Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints

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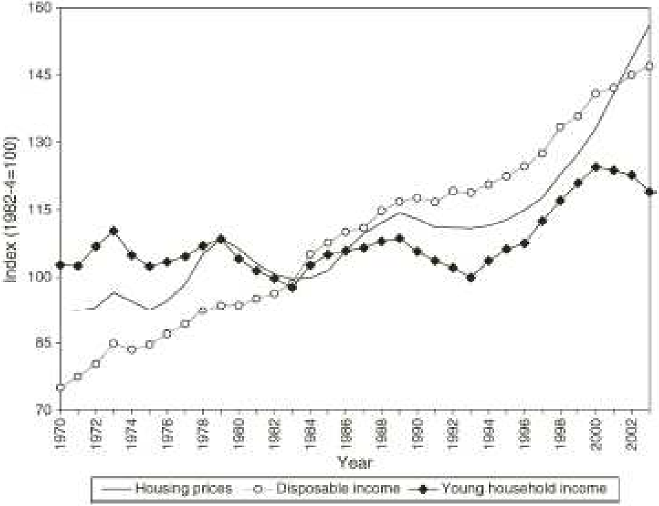
Objective

- Investigate the role of down-payment constraints in housing market fluctuations

Modeling strategy

- Life-cycle model with two types of homes ("starter" homes and "trade-up" homes) and down-payment constraint on borrowing

Housing prices and the income of young households



Model

- **Population.** Measure one of agents born every period. Each agent lives for 4 periods. Agents are indexed by the endowment stream i and their preferences m for houses relative to flats at age 4.
- **Commodities.** Numeraire consumption good and 2 types of dwellings: starter homes ("flats") and trade-up homes ("houses").
- **Endowments.** At age $j = 1, ..., 4$, agent (i, m) receives an endowment of $e_j(i)$ units of the numeraire good.
- **Technology.** Flats and houses are in fixed supply at measures S_F and S_H. Storage technology for the numeraire good allows agents to save at interest rate r.

Model

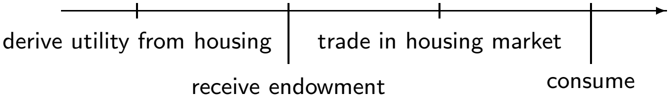
- **Preferences.** $\sum_{j=1}^{4} c_j + U(h_2, \frac{1}{2}) + U(h_3, \frac{1}{2}) + U(h_4, m)$

$$U(h,m) = \begin{cases} -\Delta & \text{if } h = \emptyset, \\ 0 & \text{if } h = F, \\ u(m) & \text{if } h = H. \end{cases}$$

- **Budget constraint.** $c_j + \frac{w_{j+1}}{1+r} + p_{h_{j+1}} = e_j + w_j + p_{h_j}$
- **Credit constraint.**

$$\frac{w_{j+1}}{1+r} \geq (\gamma - 1) p_{h_{j+1}} \;\Rightarrow\; e_j + p_{h_j} + w_j \geq \gamma p_{h_{j+1}}$$

- **Timing for period j.**



Parameter assumptions

- The supplies of flats and houses satisfy $\frac{5}{2} < S_F + S_H < 3$ and $\frac{1}{2} < S_H < 1$.
- $\gamma \geq \frac{r}{1+r}$
- Assumptions on endowment profiles:

$$e_1(0) = 0, \tag{1}$$
$$e_2(0) > e_1(3 - S_F - S_H), \tag{2}$$
$$e_2(i) > e_1(i) \text{ for all } i \in [0, 1], \tag{3}$$
$$e_3(0) > e(1), \tag{4}$$
$$e(1) > \max\{e_1(1), e(3 - S_F - S_H)\} + r\gamma^{-1} e_1(3 - S_F - S_H) \tag{5}$$

where $e(i) = (1 + r)e_1(i) + e_2(i)$

- Assumptions on the parameters of the agents' utility functions:

$$0 > u(1 - S_H), \tag{6}$$
$$u(1/2) > (1 + r^2)r\gamma^{-1}[e(1) - e_1(3 - S_F - S_H)], \tag{7}$$
$$\Delta > u(1/2) + (1 + r)^2 r\gamma^{-1} e_1(3 - S_F - S_H). \tag{8}$$

Recursive competitive equilibrium

The state of the economy at the beginning of a period is given by a collection $x = (x_2^w, x_3^w, x_4^w, x_2^h, x_3^h, x_4^h)$ of measurable functions.

A recursive competitive equilibrium consists of a **set** X **of spaces** x, **decision rules** $h_1(i,x), h_2(i,w,h,x)$, and $h_3(i,m,w,h,x)$ for housing purchases, **value functions** $v_1(i,x), v_2(i,w,h,x), v_3(i,m,w,h,x)$, and $v_4(i,m,w,h,x)$, **property price functions** $p_F(x)$ and $p_H(x)$, and a **law of motion** $\phi : X \to X$ for the state of the economy, such that:

- Given the law of motion for the state of the economy and the price functions, the decision rules of the agents solve their maximization problems and generate the respective value functions;
- Housing markets clear;
- The law of motion of the state of the economy is generated by agents' decision rules.

Steady-state equilibrium

There is a unique steady-state equilibrium.

