Financing Investment

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Sargent's Reading Group

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Motivation

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- **Problem**: is this conclusive evidence?
- **Analysis**: specification of a structural model of investment under financial constraints consistent with several empirical regularities about firm behavior observed in the data.

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- Financing constraints are **not necessary**.
- Cash flow effects can arise because of the **artificial correlation** induced by the underlying technology shocks or because of **mis-specification** (fitting a linear equation to a nonlinear decision rule).
- Financing constraints **are** important: doubts on standard investment regressions; importance of equilibrium modeling.

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- **Household**: one representative.
- **Financial intermediary**: one; provides funds to firms at some cost.

Timing

		exits, sells $k \equiv (1-\delta)k_{-1}$
Incumbent with	$\nearrow$	
k_{-1} and z_{-1}	$\longrightarrow$	stays in, observes w, draws z with $Q(z \mid z_{-1})$
Entrant with	$\longrightarrow$	enters, observes w, draws z from $\phi(z)$
no capital	$\searrow$	
		stays out

t $\qquad\qquad\qquad\qquad$ $t+1$

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		| $\pi(k,z;w) = \max_l \left[F(k,l;z) - wl - f\right]$	|
		| $i(k,k') = k' - (1-\delta)k,$ investment	|
		| $\lambda = \lambda\Big(i(k,k') - \pi(k,z;w)\Big),$ financ. cost	|
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- incumbents' shocks are uncorrelated across firms, have a common stationary and monotone (incr.) Markov trans. func. $Q(z'|z) : Z \times \Im_Z \to [0, 1]$ satisfying the Feller property; entrants draw independently from $\phi(z)$.

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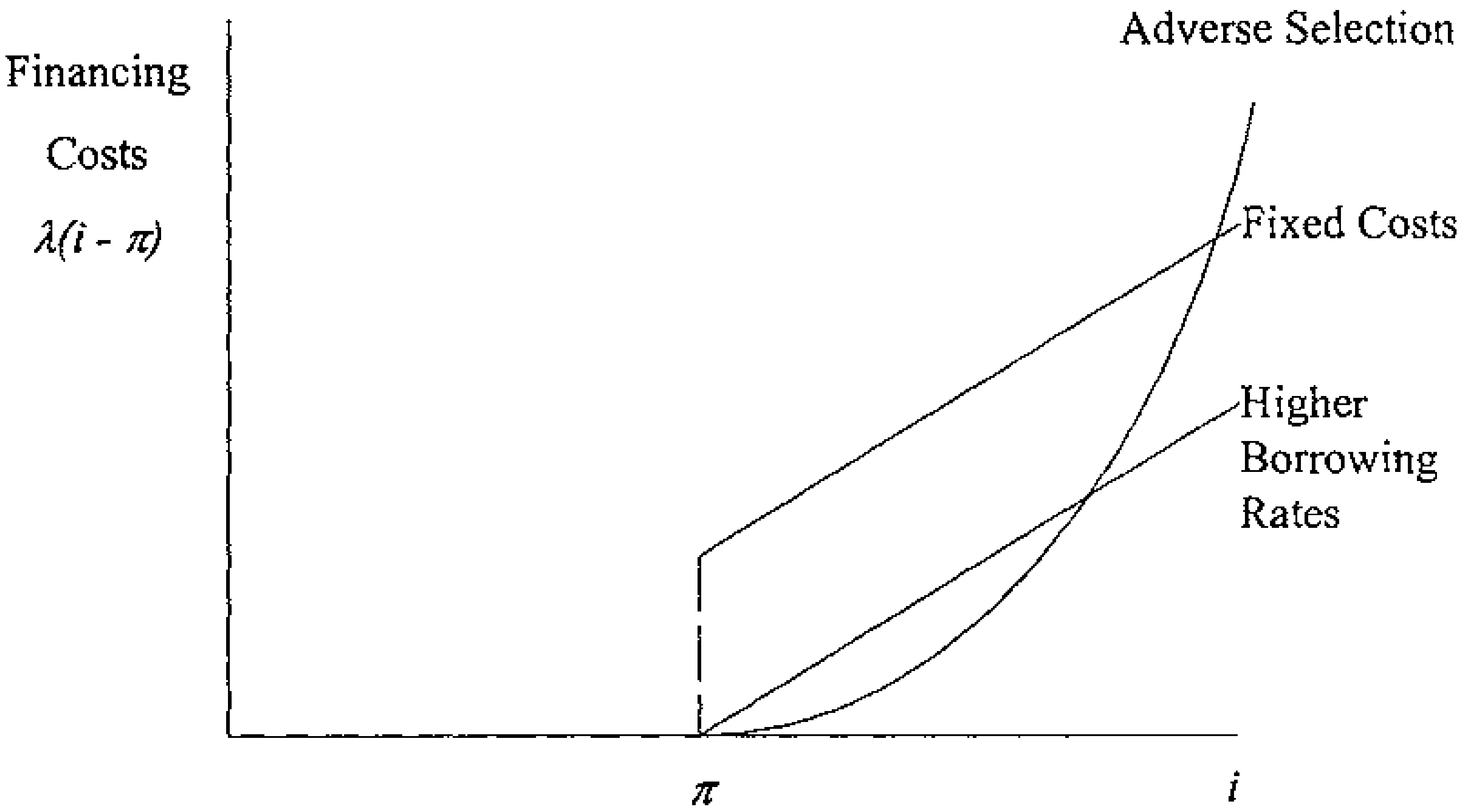
- the financing cost function $\lambda : \mathbb{R} \to \mathbb{R}_+$, satisfies: $\forall a \leq 0, \lambda(a) = 0$; $\forall a > 0$, $\lambda(a) > 0$, $\lambda'(a) > 0$.

