# The Design of Optimal Education Policies

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# Motivation



- To study the features of optimal education and tax policy in a setting where:

  1. individual ability is private information
  2. individual returns to education are uncertain
  3. the distribution of returns depends on ability

- Key features:

  1. role of parental income with borrowing constraints
  2. interaction between parental income and ability

# Findings



### Education Levels

- High Ability: higher than autarky, independent of parental income
- Low Ability: same as autarky, increasing in parental income

### Education Financing

- education of high ability ind is subsidized by fees of low ability ind and taxes
- education subsidy increases with parental income for fixed ability
- high ability / high income households contribute least to education budget
- med ability / low income households contribute most to education budget

### 2 mechanisms at work

1. provide incentives for high ability ind to be educated
2. govt intervention allows poor households to overcome borrowing constraints, they thus benefit most and are willing to pay most for education through deferred fees

# Environment



- Two period model
- Two generations: mothers and daughters

## Timing

$t = 0$

- Mothers receive income, $Y \in \{Y_1 \ldots Y_n\}$ *-public info*
- Mothers learn daughter's ability, $\theta \in [\underline{\theta}, \overline{\theta}]$ - *private info*
- Mothers choose consumption, $c$, education level of daughter, $e$ and an intergenerational transfer, $t$

$t = 1$

- Daughter's income, $Y^D$, realized from distribution $\pi(\theta, e)$

# Preferences



- Mothers maximize current utility, $u(c)$, plus expected resources available to the daughter
  $u' > 0, u'' < 0$, $\lim_{c \to 0} = +\infty$
  $u'(c^*) = 1$ for some $c^*$
- Let $y(\theta, e) = \sum_{i=1}^{n} Y_i \pi_i(\theta, e)$ be expected income of daughter
  $y_e, y_\theta, y_{\theta e} > 0$, $y_{ee}, y_{\theta\theta} \leq 0$
  $\lim_{e \to 0} = +\infty$
- Mother maximizes

