(i) $v_t \rightarrow 0 \quad a.s.$

(ii) $\tau_t^l, \tau_t^k \rightarrow 0 \quad a.s.$

(iii) $b_s' \rightarrow -\underline{M}_s^n \quad a.s$

Capital Ownership and Structure of Government Liabilities

Now allow the government (and consumers) to trade:

(i) Risk-free bond with return $R^{s_-} = \frac{\theta}{\beta E\{u_{c,s}|s_-\}}$

(ii) Capital, whose return in state s is $1 + (1 - \tau^k) F_{k,s}$

(iii) Assets in zero-net supply indexed by $i \in I_{s_-}$, whose return in state s when previous state was s_- is R_s^{i,s_-}

Total liability the government has to repay or finance in state s is:

$$\sum_{i \in I_{s\_}} x_i \left(R_s^{i,s} - \frac{\theta}{\beta E\left\{u_{c,s}|s_-\right\}} \right) + k_g \left(1 + (1 - \tau^k) F_{k,s} - \frac{\theta}{\beta E\left\{u_{c,s}|s_-\right\}} \right) + \widetilde{b} \frac{1}{\beta E\left\{u_{c,s}|s_-\right\}}$$

where $\widetilde{b}$ is the value of government's net financial position.

Government CAPM (GCAPM)

The GCAPM Euler equations are:

$$\begin{aligned} E\left\{\beta R^{s_-} u_{c,s} v_s | s_-\right\} &= \theta v_{s_-} \\ E\left\{\beta u_{c,s}\left[1+(1-\tau^k)F_{k,s}\right] v_s | s_-\right\} &= \theta v_{s_-} \\ E\left\{\beta R_s^{i,s_-} u_{c,s} v_s | s_-\right\} &= \theta v_{s_-} \end{aligned}$$

$\{\beta^t u_{c,t} v_t\}$ used as pricing kernel for the government

Public risk premium to returns that covary negatively with adverse shocks to government

$$E\left\{R_s^{i,s_-} | s_-\right\} = R^{s_-} - \frac{Cov\left\{R_s^{i,s_-}, u_{c,s} v_s\right\}}{E\left\{u_{c,s} v_s | s_-\right\}}$$

Simulation Results

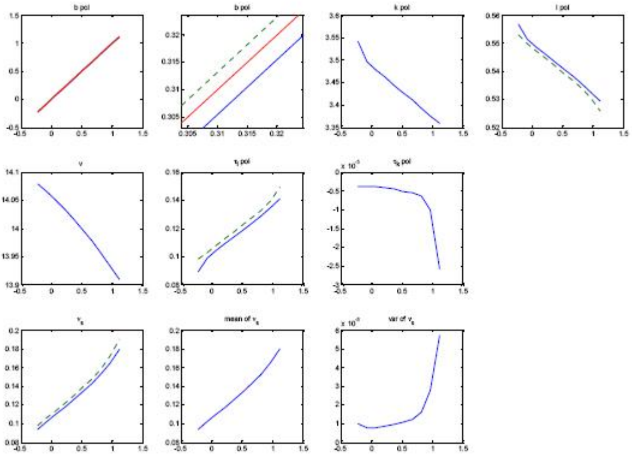

Figure 2: Policy functions, quasi-linear preferences, government expenditure shocks, s_- low.