Financing costs



Market value of an active firm

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$$\begin{aligned} p(k,z;w) &= \max_{k'\geq 0}\Big\{\pi(k,z;w) - i(k,k') - \lambda(k,k',z;w) \\ &\quad + \beta \max\left((1-\delta)k', \int p(k',z';w)Q(dz'|z)\right)\Big\} \end{aligned}$$

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- **Proposition 1:** There is a unique function $p(k,z;w)$ that satisfies the functional above.
- **Proposition 2:** The value function $p(k,z;w)$ is continuous and increasing in (k,z); continuous and strictly decr. in w.

Decision rules

- **Capital accumulation:**

$$
\begin{aligned}
k(k,z;w) &= \min\left\{ \operatorname*{arg\,max}_{k'\geq 0} \left\{ \pi(k,z;w) - i(k,k') - \lambda(k,k',z;w) \right.\right.\\
&\qquad \left.\left. + \beta \max\left((1-\delta)k', \int p(k',z';w)Q(dz'|z) \right) \right\}\right\}
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- **Exit:**

$$x(k,z;w) = \begin{cases} 1 & \text{if } z > z^* \text{ (stay)} \\ 0 & \text{if } z \leq z^* \text{ (exit)} \end{cases}, \text{ where}$$

$$z^*(k,z;w) = \min\left\{\inf\left\{z : \int p(k',z';w)Q(dz'|z) \geq (1-\delta)k'\right\}, \bar{z}\right\}$$

Free Entry

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- **Level of entry** B**:** determined by the market-clearing condition.

Aggregation

- **Mass of firms in state** (k, z)**:** $\mu(k, z)$.
- **Invariant distribution:** $\mu' = \mu = \mu^*$.

Aggregation

- **Output and labor demand:**

$$Y(\mu, B; w) = \int (y(k, z; w) - f)\, x(k, z; w)\mu(dk, dz) - Bf$$

$$L(\mu, B; w) = \int l(k, z; w) x(k, z; w)\mu(dk, dz) + B \int l(0, z; w)\phi(dz)$$

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- **Profits, investment, financing costs:** I, Π, Λ.
- **Proposition 3:** All aggregate quantities $Y(\mu, B; w), L(\mu, B; w), \Pi(\mu, B; w), I(\mu, B; w), \Lambda(\mu, B; w)$ are jointly homogeneous of degree one in B and μ.