$$\max_{e,t} \{u(Y_i - t - ke) + y(\theta, e) + t\} \tag{1}$$

subject to

$$t \geq 0$$

# Autarky



- Let $e^S(\theta; k)$ satisfy

$$y_e(\theta, e^S(\theta; k)) = k$$

- Let $e^C(\theta, Y_i; k)$ satisfy

$$y_e(\theta, e^C(\theta, Y_i; k)) = ku'(Y_i - ke^C(\theta, Y_i; k))$$

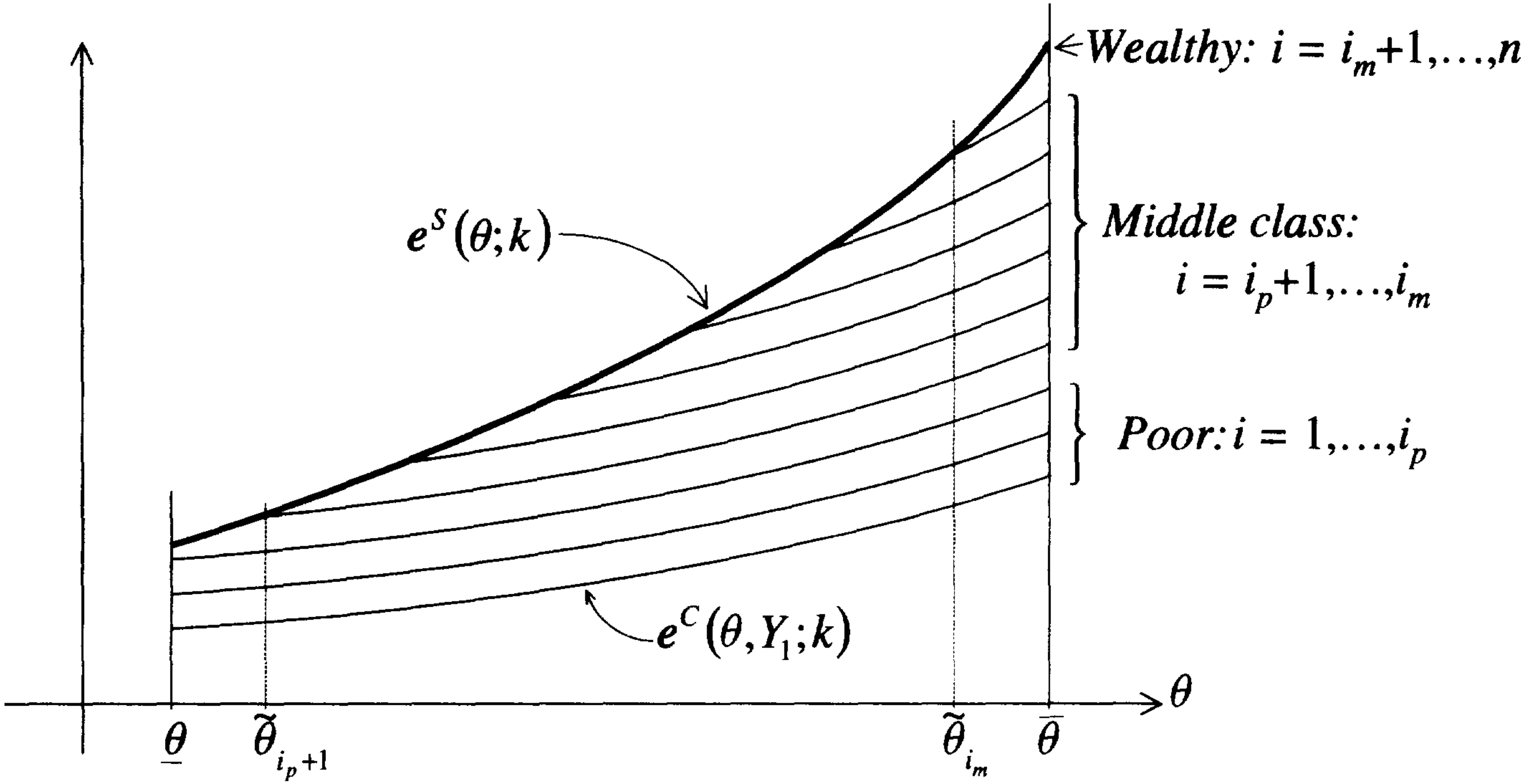
**Proposition 1** The autarkic solution is

$$e^P(\theta, Y_i; k) = \min\left\{e^S(\theta; k), e^C(\theta, Y_i; k)\right\}$$
$$t^P(\theta, Y_i; k) = \max\left\{Y_i - c^* - ke^S(\theta; k), 0\right\}$$

- Let $P(\theta, Y)$ be maximized utility

# Autarkic Solution





# Government Policy



The government has 3 available policy instruments:

- $\tau_i$: tax on income level $Y_i$
- $f_i(\theta)$: up-front fee, conditional on ability
- $m_i(\theta)$: deferred fee, conditional on ability

A **policy** is a set $\{\tau_i, e_i(\theta), f_i(\theta), m_i(\theta)\}_{i=1}^n$

Given a policy, a mother's utility is:

$$U_i(\theta) = u(Y_i - \tau_i - f_i(\theta)) + y(\theta, e_i(\theta)) - m_i(\theta)$$

Hence alternate definition of a policy is:

$$\{\tau_i, e_i(\theta), f_i(\theta), U_i(\theta)\}_{i=1}^n$$

# Government Objective



- Let $H$ be distribution of mothers' income: $Pr(Y = Y_i) = h_i$
- Let $\Phi$ be distribution of daughters' ability, with density $\phi(\theta)$

- The planner's problem is to maximize

$$\sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\overline{\theta}} U_i(\theta)\phi(\theta)d\theta \tag{2}$$

subject to the following constraints . . .

# Individual Constraints



## Upper Bound on Taxes

$$\tau_i \leq \tau_i^{\max} \qquad i = 1 \ldots n \tag{3}$$

## Incentive Compatibility

$$\theta = \arg\max_{\hat{\theta}} \left\{ u(Y_i - \tau_i - f_i(\hat{\theta})) + y(\theta, e_i(\hat{\theta})) - m_i(\hat{\theta}) \right\} \tag{4}$$

A set of sufficient conditions is

$$\begin{aligned} \dot{U}_i(\theta) &= y_\theta(\theta, e_i(\theta)) \\ \dot{e}_i(\theta) &\geq 0 \end{aligned}$$

