Rish Sharing in Private Information Models with Asset Accumulation

Explaining the Excess Smoothness of Consumption

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Sargent Reading Group Presentation - Greg Kaplan

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Aim

Derive empirical implications of relationship between consumption and income in models with private information

Introduction

1. Relative to self-insurance, economies exhibit ‘excess smoothness’. But not ‘excess sensitivity’!
    - Euler equation still holds....
    - ... hence intertemporal budget constraint must not
2. Apply tests of intertemporal budget constraints developed in Hansen, Roberds and Sargent (1991) to micro-data and interpret as tests of market structure.
3. Additional contributions:
    - Provide structural interpretation to insurance coefficients in Blundell, Pistaferri and Preston (2007)
    - Reconcile results in Allen-Cole-Kocherlakota and Abraham-Pavoni

Model: Information and Technology

- Large no., ex-ante identical agents each live $T \leq \infty$ periods
- Individual production technology:

$$x_t = f(\theta_t, e_t)$$

- $e_t \in E \subset \mathbb{R}$ is agent's effort level
- $\theta_t \in \Theta \subset \mathbb{R}$ is random walk shock (skill level):

$$\theta_t = \theta_{t-1} + v_t^p$$

- e_t, θ_t : private information
- x_t : publicly observed
- f : continuous, increasing in both arguments

Model: Preferences and Feasibility

$$\sum_{t=1}^{T} \delta^{t-1} u\left(c_t, e_t\right)$$

- Revelation principle, f increasing $\Rightarrow$ condition on θ^t alone
- Reporting strategy: $\sigma = \{\boldsymbol{\sigma}_t\}_{t=1}^{T}$ where $\boldsymbol{\sigma}_t : \Theta^t \to \Theta$
- Truthful reporting strategy, $\boldsymbol{\sigma}_t^*\left(\theta^t\right) = \theta_t$
- An **allocation** (α, c, x) is triplet $\{\mathbf{e}_t, \mathbf{c}_t, \mathbf{x}_t\}_{t=1}^{T}$ of θ^t measurable functions $\in \Omega$

$$\Omega = \left\{(\alpha, c, x) : \mathbf{e}_t\left(\theta^t\right) \in E, \mathbf{c}_t\left(\theta^t\right) \in C, \mathbf{x}_t\left(\theta^t\right) = f\left(\theta_t, \mathbf{e}_t\left(\theta^t\right)\right)\right\}$$

- Resource feasibility:

$$\mathbb{E}\left[c_t\right] \leq \mathbb{E}\left[y_t\right] \quad \forall \quad t$$

Market Arrangements

$$\max_{(\alpha,c,x)\in\Omega} \mathbb{E}\left[\sum_{t=1}^{T} \delta^{t-1} u\left(c_t, e_t\right)\right] \quad \text{subject to:}$$

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Full Information

$$\int_{\Theta^t} \left(\mathbf{c}_t\left(\theta^t\right) - \mathbf{y}_t\left(\theta^t\right)\right) d\mu^t = 0$$

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$$b_0 = 0, \quad \lim_{t\to T}\left(\prod_{n=1}^{t} q_n\right) \mathbf{b}_t\left(\theta^{t-1}\right) = 0$$

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Moral Hazard, Monitorable Assets

$$\mathbb{E}\left[\sum_{t=1}^{T} \delta^{t-1} u\left(c_t, e_t\right) \quad ; \quad \sigma^*\right] \geq \mathbb{E}\left[\sum_{t=1}^{T} \delta^{t-1} u\left(c_t, e_t\right) \quad ; \quad \sigma\right]$$

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Moral Hazard, Hidden Assets

$$\mathbb{E}\left[\sum_{t=1}^{T} \delta^{t-1} u\left(c_t, e_t\right) \quad ; \quad \sigma^*\right] \geq \mathbb{E}\left[\sum_{t=1}^{T} \delta^{t-1} u\left(\hat{c}_t, e_t\right) \quad ; \quad \sigma\right]$$

where $\hat{c}_t$ satisfies self-insurance budget constraint

A Few Things to Note

- If $u(c,e) = u(c) - v(e)$ then in moral hazard model with monitorable assets, the Rogerson condition holds:

$$\begin{aligned} q_t \mathbb{E}_t \left[\frac{1}{u'(c_{t+1})} \right] &= \delta \frac{1}{u'(c_t)} \\ u'(c_t) &\leq \frac{\delta}{q_t} \mathbb{E}_t \left[u'(c_{t+1}) \right] \end{aligned}$$

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- But in other three models, the EE holds:

$$u_c(c_t, e_t) = \frac{\delta}{q_t} \mathbb{E}_t \left[u_c(c_{t+1}, e_{t+1}) \right]$$

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- What is the distinguishing feature between models? The version of the inter-temporal budget constraint holds.
- Possible that BC based on a single asset is violated, while EE holds, generating "excess smoothness" and partial insurance
- Additional insurance can only be generated if intertemporal budget constraint is violated.