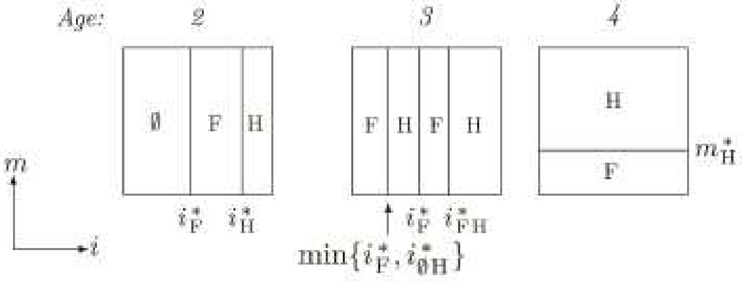
- The price of flats in this equilibrium is $p_F^* = \gamma^{-1} e_1(3 - S_F - S_H)$
- The price of houses, p_H^*, solves:

$$3 - S_H = e_1^{-1}(\gamma p_H^*) + \min\{e_1^{-1}(\gamma p_F^*), e^{-1}(\gamma p_H^*)\} + e^{-1}(r p_F^* + \gamma p_H^*) - e_1^{-1}(\gamma p_F^*) + u^{-1}(r[p_H^* - p_F^*])$$

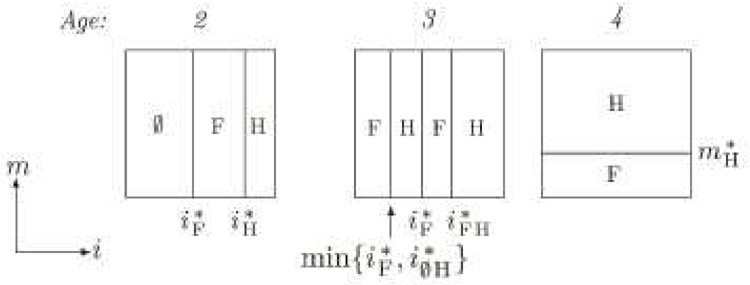
- The critical allocation of properties at the beginning of each period is determined by the critical endowment indices i_F^*, i_H^*, $i_{\emptyset H}^*$, i_{FH}^* and the critical preference index m_H^*.
- The steady-state measures of flats and houses bought and sold each period are

$$n_F^* = i_H^* - i_F^* + \min\{i_F^*, i_{\emptyset H}^*\} + (i_F^* - \min\{i_F^*, i_{\emptyset H}^*\} + 1 - i_{FH}^*) m_H^*$$
$$n_H^* = 1 - i_F^* - \min\{i_F^*, i_{\emptyset H}^*\} - i_{FH}^* + (\min\{i_F^*, i_{\emptyset H}^*\} + i_{FH}^* - i_F^*)(1 - m_H^*)$$

Steady-state equilibrium allocation of properties



Steady-state equilibrium allocation of properties



$$n^*_F = i^*_H - i^*_F + \min\{i^*_F, i^*_{\emptyset H}\} + (i^*_F - \min\{i^*_F, i^*_{\emptyset H}\} + 1 - i^*_{FH})m^*_H$$
$$n^*_H = 1 - i^*_F - \min\{i^*_F, i^*_{\emptyset H}\} - i^*_{FH} + (\min\{i^*_F, i^*_{\emptyset H}\} + i^*_{FH} - i^*_F)(1 - m^*_H)$$

Permanent change in endowments

- **Proposition 2.** The steady-state price of flats is proportional to endowments:

  $$p_F^{**} = z p_F^{*}.$$

  The steady-state price of houses changes less than proportionally with endowments, but more, in absolute terms, than the prices of flats:

  $$p_H^* + (z-1)p_F^* < p_H^{**} < z p_H^* \text{ if } z > 1$$
  $$z p_H^* < p_H^{**} < p_H^* + (z-1)p_F^* \text{ if } z < 1$$

- **Proposition 3.** For z sufficiently close to 1, an economy with endowment profiles $\{z e_j\}_{j=1}^4$ and initial conditions x^* admits a recursive competitive equilibrium.

Price and volume dynamics

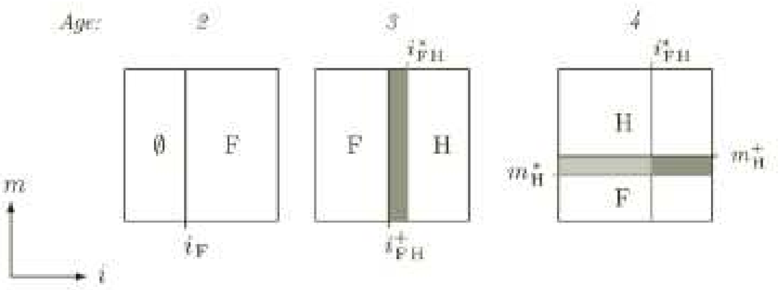
- **Proposition 4.** Suppose that all houses purchases are repeat purchases. Then, the price of houses overshoots its new steady-state level (i.e. $|p_H^+ - p_H^*| > |p_H^{**} - p_H^*|$) if and only if

$$ze_1(i_{FH}^{**}) < p_F^{**},$$

that is, in the new steady state the price of flats exceeds the age 1 wealth of those households that are the marginal house buyers at age 2.

If the price of houses overshoots, transaction volumes move with prices.

Allocations of properties



Price and volume dynamics

- **Proposition 5.** The price of houses overreacts to the change in endowments (i.e. $|p_H^+ - p_H^*| > |z-1|p_H^*$) if
  (i) $e_1(i_{FH}^*) < p_F^*$ that is, in the initial steady state the price of flats exceeds the age 1 wealth of those households that are the marginal house buyers at age 2;
  (ii) the measure of age 2 first-time buyers of houses in the initial steady state, $i_F - \min\{i_F, i_{\emptyset H}^*\}$, is sufficiently small;
  (iii) the distribution function of age 4 utility premiums for houses relative to flats, u^{-1}, increases at a sufficiently low rate around the user cost differential between houses and flats in the initial steady state, $r[p_H^* - p_F^*]$.

  If the house price overreacts, transaction volumes move with prices.