## Participation Constraints

$$U_i(\theta) \geq P(\theta, Y_i - \tau_i) \qquad \forall \theta \in [\underline{\theta}, \overline{\theta}] \tag{5}$$

# Aggregate Constraints



**Liquidity Constraint** $(\lambda)$

$$\sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\overline{\theta}} k e_i(\theta)\phi(\theta)d\theta \leq \sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\overline{\theta}} f_i(\theta)\phi(\theta)d\theta + \sum_{i=1}^{n} h_i \tau_i$$

"education must be financed out of upfront fees and taxes"

**Budget Constraint** $(\beta)$

$$-\sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\overline{\theta}} m_i(\theta)\phi(\theta)d\theta \leq 0$$

Combined with liquidity constraint implies "resources spent on education not more than total resources of households"

**Education Constraint** $(\sigma)$

$$\sum_{i=1}^{n} h_i \int_{\underline{\theta}}^{\overline{\theta}} e_i(\theta)\phi(\theta)d\theta = E$$

# Optimal Education Policy



Let $e^*(\theta; \beta, x)$ solve

$$y_e(\theta, e, E) - x = \frac{\beta - 1}{\beta} \frac{1 - \Phi(\theta)}{\phi(\theta)} y_{e\theta}(\theta, e)$$

**Proposition 3** The optimal education policy satisfies

$$e_i(\theta) = max\left\{e^*(\theta; \beta, k^*), e^P(\theta, Y_i - \tau_i; k)\right\} \tag{6}$$

where $k^* = \frac{\beta+\lambda}{\beta}k - \frac{\sigma}{\beta}$

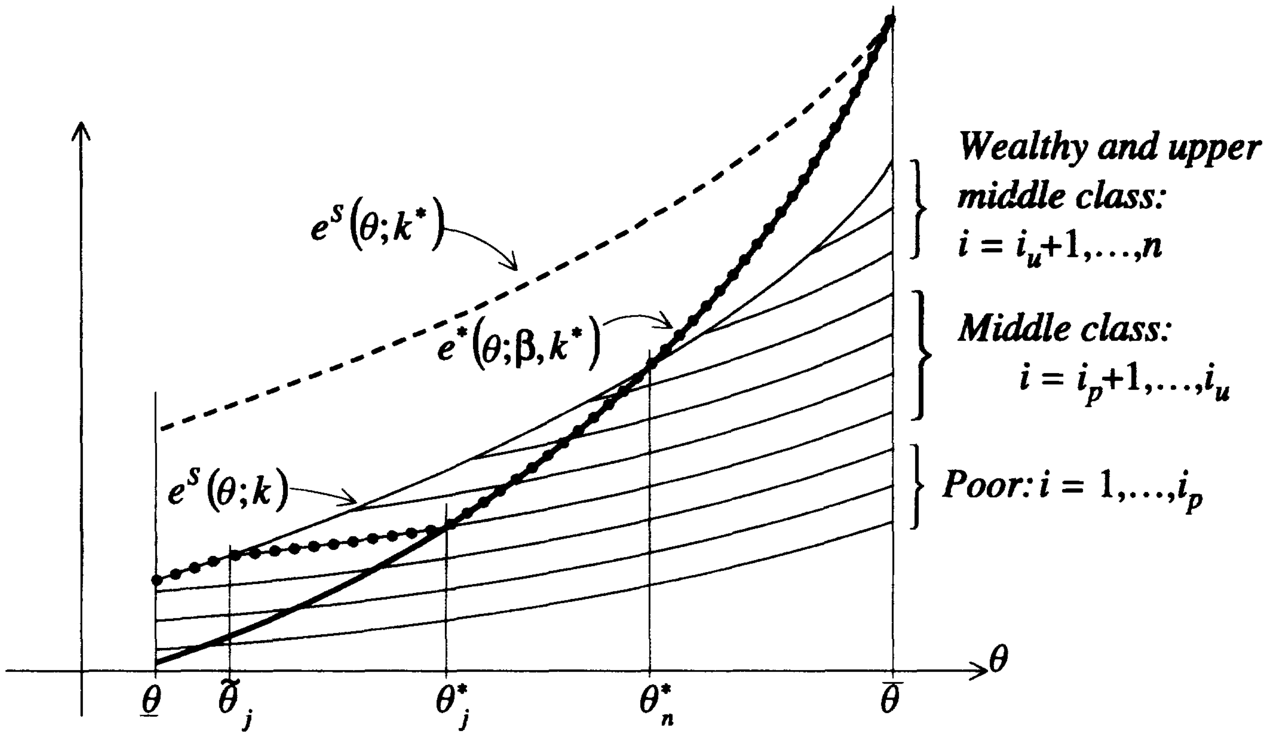
- $k^* \simeq$ social unit cost of education
- households divided into two groups: one group who receives same education they would privately, one group who receives more
- latter group (high ability) is non-empty
- high ability education does not depend on income

