Discussion of: “Optimal Taxation without state-contingent debt”, by Aiyagari, Marcet, Sargent and Seppälä (JPE ’02)

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Motivation, Setting and Goal of the Paper

- Barro ('79): Taxes follow a Random Walk.
  - Analogy w/ consumption smoothing model (CW ('00)).
  - Underlying key assumption: Incomplete Markets.
- LS ('83): Taxes inherit LoM of g_t.
  - Insurance "across states", not "across time".
  - Underlying key assumption: Complete Markets.
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  - Insurance "across states", not "across time".
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- Historical Data: Closer to Barro ('79).
- AMSS (JPE '02): Study optimal Ramsey taxation (distortionary labor taxes) under incomplete markets with 1 period risk free debt:
  - Taxes: LS '83 component + "near unit root".
  - Characterize conditions for LoM of taxes.
  - Welfare Implications: Incomplete markets add add'l restrictions due to the fact that debt is *not* state-contingent.

The Model

- $g_t \sim ([g_{min}, g_{max}], P, \pi)$, $g^t \equiv (g_0, \ldots, g_t)$.
- b_t^G: 1 period risk-free g^t-measurable debt.

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- HH Problem:

$$\max_{c_t, x_t, b_t^G} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t, x_t) \right]$$

$$s.t.\ p_t^b b_t^G + c_t \leq (1 - \tau_t)(1 - x_t) + b_{t-1}^G + T_t, \ \forall t.$$

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- GOV Problem:

$$\begin{aligned} b_{t-1}^G + T_t &\leq s_t + p_t^b b_t^G,\ s_t = \tau_t(1 - x_t) - g_t, \forall t, \\ b_t^G &\in [\underline{M}, \overline{M}]. \end{aligned}$$

- $[\underline{M}, \overline{M}]$: Natural debt limits (N.D.L.) or "Ad-hoc" (A-H.D.L.).

The Model (cont.)

- **Def.** An eqm is $\{(c_t, x_t, g_t), (\tau_t, b_t^G), p_t^b\}_t$ s.t.
  1. Given $\{(\tau_t, b_t^G), p_t^b\}_t$, $\{c_t, x_t, g_t\}_t$ solves HH problem.
  2. Given $\{p_t^b\}_t$, $\{(\tau_t, b_t^G)\}_t$ solves GOV problem.
  3. $c_t + g_t + x_t = 1$ for all t.
- **Def.** The Ramsey problem is to choose an eqm. that maximizes $E_0\left[\sum_{t=0}^{\infty} \beta^t U(c_t, x_t)\right]$.

Primal Approach

- $T_t = 0$.
- Primal Approach:

$$\max_{\{c_t, x_t\}_t} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(c_t, x_t) \right] \quad s.t.$$

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U_{c,t} s_t \right] = U_{c,0} b^G_{-1} \qquad (LS - IC),$$

$$s_t = \frac{U_{c,t} - U_{x,t}}{U_{c,t}} (c_t + g_t) - g_t, \;\; c_t + g_t + x_t = 1.$$

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$$\underline{M} \leq E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{U_{c,t+j}}{U_{c,t}} s_{t+j} \right] \leq \overline{M} \qquad (AMSS - DL),$$

$$E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{U_{c,t+j}}{U_{c,t}} s_{t+j} \right] \;\; is \;\; g^{t-1} - measurable \qquad (AMSS - MC),$$

Primal Approach (cont.)

- **Obs:** AMSS-MC: b^G_{t-1} is g^{t-1} measurable, then $E_t\left[\sum_{j=0}^{\infty}\beta^j U_{c,t+j}s_{t+j}\right]$ too.
  - In LS ('83) this holds automatically.
  - $(g^t)_t$ conditions due to non-state contingency of debt.
  - Welfare under I.M. $\leq$ Welfare under C.M.

