The Business Cycle and the Life Cycle

Presentation for Sargent Reading Group

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A Representative Agent Model

Preferences

$$\sum_{t=0}^{\infty} \beta^t \left[\log c_t - \frac{\omega}{\gamma} H_t^{\gamma} \right]$$

$$\begin{aligned} C_t &= \left[\psi C_{mt}^{\xi} + (1-\psi) C_{nt}^{\xi} \right]^{\frac{1}{\xi}} \\ H_t &= H_{mt} + H_{nt} \end{aligned}$$

Technology

$$\begin{aligned} Y_{mt} &= z_t K_{mt}^{\theta} \left[(1+g)^t H_{mt} \right]^{1-\theta} \\ \log z_{t+1} &= \rho \log z_t + \varepsilon_{t+1} \\ Y_{nt} &= K_{nt}^{\eta} \left[(1+g)^t H_{nt} \right]^{1-\eta} \end{aligned}$$

$$\begin{aligned} K_{mt+1} &= (1-\delta_m) K_{mt} + I_{mt} \\ K_{nt+1} &= (1-\delta_n) K_{nt} + I_{nt} \end{aligned}$$

A Representative Agent Model

Government

Labor Income Tax: τ_h
Capital Income Tax: τ_k

Resource Constraints

$$\begin{aligned} Y_{mt} &= C_{mt} + G_t + I_{mt} + I_{nt} \\ Y_{nt} &= C_{nt} \end{aligned}$$

Parameters

β	θ	η	δ_m	δ_n
0.954	0.30	0.27	0.065	0.057

g	ρ	σ_ε	τ_h	τ_k
0.018	0.806	0.0139	0.25	0.50

A Representative Agent Model

Standard Deviation of Market Hours Relative To Output

	$\gamma = 1$	$\gamma = 2$	$\boldsymbol{\gamma = 2.5}$	$\gamma = 3.5$	$\gamma = 4$	$\gamma = 11$
$\xi = 0$	.66	.48	.45	.43	.40	.35
$\xi = 0.2$	.67	.50	.47	.45	.43	.38
$\xi = 0.4$	.68	.53	.51	.49	.47	.44
$\boldsymbol{\xi = 0.5}$	.69	.56	**.54**	.52	.50	.47
$\xi = 0.6$	.71	.59	.57	.56	.54	.52
$\xi = 0.8$	.78	.71	.70	.69	.69	.68

- **Data:** .80

Is This Hiding Something?

- Model can not account for magnitude of business cycle fluctuations in hours
- Is this accross all sub-groups or just some?
- Where should we enrich the model in order to understand this fact?

Is This Hiding Something?

Fluctuations by Age

Relative cyclical fluctuations of hours by age group

	16-19	20-24	25-34	35-44	45-54	55-64	65+
σ_h/σ_{Y_m}	2.23	1.23	0.86	0.64	0.57	0.59	1.26
% of H_m	4	11	26	25	20	12	2
% of σ_{H_m}	11	16	28	19	14	9	3

- Shape **not** driven by differences in sectoral composition

Is This Hiding Something?

Fluctuations by Education

	<HS	HS	C-d/o	C-grad
σ_h/σ_{Y_m}	1.20	0.84	0.67	0.23

Fluctuations by Gender

	Males	Females
σ_h/σ_{Y_m}	0.87	0.73

A Life-Cycle Model

Demographics

- Overlapping generations, T period life, annual, retire at T_R

Preferences

$$\sum_{a=0}^{T} \beta^a U^a \left(c_{m,t+a}^{a+1}, c_{n,t+a}^{a+1}, h_{m,t+a}^{a+1}, h_{n,t+a}^{a+1} \right)$$

$$U^a = \left[\psi^a c_m^{\xi} + \left(1 - \psi^a\right) c_n^{\xi} \right]^{\frac{1}{\xi}} - \frac{\omega^a}{\gamma} \left(h_{mt} + h_{nt} \right)^{\gamma}$$

Individual Constraints

$$\begin{aligned}
c_{nt}^a &= \left(k_{nt}^a\right)^{\eta} \left[\left(1+g\right)^t h_{nt}^a \right]^{1-\eta} \\
c_{mt}^a + i_{mt}^a + i_{nt}^a &= \left(1-\tau_h\right) w_t e^a h_{mt}^a + \left(1-\tau_k\right) r_t k_{mt}^a + \tau_t \\
k_{mt+1}^{a+1} &= \left(1-\delta_m\right) k_{mt}^a + i_{mt}^a \\
k_{nt+1}^{a+1} &= \left(1-\delta_n\right) k_{nt}^a + i_{nt}^a
\end{aligned}$$

$$k_{ma}^{T+1} = 0, \quad k_{nt}^{T+1} = k_{n0}$$

A Life-Cycle Model

Technology

$$
\begin{aligned}
Y_{mt} &= z_t K_{mt}^{\theta} \left[(1+g)^t E_{mt} \right]^{1-\theta} \\
E_{mt} &= \sum_{a=1}^{T} e^a h_{mt}^a
\end{aligned}
$$

