# *Labor Income and Predictable Stock Returns*

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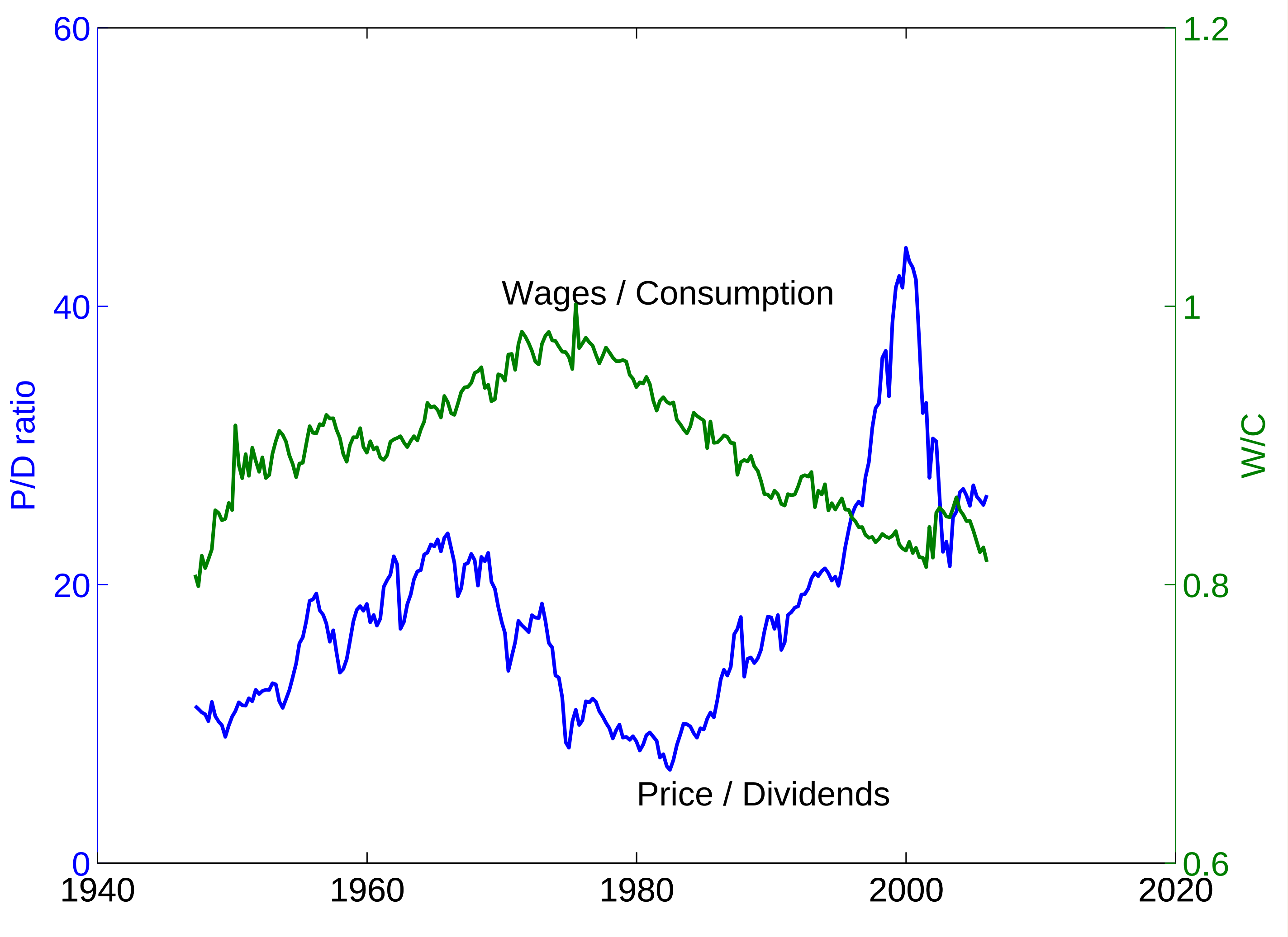
# Background

There appears to be a negative low frequency relation between:

- the share of consumption finance by labor income and
- the valuation of the stock market.



# Background (continued)

Sorting stocks into portfolios according to size and book-to-market, Fama & French (1993) find that

- stocks in small firms outperform those of large firms.
- stocks with low book-to-market (value stocks) outperform those with high book-to-market (growth stocks).

Santos and Veronesi's proposes a general equilibrium model which can account for both

- The negative relation between $W/C$ and expected excess returns, and
- The relation found by Fama and French

# Key mechanism (for market portfolio)

- Consumption is financed by wage income and various dividends.
- The share from each of these income streams is mean reverting.
- When the share of consumption financed by dividends is low, the covariance of dividends with consumption is also low, so investors require less excess return to hold stock.

