Equilibrium Unemployment

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- Flows into and out of unemployment.

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- Such a model would be an ideal laboratory to examine such questions as the impact of unemployment insurance and the cost of business cycles fluctuations.

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- exogenous borrowing constraint

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- If he rejects it he gets UI benefits μ and his next ε' is drawn from $H(\varepsilon')$

The Problem of a Worker

- A worker first chooses k and l as follows

$$Y(\varepsilon, \lambda; Z) = \max_{k,l} \left[O\left(k, l; \varepsilon, \lambda\right) - \left(R(\lambda; Z) + \delta\right)k - d(l) \right]$$

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The Choice problem of a worker is

$$\begin{aligned}
W\left(a, \varepsilon, \lambda; Z\right) &= \max_{c,a'} \{ U\left(c\right) + \beta \int \max\left[W\left(a', \varepsilon', \lambda'; Z'\right), S\left(a', \lambda'; Z'\right)\right] \\
&\quad \times dG\left(\varepsilon' | \varepsilon\right) dF\left(\lambda' | \lambda\right) d\varepsilon' d\lambda' \} \\
s.t \quad c + a' &= Y(\varepsilon, \lambda; Z) + \left[1 + R(\lambda; Z)\right] a - T\left(\lambda; Z\right), \\
a' &\geq \bar{a} \\
Z' &= \mathbf{T} Z
\end{aligned}$$

The Problem of a Searcher

The Choice problem of a searcher is :

$$\begin{aligned} S(a,\lambda;Z) &= \max_{c,a'}\{U(c)+\beta\int \max\left[W\left(a',\varepsilon',\lambda';Z'\right),S\left(a',\varepsilon';Z'\right)\right] \\ &\quad \times dH(\varepsilon)\,dF\left(\lambda'|\lambda\right)d\varepsilon' d\lambda'\} \\ s.t \quad c+a' &= [1+R(\lambda:Z)]\,a+\mu, \\ a' &\geq \bar{a} \\ Z' &= \mathbf{T}Z \end{aligned}$$

The decision rule governing whether someone works or not is:

$$\Omega\left(a,\varepsilon,\lambda;Z\right)=\left\{\begin{array}{c}1\ \mathit{if}\ \ W\left(a,\varepsilon,\lambda;Z\right)\geq S\left(a,\varepsilon;Z\right)\\ 0\ \mathit{otherwise}.\end{array}\right\}$$

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The government maintains a balanced budget each period:

$$\mu \int [1 - \Omega(a, \varepsilon, \lambda; Z)] \, dZ(a, \varepsilon) \, da d\varepsilon = \tau \int \Omega(a, \varepsilon, \lambda; Z) \, dZ(a, \varepsilon) \, da d\varepsilon$$

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The capital market clears

$$\int K(\varepsilon, \lambda; Z) \, \Omega(a, \varepsilon, \lambda; Z) \, dZ(a, \varepsilon) \, dad\varepsilon = \int adZ(a, \varepsilon) \, dad\varepsilon$$

The law of motion for the economy-wide distribution of wealth, or Z'=**T**Z, is described by:

$$\begin{aligned} Z'\left(a',\varepsilon'\right) &= \int\{I\left(A\left(a,\varepsilon,\lambda;Z\right)-a'\right)\left[\Omega\left(a,\varepsilon,\lambda;Z\right)G\left(\varepsilon'|\varepsilon\right)\right. \\ &\left.+\left(1-\Omega\left(a,\varepsilon,\lambda;Z\right)\right)dH\left(\varepsilon'\right)\right]dZ\left(a,\varepsilon\right)dad\varepsilon\}, \end{aligned}$$

where I(x)=1 if $x \leq 0$ and I(x)=0 otherwise.