# Symmetric Information



**Corollary 2** If the government can costlessly observe $\theta$ then the education policy satisfies

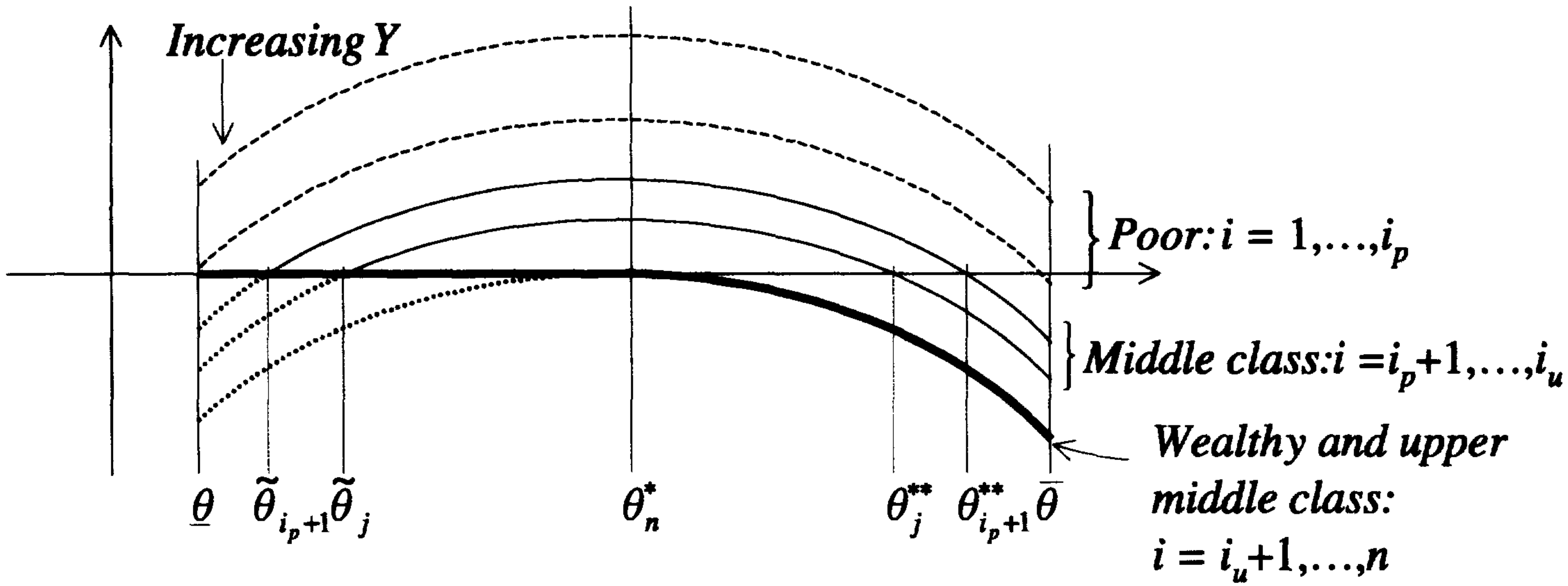
$$e_i(\theta) = e^S(\theta; k^*)$$



# Households' Financial Contribution



- Let $z_i(\theta) = f_i(\theta) + m_i(\theta) - ke_i(\theta)$
  $=$ net contribution to education budget



# Limitations and Extensions



- uncertainty over returns - risk neutrality
- static model - intergenerational dynamic contract
- subset of class of policies considered eg income-contingent loans are ruled out since $m_i(\theta)$ can not depend on daughter's income
- correlate ability and parental income

$\Rightarrow$ study dynamic model with risk aversion and its decentralization in a model similar to Farhi and Werning (2005)