Characterizing Equilibria

- Constrained efficient allocations in each environemtn can be replicated by a plan on lump-sum transfers $\left\{\boldsymbol{\tau}_t\left(\theta^t\right)\right\}_{t=1}^T$
- Focus on situations in which optimal consumption plan is x^t-measurable. Then optimal transfers are functions of x^t only, $\boldsymbol{\tau}_t^*\left(x^t\right)$
- Hence budget constraint of agent is

$$c_t + q_t b_{t+1} = x_t + \boldsymbol{\tau}_t^*\left(x^t\right) + b_t$$

The Allen-Cole-Kocherlakota Economy

Assumptions:

1. $u(c, e) = u(c - e)$
2. $x_t = f(\theta_t, e_t) = \theta_t + e_t$

$\Rightarrow$Linearity implies we can set $e_t \equiv 0$, WLOG. Then since all variations in θ_t induce variation in x_t, $\mathbf{c}_t$ will be x^t-measurable

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Final Period, $t = T < \infty$

$$\begin{aligned} u(c_T - e_T) &= u\left(\theta_T + \boldsymbol{\tau}_T^*\left(x^{T-1}, x_T\right) + b_T\right) \\ &= u\left(\theta_T + \boldsymbol{\tau}_T^*\left(x^{T-1}, \theta_T + e_T\right) + b_T\right) \end{aligned}$$

$$\Rightarrow \boldsymbol{\tau}_T^*\left(x^T\right) \text{ not a function of } x_T$$

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Penultimate Period, $t = T-1$

From FOC for agent's effort choice, evaluated at $e_T^* = e_{T-1}^* = 0$

$$\frac{\partial \boldsymbol{\tau}_{T-1}^*\left(x^{T-1}\right)}{\partial x_{T-1}} + \delta \frac{\partial \boldsymbol{\tau}_T^*\left(x^T\right)}{\partial x_{T-1}} \mathbb{E}_{T-1}\left[\frac{u'\left(c_T^*\right)}{u'\left(c_{T-1}^*\right)}\right] = 0$$

From Euler equation $\mathbb{E}_{T-1}\left[\frac{u'\left(c_T^*\right)}{u'\left(c_{T-1}^*\right)}\right] = \frac{q_{T-1}}{\delta}$

$$\frac{\partial \boldsymbol{\tau}_{T-1}^*\left(x^{T-1}\right)}{\partial x_{T-1}} + q_{T-1}\frac{\partial \boldsymbol{\tau}_T^*\left(x^T\right)}{\partial x_{T-1}} = 0$$

$$\Rightarrow \boldsymbol{\tau}_{T-1}^*\left(x^{T-1}\right) + q_{T-1}\boldsymbol{\tau}_T^*\left(x^{T-1}, \cdot\right) \text{ not a function of } x_{T-1}$$

$\Rightarrow$ implies that inter-temporal budget constraint for risk-free asset holds. Hence no additional insurance possible.

The Case with Some Risk Sharing

Assumptions:

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From FOC for agent's effort choice

$$1 + \delta \frac{\partial \boldsymbol{\tau}_T^*\left(x^{T-1}, x^T\right)}{\partial x_T} = \frac{1}{f_e}$$

Recall in ACK, we had $f_e = 1$. But risk sharing imples $\frac{\partial \boldsymbol{\tau}_T^*\left(x^{T-1}, x^T\right)}{\partial x_T} < 0$. So risk-sharing is possible if $f_e > 1$

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Intuition: To make agents share risk, must ensure that agent does not shirk, ie reduce effort. If $f_e > 1$, then there is room to take resources away from agent without them shirking

A Closed Form Solution

Assumptions:

1. $u(c, e) = u(c - e)$
2. $x_t = f(\theta_t, e_t) = \theta_t + a \min\{e_t, 0\} + b \max\{e_t, 0\}$

When u is quadratic and $\delta = q$ we get

$$\Delta c_{t+1} = \frac{1}{a} v_{t+1}^{p}$$

When $a = 1$, get back to PIH, when $a \rightarrow \infty$, approach complete markets.

Extensions: CRRA utility, temporary shocks

Empirical Contribution

Test 1

$$\Delta y_t = \rho_1(L) w_{1t} + \rho_2(L) w_{2t}$$
$$\Delta c_t = \pi w_{1t}$$

$$H0 : \pi = \rho_1(q)$$
$$H1 : \pi < \rho_1(q)$$

Note $\frac{\rho_1(q)}{\pi} = a$. P-values for excess smoothness test range from

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Note $\frac{\rho_1(q)}{\pi} = a$. P-values for excess smoothness test range from

Test 2

$$\Delta var(c_t) = \frac{1}{a^2}\Delta var(x_t)$$

$$\begin{aligned}H0 &: a = 1 \\ H1 &: a > 1\end{aligned}$$

Implied a range from 1.7 to 5.2