Risk-Sharing Between and Within Families

Econometrica (1996)

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Outline and Motivation

- Altug and Miller (1990): test based on exogeneity of lagged wage changes to consumption changes
- No power against alternative of self-insurance in some circumstances
- Test intra-family risk sharing

A Complete Markets Model

- Discrete time, $t = 0...$
- Decision making unit: household k in family i.
- Choose $\{C_{ik}(s^t), L_{ik}(s^t)\}$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t u_{ik}\left(C_{ik}(s^t), L_{ik}(s^t); s^t\right) \Pr(s^t)$$

subject to :

$$\sum_{t=0}^{\infty} \sum_{s^t} q(s^t)\, C_{ik}(s^t) = A_{ik} + \sum_{t=0}^{\infty} \sum_{s^t} w_{ik}(s^t) \left[1 - L_{ik}(s^t)\right]$$

A Complete Markets Model

- FONC for consumption:

$$\Pr\left(s^t\right)\beta^t \frac{\partial u_{ik}\left(C_{ik}\left(s^t\right), L_{ik}\left(s^t\right); s^t\right)}{\partial C_{ik}\left(s^t\right)} = q\left(s^t\right)\Lambda_{ik}$$

- Take logs:

$$\theta_{ik}\left(s^t\right) = \lambda_{ik} + p\left(s^t\right)$$

$$\begin{aligned}
\theta_{ik}\left(s^t\right) &= \log\left[\frac{\partial u_{ik}\left(C_{ik}\left(s^t\right), L_{ik}\left(s^t\right); s^t\right)}{\partial C_{ik}\left(s^t\right)}\right] \\
\lambda_{ik} &= \log\left[\Lambda_{ik}\right] \\
p\left(s^t\right) &= \log\left[\frac{q\left(s^t\right)}{\Pr\left(s^t\right)}\right] - t\log\beta
\end{aligned}$$

Family Risk Sharing

- Full set of A-D securities are traded between households within the same families
- Assets may or may not be traded between families

$$\theta_{ik}\left(s^t\right) = \lambda_{ik} + p_i\left(s^t\right)$$

$$\begin{aligned} \text{full risk-sharing} &: \quad \theta_{ikt} = \lambda_{ik} + p_t \\ \text{family risk-sharing} &: \quad \theta_{ikt} = \lambda_{ik} + p_{it} \end{aligned}$$

Consumption Equation

$$
\begin{aligned}
u(C, L) &= \frac{C_{ikt}^{1-\rho_0}}{1-\rho_0} L_{h,ikt}^{\rho_h \rho_0} L_{w,ikt}^{\rho_w \rho_0} \exp\{\rho_0 \eta_{ikt}\} + g\,(\text{other goods}) \\
\eta_{ikt} &= b_{ikt}^{'}\beta + \varepsilon_{ikt}
\end{aligned}
$$

$$
\begin{aligned}
C &: \quad \text{food consumption} \\
L_h &: \quad \text{husband's leisure} \\
L_w &: \quad \text{wife's leisure}
\end{aligned}
$$

- Differentiate and re-arrange:

$$
\begin{aligned}
c_{ikt} &= z_{ikt}^{'}\delta + \theta_{ikt} + \varepsilon_{ikt} \\
z_{ikt}^{'} &= \left(b_{ikt}^{'}, l_{h,ikt}, l_{w,ikt}\right) \\
\delta' &= (\beta', \rho_h, \rho_w)
\end{aligned}
$$

- If leisure is freely chosen or measured with error then an instrument will be required for estimation

Assumptions on Preference Shocks

$$\begin{aligned} E\left(\varepsilon_{ikt}\right) &= 0 \\ \varepsilon_{ikt} &\perp x_{ihs} \end{aligned}$$

where $x'_{ikt} = \left(b'_{ikt}, g'_{ikt}, w'_{ikt}\right)$.

- g'_{ikt} : vector of instruments for leisure

Sample Selection Mechanism

- Assume $\exists$ function $\phi_t(v_{ik}, x_{ik})$ such that a household-year observation is observed if

$$\phi_t(v_{ik}, x_{ik}) \geq 0$$

where v_{ik} is the permanent component of the error term (preference shock) ε_{ikt}

- Since

$$E\left[\Delta\varepsilon_{ik,t+1} | v_{i,} x_i\right] = 0$$

then for any given "family type":

$$\begin{aligned} E\left[\Delta\varepsilon_{ikt}\right] &= 0 \\ E\left[x_{ihs} \cdot \Delta\varepsilon_{ikt}\right] &= 0 \end{aligned}$$