Calibration

The instantaneous utility function is:

$$U\left(\tilde{c} - D\left(l\right)\right) = \frac{\left(\tilde{c} - l^{1+\theta}/\left(1+\theta\right)\right)^{1-\sigma} - 1}{1-\sigma}, \ \theta > 0, \ \sigma > 0$$

The production function is:

$$O\left(k, l; \varepsilon, \lambda\right) = \exp\left(\lambda + \varepsilon\right) k^{\alpha} l^{1-\alpha}$$

Income Process Calibration

Aggregate shocks.

$$\lambda' = \rho_\lambda \lambda + \xi, \quad \xi \sim N\left(0, \sigma_\lambda^2\right)$$

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Finally unemployment compensation is set to $\mu = \eta y*$

Calibration parameters

Table 1
Baseline parameterization

Parameter	Benchmark value	Description
Preferences		
β	$1/1.06^{1/8}$	Time preference
σ	2	Risk aversion
θ	10	Inverse of labor supply elasticity
Technology		
α	0.36	Share of capital in production
δ	0.006	Depreciation rate
Policy		
η	0.5	Replacement ratio (UI)
Shocks		
Aggregate		
ρ_λ	0.98	Persistence, Aggregate shock
σ_λ	0.009	Std. dev. innovation, Agg. shock
Workers		
ρ_ε	0.9	Persistence, Worker's shock
σ_ε	0.052	Std. dev. innovation, Worker's shock
Searchers		
σ_ν	0.085	Std. dev. innovation, Searcher's shock

Income dynamics

Heaton and Lucas(1996)

$$\ln\left(y_{it}/y_{it-1}\right) = \upsilon_0 + \upsilon_1 \ln\left(y_{it-1}/y_{it-2}\right) + \upsilon_2 \ln\left(\mathbf{y}_t/\mathbf{y}_{t-1}\right) + \mu_{it}$$

Table 3
Income distribution

Parameter	Data[a]	Model
υ_0	−3.564	−3.459
υ_1	0.527	0.501
υ_2	0.081	0.067
Std. dev. (u_{it})	0.241	0.186

[a] Heaton and Lucas (1996, Table A2).

Hubbard et al's (1995)

$$\ln\left(y_{it}\right) = \upsilon_1 \ln\left(y_{it-1}\right) + \mu_{it}$$

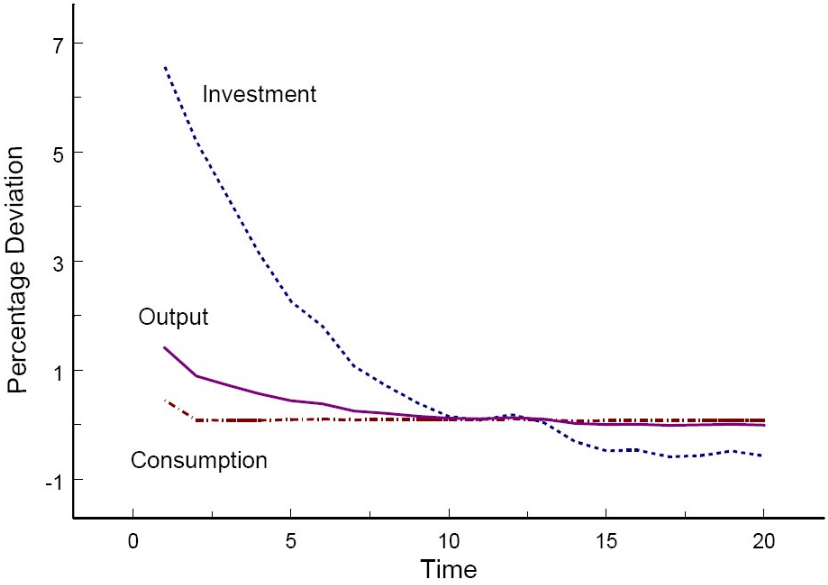
Model $\upsilon_1 = 0.5$ and σ_{μ_i}=0.19 vs Data $\upsilon_1 = 0.95$ and σ_{μ_i}=0.14

Comparative Statics

Table 4
Comparative statics exercises

Benchmark value	New value	Unemployment rate
Worker		
$\sigma_\varepsilon = 0.025$	$\sigma_\varepsilon = 0.052 \times 0.80 = 0.042$	4.9
	$\sigma_\varepsilon = 0.052 \times 1.2 = 0.062$	7.2
$\rho_\varepsilon = 0.9$	$\rho_\varepsilon = 0.9 \times 0.95 = 0.855$	5.5
	$\rho_\varepsilon = 0.9 \times 1.05 = 0.945$	6.3
Searcher		
$\sigma_\nu = 0.085$	$\sigma_\nu = 0.085 \times 0.80 = 0.68$	5.7
	$\sigma_\nu = 0.085 \times 1.2 = 0.102$	6.8
Government		
$\eta = 0.5$	$\eta = 0.7$	13.9