# Model

Representative agent has a standard CRRA utility function:

$$U(C_t, t) = \begin{cases} \frac{e^{-\phi t} C_t^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ e^{-\phi t} \log(C_t) & \text{if } \gamma = 1 \end{cases} \tag{1}$$

Consumption financed by $n$ sources. Where labor income is the first source. The share of each source is given by:

$$s_t = \begin{pmatrix} s_t^1 \\ s_t^2 \\ \vdots \\ s_t^n \end{pmatrix} = \begin{pmatrix} W_t/C_t \\ D_t^2/C_t \\ \vdots \\ D_t^n/C_t \end{pmatrix} \tag{2}$$

To simplify notation, I'll use $W_t$ and $D_t^1$ interchangably in what follows.

## Dividend process

Let $D_t = (W_t^1, D_t^2, \cdots, D_t^n)$ be the vector of dividends. We assume that individual dividends follows

$$\frac{dD_t^i}{D_t^i} = \mu_D^i(D_t)dt + v_i' dB_t. \tag{3}$$

where $\mu_D^i(D_t)$ is some drift rate, $v_i$ is a vector of constants, and $dB_t$ be a $n \times 1$ vector of Brownian motions.

# Consumption process

It is assumed that the aggregate consumption process is given by

$$\frac{dC_t}{C_t} = \mu_c(s_t) + \sigma_c' dB_t. \tag{4}$$

where

$$\begin{aligned} \sigma_c &= (\sigma_{c,1}, 0, \cdots, 0)' \\ \mu_c(s_t) &= \bar{\mu}_c + s_t'\theta \\ \theta &= (\theta_1, \theta_2, \cdots, \theta_n)' \end{aligned}$$

$\theta_i$ is related to the instantaneous covariance between the growth rate of the share $i$ and consumption:

$$\mathrm{cov}_t\left(\frac{ds_t^i}{s_t^i}, \frac{dC_t}{C_t}\right) = \theta_i - s_t'\theta$$

For simplicity, we will assume $\theta_i = \bar{\theta}$, for $i \geq 2$.

## Dividend share processes

The vector of dividend shares follows a continuous time, vector autoregressive process with

$$ds_t = \Lambda' s_t dt + I(s_t)\Sigma(s_t)dB_t. \tag{5}$$

where $\Lambda$ is an ($n \times n$) matrix with

$$\lambda_{ij} \quad \begin{cases} \geq 0 & \text{if } i \neq j \\ = -\sum_{j \neq i} \lambda_{ij} & \text{if } i = j \end{cases} \tag{6}$$

$I(s_t)$ is a matrix with the shares along it's main diagonal and zeros elsewhere and $\Sigma(s_t)$ is an ($n \times n$) matrix whose $i$th row is

$$\sigma^i(s_t) = v_i' - \sum_{j=1}^{n} s_t^j v_j' \tag{7}$$

# Dividend share processes (cont)

The processes assumptions above guarantee

- that the shares are always positive (i.e. no asset comes to fully fund consumption);
- that they sum up to 1; and
- that consumption always equals total income

## Prices

The price of asset $i$ equals the expected discounted value of all it's dividend payments

$$P_t^i = E_t \left[ \int_t^\infty \frac{U_c(C_\tau)}{U_c(C_t)} D_\tau^i d\tau \right]. \tag{8}$$

Under the process assumptions above the vector of prices $P_t = (P_t^1, \ldots, P_t^n)'$ is given by

$$P_t = b' D_t \tag{9}$$

where

$$\begin{aligned} b &= (I(\tilde{\phi}) - \Lambda')^{-1} \\ \tilde{\phi}^i &= \phi - (1-\gamma)\bar{\mu}_c + \frac{1}{2}\gamma(1-\gamma)\sigma_c'\sigma_c - (1-\gamma)\theta^i \end{aligned}$$

Hence current price of an asset depends on all dividends. Numerically, the $b_{ii}$ terms turn out to be to be an order of magnitude greater than the off diagonal elements of $b$.

# Excess returns

The expected excess return of asset $i$ is given by

$$E_t[dR_t^i] = \gamma \left( \sigma_c' \sigma_c + \frac{b_i' I(s_t)(\theta - s_t' \theta)}{b_i' s_t} \right) \tag{10}$$

# Dividend share processes (simplified)

We now assume all financial assets are “unconditionally identical” by assuming $\theta_i = \bar{\theta}$ for $i = 2, \ldots, n$ and that $\lambda_{ij} = a\bar{s}^j$ for $i \neq j$. This results in the simpler dividend share processes:

$$ds_t^i = a(\bar{s}_t^i - s_t^i) + s_t^i \sigma^i(s_t) dB_t. \tag{11}$$

where

$$\sigma^i(s_t) = v_i' - \sum_{j=1}^{n} s_t^j v_j' \tag{12}$$

Also, it follows for the financial assets that

$$\text{cov}_t\left(\frac{ds_t^i}{s_t^i}, \frac{dC_t}{C_t}\right) = -(\bar{\theta} - \theta^w) s_t^w$$

Santos & Veronesi find that this covariance is negative in the data.