- Individual decision rules depend on distributions of market and non-market capital, μ_{mt}, μ_{nt}.
- Standard definition for recursive competitive equilibrium with aggregate state, $S_t = (\mu_{mt}, \mu_{nt}, z_t)$

Calibration

T	55	ρ	0.895
T_R	45	σ_ε	0.0153
θ	0.30	τ_h	0.25
η	0.21	τ_k	0.50
δ_m	0.0654	γ	2.5
δ_n	0.0568	ξ	0.45
g	0.0184	β	0.967

- subtract housing services from market output
- 1962-2000 (CPS time period for fluctuations by age)
- e^a : match lifecycle profile of male wages
- ψ^a, ω^a : match lifecycle profile of home and market wages

Calibration

Figure 1: Household Profile for Market Hours

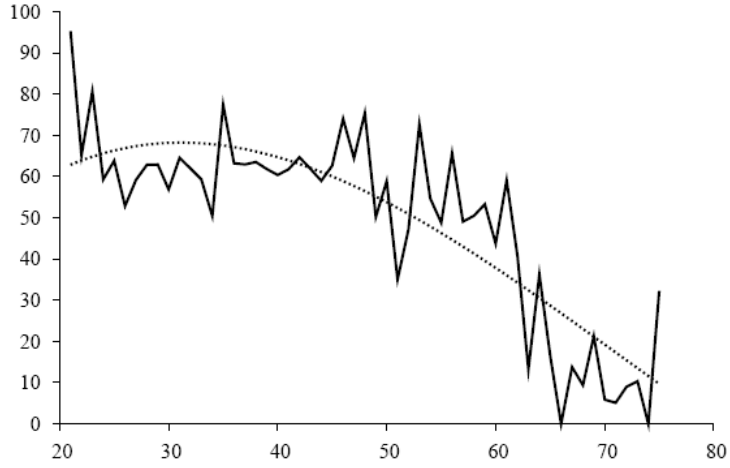


Figure 2: Household Profile for Home Hours

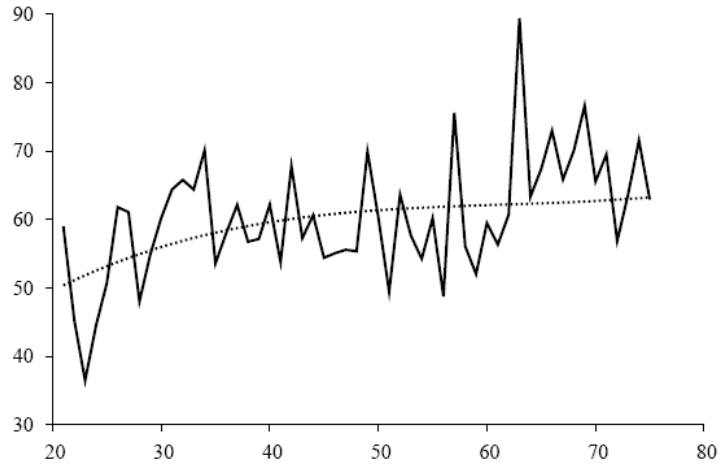


Figure 3: Calibrated Profile for ψ

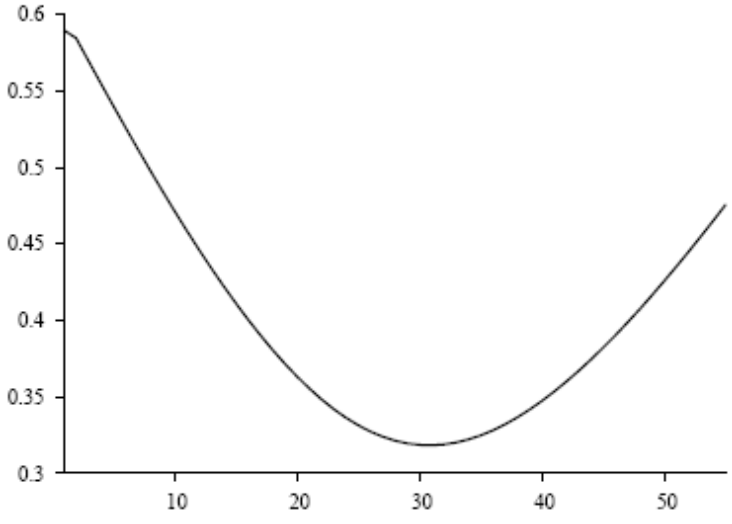
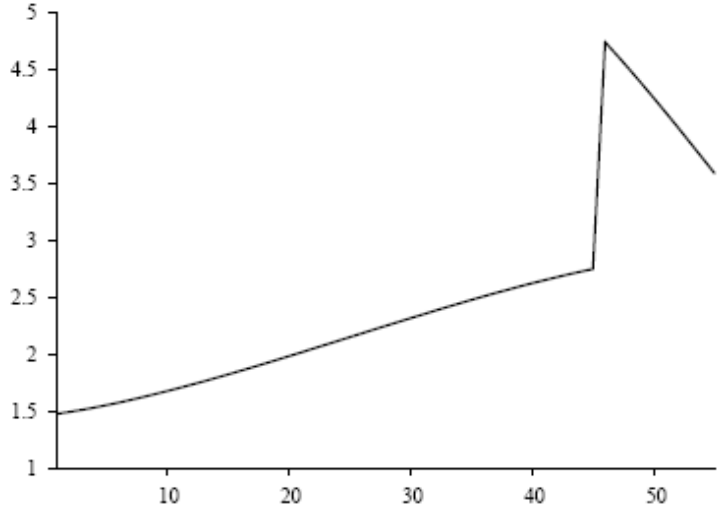


Figure 4: Calibrated Profile for ω



What Do We Find?

	Data	Model
	0.80	**0.47**
16-19	2.23	
20-24	1.23	0.39
25-34	0.86	0.35
35-44	0.64	0.35
45-54	0.57	0.46
55-64	0.59	0.97
65+	1.26	

- Ages 55-59 has volatility $= 0.73$
- Pattern is robust to calibration strategy

Understanding the Pattern

- Exogenous age differences:
  - planning horizon
  - weight on home vs market consumption
  - disutility of work
  - labor efficiency
- Endogenous differences due to accumulation of capital
- Partial equilibrium experiments fixing wage or renta rate processes

Understanding the Pattern

