# On the Aggregate Labor Supply

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# Paper in a Nutshell

Q: What determines the aggregate labor supply elasticity when individual labor supply is indivisible and markets are incomplete?

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A: Shape of the reservation wage distribution

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Let $\Phi \left( \hat{w} \right)$ be distribution of reservation wages. Aggregate labor supply:

$$H \left( w \right) = \int_{\hat{w}=0}^{w} d\Phi \left( \hat{w} \right) = \Phi \left( w \right)$$

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1. $\Gamma(w)$ determined by shape of reservation wage distribution
2. Joint distribution of $(a_i, x_i, B_i)$ is relevant
3. $\Gamma(w)$ moves with business cycle
4. $\Gamma(w) = \infty$ whenever mass of workers with $\hat{w}_i = w$

Intuition: high $\Gamma(w)$ if large mass of workers with reservation wage near current wage. Small wage movements push many workers in/out of work.

# Incomplete Markets Model with Indivisible Labor Supply

**Consumers:**

$$\max_{\{c_t, h_t\}} E \sum_{t=0}^{\infty} \beta^t \left[ \ln c_t - B h_t \right]$$

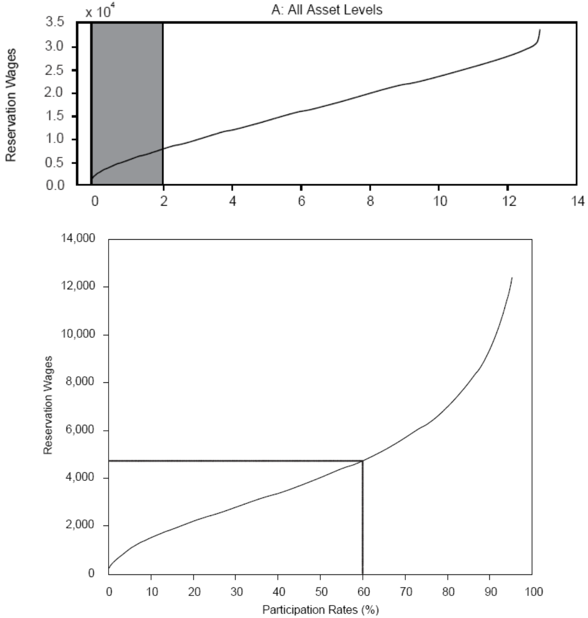
$$\begin{aligned}
\text{subject to} &\quad : \\
c_t + a_{t+1} &= W_t x_t h_t + R_t a_t \\
\ln x_{t+1} &= \rho_x \ln x_t + \varepsilon_x \\
a_{t+1} &\geq \bar{a}, \quad h_t \in \{0, 1\}
\end{aligned}$$

**Production:**

$$\begin{aligned}
Y_t &= \lambda_t L_t^{\alpha} K_t^{1-\alpha} \\
\ln \lambda_{t+1} &= \rho_\lambda \ln \lambda_t + \varepsilon_\lambda
\end{aligned}$$

$\Rightarrow$ Stochastic competitive equilibrium, calibrated to quaterly data, choose $B$ so that steady state employment is 60%.

# Reservation Wage Distribution



# Implied Labor Supply Elasticity

**Table 3 Labor Supply Elasticity Implied by the Reservation Wage Distribution**

| | Employment Rate | |
|---|---|---|
| $E = 58\%$ | $E = 60\%$ | $E = 62\%$ |
| 1.12 | 1.05 | 0.97 |

Notes: The numbers reflect the elasticity of the labor market participation rate with respect to reservation wage (evaluated at employment rates of 58, 60, and 62 percent) based on the reservation wage distribution in the steady state.

# Comparison with Representative Agent Economy

$$u\left(C, H\right) = \ln C - B\frac{H^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}}$$

$$\Longrightarrow \Gamma\left(w\right) = \gamma$$

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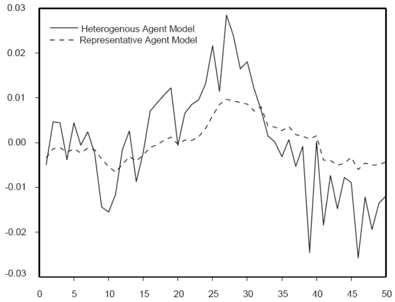
$\gamma = 0.4$

**Figure 5 Total Hours Worked from the Models**




Is this a fair comparison?

# Final Thoughts

**Basic story:** $\Gamma(w)$ is large because many (employed and unemployed) workers are marginal. i.e. at their current wages, it would only take a small wage change for them to change their employment decision.

- Is the spirit so different from a continuous choice set up with high individual labor supply elasticity?
- If wages are close to reservation wages, suggests that bargaining power of workers is very low.
- Suggests that agents have high outside options - eg home production, unemployment benefits. Similar to finding in search and matching literature