# Prices-dividend ratios (log utility case)

If we also assume log utility ($\gamma = 1$), the price dividend ratio of asset $i$ is given by

$$\frac{P_t^i}{D_t^i} = \left(\frac{1}{\phi}\right)\left(\frac{1}{\phi + a}\right)\left[\phi + a\left(\frac{\bar{s}^i}{s_t^i}\right)\right]. \tag{13}$$

We arrive at the P/D ratio for the aggregate stock market by summing over the assets 2 to $n$:

$$\begin{aligned} \frac{P_t^M}{D_t^M} &= \frac{\sum_{i=2}^n P_t^i}{\sum_{i=2}^n D_t^i} \\ &= \left(\frac{1}{\phi}\right)\left(\frac{1}{\phi + a}\right)\left[\phi + a\left(\frac{1 - \bar{s}^w}{1 - s_t^w}\right)\right] \\ &= \left(\frac{1}{\phi}\right)\left(\frac{1}{\phi + a}\right)\psi(s_t^w). \end{aligned} \tag{14}$$

The relative price of the total wealth portfolio, $P_t^{TW}/C_t$ is given by

$$\frac{P_t^{TW}}{C_t} = \frac{1}{\phi} \tag{15}$$

# Excess returns (log utility case)

The expected excess return of asset $i$ is given by

$$E_t[dR_t^i] = \sigma_c'\sigma_c + \left[\frac{\bar{\theta} - \theta_w}{1 + \frac{a}{\phi}\frac{\bar{s}_i}{s_i}}\right] s_t^w; \tag{16}$$

that of the market portfolio by

$$E_t[dR_t^i] = \sigma_c'\sigma_c + (\bar{\theta} - \theta_w)\left(\frac{s_t^w(1 - s_t^w)}{\phi(1 - s_t^w) + a(1 - \bar{s}^w)}\right). \tag{17}$$

## A conditional CAPM representation (simplified case)

The expected excess return of asset $i$ is given by

$$E_t[dR_t^i] = \beta^{w,i}(s_t)E_t[dR_t^w] + \beta^{M,i}(s_t)E_t[dR_t^M] \quad (18)$$

where $\beta^{w,i}(s_t)$ and $\beta^{M,i}(s_t)$ are the multiple regression coefficients,

$$\begin{bmatrix} \beta^{w,i}(s_t) \\ \beta^{M,i}(s_t) \end{bmatrix} = (\Sigma^{wM})^{-1} \begin{bmatrix} \mathrm{cov}_t(dR_t^i, dR_t^w) \\ \mathrm{cov}_t(dR_t^i, dR_t^M) \end{bmatrix}$$

and where that of the market portfolio by

$$E_t[dR_t^i] = \sigma_c'\sigma_c + (\bar{\theta} - \theta_w)\left(\frac{s_t^w(1 - s_t^w)}{\phi(1 - s_t^w) + a(1 - \bar{s}^w)}\right). \quad (19)$$

## Simulation exercise

Run the model for 10,000 periods with 200 shares and the following parameters:

| $\gamma$ | $\phi$ | $\mu_c$ | $\sigma_c$ | $\bar{s}^w$ | $\bar{s}^i$ | $a$ |
|---|---|---|---|---|---|---|
| 60 | 0.0984 | 0.0223 | 0.028 | 0.8637 | 0.0007 | 0.0892 |
| Covariance matrix $\Sigma$ | | | | | | |
| $\nu^{w,1}$ | $\nu^{w,2}$ | $\nu^{w,i\geq 3}$ | $\nu^{i,1}$ | $\nu^{i,i}$ | $\nu^{i,j}1_{i\neq j,1}$ | |
| -0.0089 | -0.0826 | 0 | 0.0567 | 0.6934 | 0 | |

Unconditional moments from simulation

| $E[s_t^w]$ | $\text{Std}(s_t^w)$ | $\text{Corr}(ds_t^w, dc_t)$ | $E[R_t^M]$ | $\text{Std}(R_t^M)$ | $E[r_t^f]$ | $\text{Std}(r_t^f)$ |
|---|---|---|---|---|---|---|
| 0.8637 | 0.0556 | -0.2412 | 0.0753 | 0.00621 | 0.03 | 0.0007 |

# FAMA French regressions

| | Cons | $R^M$ | $s^w R^M$ | $cay R^M$ | $\ln(D/R) R^M$ | SMB | HML | $\hat{R}^2$ |
|---|---|---|---|---|---|---|---|---|
| LL (01) | $V$ | $V$ | | $V^\star$ | | | | 31 % |
| | $V^\star$ | $V$ | $V^\star$ | $V$ | | | | 60 % |
| | $V$ | $V$ | | | $V^\star$ | | | 53 % |
| | $V$ | $V$ | $V$ | | $V^\star$ | | | 55 % |
| FF (93) | $V^\star$ | $V$ | | | | $V$ | $V^\star$ | 68 % |
| | $V^\star$ | $V$ | $V$ | | | $V$ | $V^\star$ | 67 % |