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$$\mathcal{L} = E_0\left[\sum_{t=0}^{\infty}\beta^t\left(U(c_t, 1-c_t-g_t) - \psi_t U_{c,t}s_t + U_{c,t}\left(\nu_{1t}\overline{M} - \nu_{2t}\underline{M} + \gamma_t b^G_{t-1}\right)\right)\right]$$

$\psi_t = \psi_{t-1} + \nu_{1t} - \nu_{2t} + \gamma_t$ *and* $\psi_{-1} = 0.$

- FONC:

$$x_t : U_{c,t} - U_{x,t} - \psi_t\kappa_t + (U_{cc,t} - U_{cx,t})\left(\nu_{1t}\overline{M} - \nu_{2t}\underline{M} + \gamma_t b^G_{t-1}\right) = 0$$

$$\kappa_t = (U_{cc,t} - U_{cx,t})\, s_t + U_{c,t}s_{c,t}$$

$$b^G_t : E_t\left[U_{c,t+1}(\psi_{t+1} - \psi_t - \nu_{1t+1} + \nu_{2t+1})\right] = 0$$

Primal Approach (cont.)

- **Obs: (1)** $FONC(x_t) \Rightarrow$ Taxes "almost" a nonlinear function of ψ_t.
- **Obs: (2)** $FONC(b_t^G) \Rightarrow \psi_{t+1}$ "almost" a Martingale.
- **Obs: (3)** In LS ('83) $\psi_{t+1} = \psi_t = \psi = \gamma_0$.
  - $FONC(x_t)$: $U_{c,t} - U_{x,t} - \psi \kappa_t = 0$. No history dependence.

LoM of Taxes

- Taxes are linked to ψ_t ("almost" a martingale). Thus we study LoM of ψ_t.
- 3 possibilities:
  - (I) $\psi_t \rightarrow 0$ (1st Best).
  - (II) $\psi_t \rightarrow \psi_\infty < 0$ (LS ('83) eqm.).
  - (III) $\psi_t \Rightarrow \psi_\infty$ a R.V ($\approx$ Barro ('79)).

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- If $U_{c,t} = 1$ w/ N.D.L. then:
  - (I) if g_t R.V.
  - (II) if $g_t \to g_\infty$.
- If $U_{c,t} = 1$ w/ A-H.D.L. then:
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- If $U_{c,t} = 1$ w/ A-H.D.L. then:
  - (III).
- If $U_{c,t} \neq 1$ w/ N.D.L. then:
  - Typically martingale approach not conclusive.
  - With "sufficient randomness" (II) is ruled out.

LoM of Taxes: General Case

- Assuming $\nu_{it} = 0$

$$\psi_t = E_t\left[\frac{U_{c,t+1}}{E_t[U_{c,t+1}]}\psi_{t+1}\right] \iff$$

$$\theta_t\psi_t = E_t\left[\theta_{t+1}\psi_{t+1}\right], \ \log(\theta_t) = \sum_{\tau=1}^{t} \log(U_{c,\tau}) - \log(E_{\tau-1}[U_{c,\tau}])$$

- $\theta_t\psi_t$ bdd martingale, then: $\theta_t\psi_t \to \theta_\infty\psi_\infty$. When $\theta_\infty \neq 0$?
- If $U_{c,t} \neq E_t[U_{c,t+1}]$ w.p.p. then $\theta_\infty = 0$.
- **Obs:** Martingale approach not very useful.

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- If $U_{c,t} \neq E_t[U_{c,t+1}]$ w.p.p. then $\theta_\infty = 0$.
- **Obs:** Martingale approach not very useful.
- If $\frac{g_t - \tau_t(1-x_t)}{1-p_t^b}$ is "suff. random" then $\Pr(\psi_\infty < 0) = 0$

Analytical Examples

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- $\tau_t = -\psi_t R'(x_t)$ with $R(x) = (1 - H'(x))(1 - x)$.
- If $\nu_{2t} = 0$ (N.D.L.) then $E_t[\psi_{t+1}] \geq \psi_t$:
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  - ψ_t a martingale in the interior.
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- If $\nu_{2t} > 0$ (A-H.D.L.) then
  - ψ_t a martingale in the interior.
  - "reflecting boundaries".
- $g_t \to g_\infty$ (absorbing state).
  - $\psi_t \to \psi_\infty < 0$ if $g_t \to g_\infty$.

Numerical Examples

- Taxes and debt are more persistent.
- Taxes has "LS component (short memory) + Barro component (long memory)".
- Welfare: Close to C.M.: Effective Self-insurance.