The Effect of the Time Horizon

- No home production, no retirement
- $\beta E[R_t] = 1$, $\omega = 1$

Table 15: Effect of Planning Horizon: Wage Shocks, No Retirement

Age Interval	$T = 5$	$T = 15$	$T = 25$	$T = 35$	$T = 45$	$T = 55$
$20 - 24$	.026	.115	.179	.223	.252	.272
$25 - 29$		.087	.161	.214	.249	.272
$30 - 34$		.044	.129	.193	.235	.264
$35 - 39$			.087	.163	.215	.249
$40 - 44$			.042	.128	.190	.232
$45 - 49$				.087	.162	.213
$50 - 54$				.042	.128	.190
$55 - 59$					.087	.162
$60 - 64$					.042	.128
$65 - 70$						.087
$71 - 75$						.042

Understanding the Pattern

The Effect of Retirement

Table 16: Effect of Retirement: Wage Shocks

Age	$T_R = 0$	$T_R = 10$	$T_R = 15$	$T_R = 20$
$20 - 24$	.252	.253	.253	.255
$25 - 29$	.249	.254	.256	.260
$30 - 34$	.235	.247	.252	.256
$35 - 39$	.215	.235	.242	.249
$40 - 44$	.190	.225	.236	.246
$45 - 49$	.162	.219	.235	.249
$50 - 54$	.128	.222	.247	.266
$55 - 59$	.087	.256	.291	.316
$60 - 64$	.042	.390	.427	.451

Understanding the Pattern

The Effect of Lifecycle Productivity Changes

Table 17: Effect of Life-Cycle Earnings: Wage Shocks, No Retirement

Age	Standard Deviation of Hours		
	no peak	peak= 2	peak= 3
20 – 24	.272	.293	.303
25 – 29	.272	.252	.245
30 – 34	.264	.212	.191
35 – 39	.249	.175	.143
40 – 44	.232	.149	.112
45 – 49	.213	.146	.118
50 – 54	.190	.169	.162
55 – 59	.162	.186	.203
60 – 64	.128	.196	.243
65 – 69	.087	.211	.285
70 – 74	.042	.220	.323