Impulse response, positive shock



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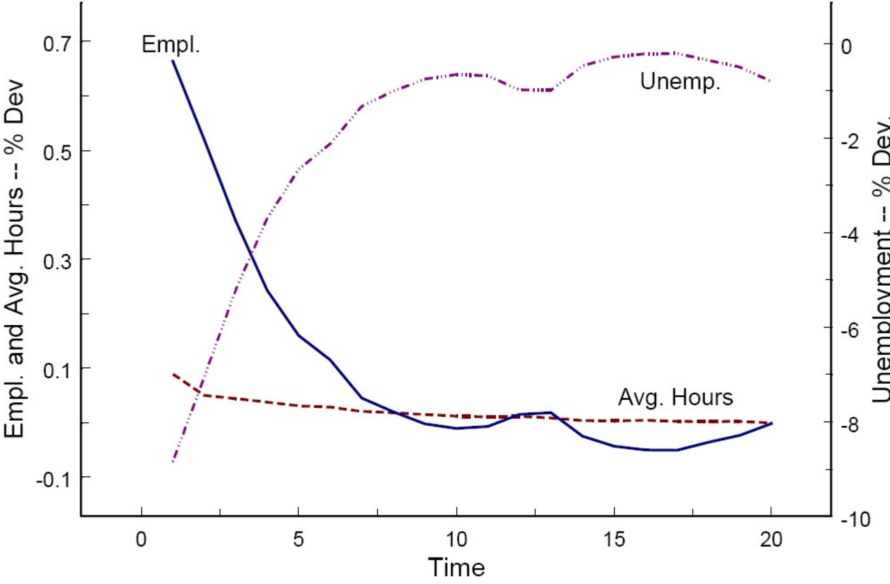

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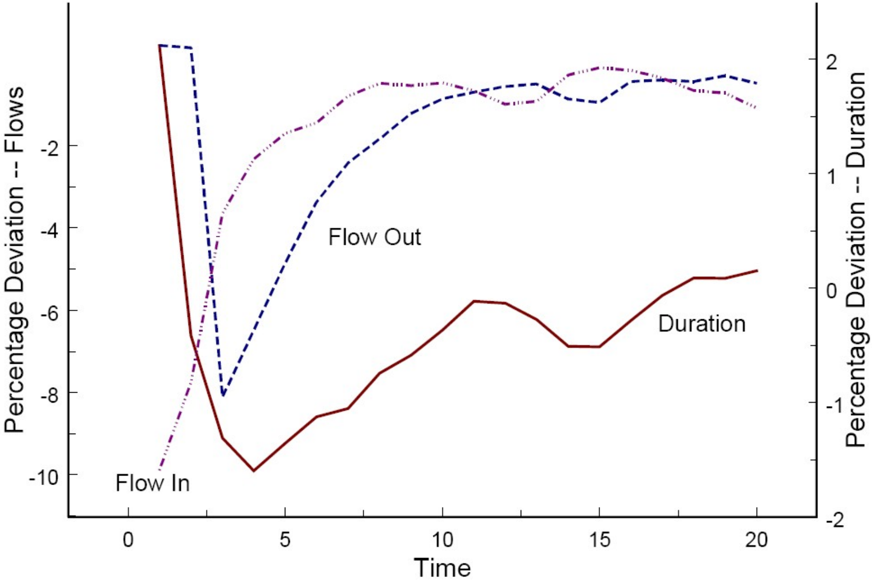

Welfare Cost of business cycle fluctuations

$$E_b\left[\sum_{t=0}^{\infty}\beta^t U\left(c_t\right)\right] = E_n\left[\sum_{t=0}^{\infty}\beta^t U\left(\varpi c_t\right)\right]$$

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$$\varpi = \left\{\frac{E_b\left[\sum_{t=0}^{\infty}\beta^t c_t^{1-\sigma}\right]}{E_n\left[\sum_{t=0}^{\infty}\beta^t c_t^{1-\sigma}\right]}\right\}^{1/(1-\sigma)}$$

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For the model economy $\varpi - 1 = 0.0056$.
This implies that the agent prefers to live in an economy with aggregate shocks.

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- Flows between employment and nonparticipation are as large as flows between employment and unemployment.
- What about the long term unemployed?
- The model abstracts from vacancies, so that the number of new job openings always equals the number of unemployed workers