Household

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$$\max_{c,l \geq 0} U(c, 1-l)$$

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- **Solutions**: $C(w, \Pi(\mu, B; w))$ and $L^s(w, \Pi(\mu, B; w))$.

Stationary Competitive Eq.

- A set of decision rules: $C(w, \Pi(\mu, B; w)), L^s(w, \Pi(\mu, B; w))$ and $k(k, z; w), l(k, z; w), x(k, z; w)$.
- A value function for each firm: $p(k, z; w)$.
- Aggregate quantities: $Y(\mu, B; w), L(\mu, B; w), \Pi(\mu, B; w), I(\mu, B; w), \Lambda(\mu, B; w)$.
- A wage rate w.
- A measure μ of firms and an entry level B.

such that:

Stationary Competitive Eq.

- The decision rules of the consumer and the decision rules of each firm as well as their value functions solve their problems.
- The free entry condition is satisfied.
- Markets clear: $L^s(w, \Pi(\mu, B; w)) = L(\mu, B; w)$ and $C(w, \Pi(\mu, B; w)) = Y(\mu, B; w) - I(\mu, B; w) - \Lambda(\mu, B; w)$.
- Aggregation is satisfied as well as the law of motion for the measure of firms with $\mu = \mu'$.
- **Proposition 5:** A stationary competitive equilibrium with positive entry exists.

Calibration

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- **Technology:**
  $y = Ae^z k^{\alpha_k} l^{\alpha_l}$, with $\alpha_k + \alpha_l = .95$.
  $z' = \rho z + \varepsilon'$, with $\varepsilon \sim N(0, \sigma)$ over $[-4\sigma, 4\sigma]$.
  $Z = [-4\sigma/\sqrt{1-\rho^2}, 4\sigma/\sqrt{1-\rho^2}]$; $\rho = .62$ and $\sigma = .15$.
  $\delta = .145$.
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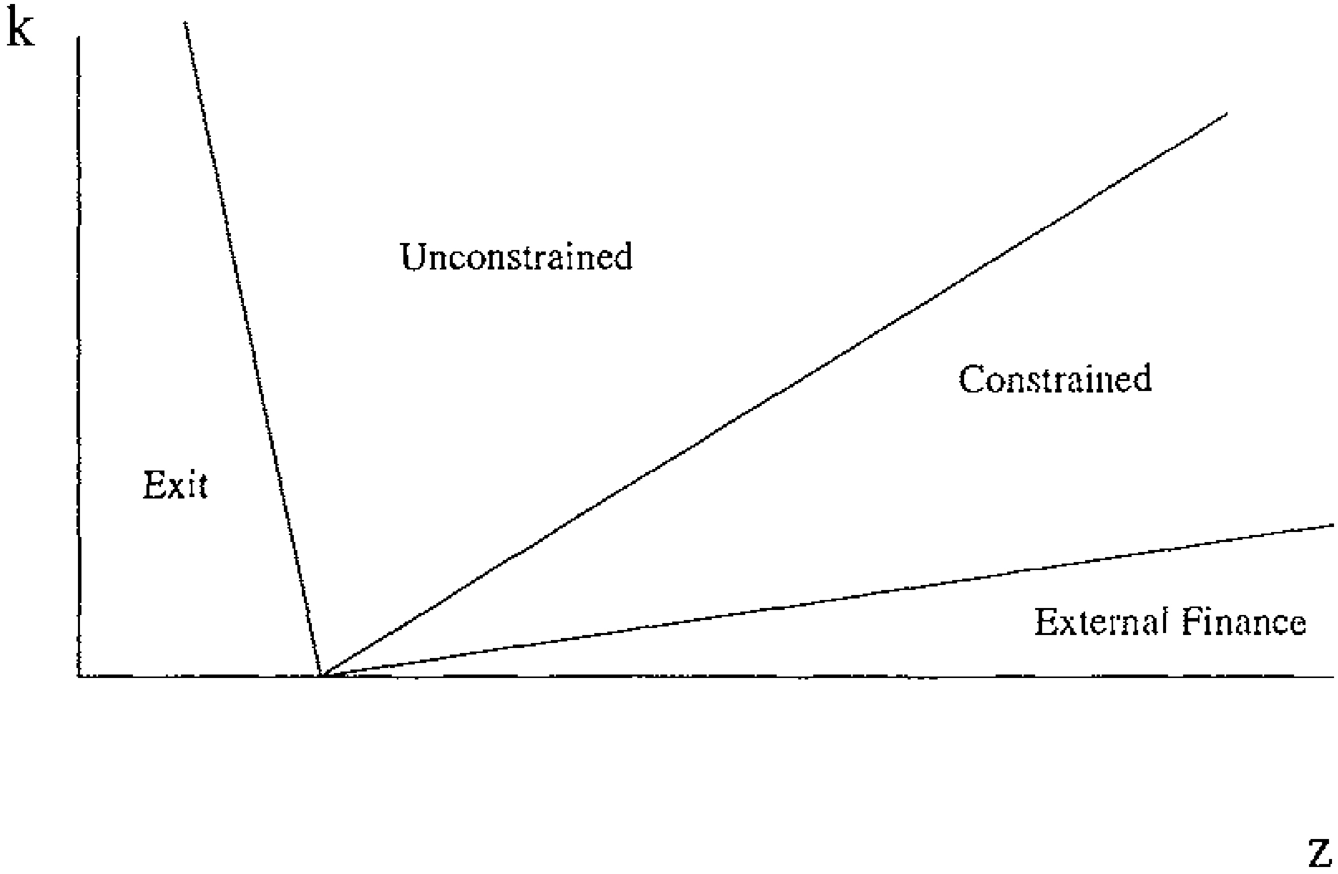
- **Financing costs:**
  $\lambda = .108 + .028 \times$ New Issues.