Full Risk Sharing Excluding Split-offs

$$\begin{aligned} E\left[\Delta c_{i1t} - \Delta z'_{i1t}\delta - d_t\pi'\right] &= 0 \\ E\left[\Delta x_{i1t} \cdot \left(\Delta c_{i1t} - \Delta z'_{i1t}\delta - d_t\pi'\right)\right] &= 0 \end{aligned}$$

- Large number of moments, collapse to give:

$$E\left[\sum_{t=1}^{T-1} \Delta x_{i1t} \cdot \left(\Delta c_{i1t} - \Delta z'_{i1t}\delta - d_t\pi'\right)\right] = 0$$

Long Lags and Self-Insurance

- Altug and Miller (1990) use the same set of moment conditions with the instruments, Δx_{i1t}, replaced with lagged wages, $w_{i1,t-1}$ and $w_{i1,t-2}$.
- Test of overidentifying restrictions has no power against alternative of self-insurance together with unforecastable component of consumption change consisting of the sum of aggregate macro component and idiosyncratic component
- Using wages changes, Δw_{i1t}, may also lead to low power if wage changes are known in advance.
- Using long lags, exploits zero correlation with future changes in exogenous variables:

$$E\left[\sum_{t=1}^{T-\tau} \Delta_\tau x_{i1t} \cdot \left(\Delta_\tau c_{i1t} - \Delta_\tau z'_{i1t}\delta - \left(d_t + \ldots + d_{t+\tau-1}\right)\pi'\right)\right] = 0$$

$$\tau = 1, 2 \ldots T-1$$

Moment Conditions

Full Risk Sharing with Family Split-offs

$$E\left[\sum_{k=1}^{K}\sum_{t=1}^{T-\tau}\Delta_\tau x_{ikt}\cdot\left(\Delta_\tau c_{ikt}-\Delta_\tau z_{ikt}'\delta-\left(d_t+\ldots+d_{t+\tau-1}\right)\pi'\right)\right]=0$$

Family Risk Sharing

Let

$$\tilde{\Delta}_\tau\xi_{ikt}=\Delta_\tau\xi_{ikt}-\frac{1}{K}\sum_{k=1}^{K}\Delta_\tau\xi_{ikt}$$

$$E\left[\sum_{k=1}^{K}\sum_{t=1}^{T-\tau}\tilde{\Delta}_\tau x_{ikt}\cdot\left(\tilde{\Delta}_\tau c_{ikt}-\tilde{\Delta}_\tau z_{ikt}'\delta\right)\right]=0$$

Data

- PSID, food consumption, 2 panels:
  1. 1968-81: replicate Altug and Miller (1990), show effect of including long lags
  2. 1985-87: maximum number of split-offs to test family risk-sharing, hours and food data refer to same period

Results 1

	1	2	3	4	5	6	7	8	9
orthogonality conditions	(2.18a, b)	(2.18a, b)	(2.18a, 19)	(2.18a, 19)	(2.18a, 19)	(2.18a, 19)	(2.18a, 19)	(2.20a, b)	(2.20a, b)[a]
long changes used?	no	no	no	no	no	no	no	yes	yes
consolidation of orth. conditions	none	none	over t	over t	over t	over t	over t	over t	over t, τ
71–74 change used?	no	yes	yes	yes	yes	yes	yes	yes	yes
instruments $(\Delta x_{j1t})^b$	set A	set A	set A	set B	set C	set D	set E	set E	set E
#orthogonality conditions	112	126	22	15	14	18	17	81	17
χ^2	118.7	172.2	61.4	44.5	5.9	6.1	9.5	93.7	22.9
degrees of freedom	102	115	11	4	2	2	1	65	1
p-value (%)	12.322%	0.044%	0.000%	0.000%	5.320%	4.798%	0.211%	1.136%	0.000%

Results 2

	1	2	3	4	5
orthogonality conditions	(2.20)	(2.21)	(2.21)	(2.21)	(2.23)
null hypothesis tested	full risk sharing	full risk sharing	full risk sharing	self insurance	fam. risk sharing
sub-sample used	original households	original & split-offs	original & split-offs	original & split-offs	original & split-offs
time-balanced?	yes	no	no	no	no
#families	697	1,251	1,251	1,251	500
#households	697	2,066	2,066	2,066	1,298
#1-yr changes	1,394	3,473	3,473	3,473	—
#2-yr changes	697	1,681	1,681	1,681	—
instruments $(\Delta x_{ikt})^a$	set E′	set E′	set F	set F	set E′
#orthogonality conditions	24	24	24	24	22
χ^2	20.0	30.4	31.6	20.2	28.9
degrees of freedom	12	12	12	12	12
p-value (%)	6.724%	0.242%	0.157%	6.374%	0.412%