Results

Firm behavior:



Results

Aggregate results:

Variable	Model
Investment share (I/Y)	0.21
Share of financing costs (Λ/Y)	0.0062
External finance/total costs	0.17
Flotation costs/external finance	0.39

Results

Cross-sectional results:

Variable	Data	Model
Matched quantities		
Average size (capital)	80.89	80.89
Investment rate I/K	0.145	0.145
Standard deviation I/K	0.139	0.160
Autocorrelation I/K	0.239	0.191
Other quantities		
Mean q	1.56	1.12
Growth rate (sales)	0.036	0.031
Average CF/K	0.292	0.221
Standard deviation CF/K	0.214	0.091
Negative investment (fraction)	0.08	0.13

Note: CF denotes cash flow

Results

Role of financing constraints:

Variable	External finance	Constrained	Unconstrained
Firms	0.07	0.63	0.30
Share of investment	0.79	0.74	−0.53
Size (capital)	55.96	171.93	298.57
Mean I/K	1.20	0.188	−0.086
Tobin's q	1.34	1.14	1.08
Marginal product of capital	0.24	0.22	0.19

Empirical implications

- **Reduced-form investment equation:**

$$\frac{i_{i,t}}{k_{i,t-1}} = b_0 + b_1 \underbrace{\frac{p_{i,t-1}}{k_{i,t-1}}}_{q_{i,t-1}} + b_2 \frac{\pi_{i,t-1}}{k_{i,t-1}} + f_t + d_i + \varepsilon_{i,t}$$

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- **Results:** cash flow effect in the data; coefficient size and no cash flow effect with artificial data; λ not necessary.

Coefficient	Data	Benchmark model	No financing constraints
b_1	0.06	2.82	8.07
	(0.01)	(0.08)	(0.10)
$\bar{R}^2$	0.12	0.53	0.84

Coefficient	Data	Benchmark model	No financing constraints
b_1	0.06	4.13	11.52
	(0.01)	(0.39)	(0.05)
b_2	0.14	−2.67	−10.19
	(0.04)	(0.77)	(0.10)
$\bar{R}^2$	0.25	0.53	0.98

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