The Interaction between Time-Nonseparable Preferences and Time Aggregation

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Econometrica(1993)

Introduction

- In a continuous-time environment the assumption that preferences are time separable is far from appealing. An individual's preferences over consumption at one instant are assumed to be unaffected by consumption the instant before.
- To identify time-nonseparabilities of preferences from discrete time data it is important to take into account time aggregation effects.
- Heaton writes a continuous-time, linear-quadratic representative consumer model with time-nonseparable preferences of several forms and finds that the data is not consistent with a model of time-separable preferences.

Time-Separable preferences and Time Aggregation

- Representative consumer with time separable preferences over consumption facing a constant interest rate equal to its pure rate of time preference.
- Hall(1978) showed that then the marginal utility of consumption follows a martingale. Furthermore, if preferences are quadratic with a constant bliss point consumption is a martingale.
- Flavin(1981) and Hayashi(1982) tested whether $E\{c(t+1) - c(t) \mid \mathcal{F}(t)\} = 0$ and rejected that hypothesis.
- Observed consumption expenditures over a period are $\overline{c} = \int_{t-1}^{t} c(\tau) d\tau$. Working(1960) showed that $\{\overline{c}(t) - \overline{c}(t-1) : t = 0, 1, 2, ...\}$ follows an MA(1) with first order autocorrelation of 0.25.
- Christiano, Eichenbaum, and Marshall(1991) showed that quarterly SA consumption data are remarkably consistent with these implications. There is little evidence against the martingale hypothesis once time averaging of the data is taken into account.

Test of the model using SA consumption data (1)

TABLE I

ESTIMATED AUTOCORRELATION FUNCTION FOR FIRST-DIFFERENCES OF PER CAPITA, SEASONALLY-ADJUSTED QUARTERLY CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES EXPONENTIAL TREND REMOVED

Order of Autocorrelation	Estimated Autocorrelation[a]	
	52, 1 to 86, 4	59, 1 to 86, 4
1	0.276 (0.068)	0.260 (0.081)
2	0.108 (0.063)	0.148 (0.067)
3	0.201 (0.080)	0.259 (0.084)
4	0.057 (0.087)	0.118 (0.095)
5	−0.202 (0.108)	−0.178 (0.130)

[a] Standard errors are in parentheses. Standard errors were calculated using one year of lags in the Newey-West (1987) procedure.

Test of the model using SA consumption data (2)

TABLE II

ESTIMATES OF MA(1) MODEL FOR FIRST-DIFFERENCES OF PER CAPITA, SEASONALLY-ADJUSTED QUARTERLY CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES FIRST-ORDER AUTOCORRELATION RESTRICTED TO 0.25

Parameter	Parameter Estimates[a]	
	52,1 to 86,4	59,1 to 86,4
μ ($\times 10^2$)	0.475 (0.052)	0.485 (0.060)
θ_0	0.224 (0.016)	0.253 (0.019)
Log-Likelihood	−36.16	−35.87

[a] Standard errors are in parentheses.

TABLE III

ESTIMATES OF MA(1) MODEL FOR FIRST-DIFFERENCES OF PER CAPITA, SEASONALLY-ADJUSTED QUARTERLY CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES UNRESTRICTED ESTIMATION

Parameter	Parameter Estimates[a]	
	52,1 to 86,4	59,1 to 86,4
μ ($\times 10^2$)	0.474 (0.053)	0.487 (0.059)
θ_0	0.224 (0.016)	0.252 (0.019)
θ_1	0.062 (0.019)	0.060 (0.023)
Log-Likelihood	−36.15	−35.83

Monthly data is not consistent with time separable preferences

ESTIMATES OF MA(1) MODEL FOR FIRST-DIFFERENCES OF PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES, 59, 1 TO 86, 12

Parameter	Estimation with 0.25 Restriction on $R(1)$[a]	Unrestricted[a]
μ ($\times 10^2$)	0.156 (0.027)	0.164 (0.017)
θ_0	0.248 (0.015)	0.216 (0.010)
θ_1	—	−0.047 (0.011)
Log-Likelihood	−97.43	−54.22

[a] Standard Errors are in parentheses.

TABLE V

ESTIMATED AUTOCORRELATION FUNCTION FOR FIRST-DIFFERENCES OF PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES, 59, 1 TO 86, 12 EXPONENTIAL TREND REMOVED

Order of Autocorrelation	Estimated Autocorrelation[a]
1	−0.211 (0.078)
2	0.052 (0.073)
3	0.104 (0.067)
4	−0.003 (0.058)
5	0.000 (0.037)
6	0.086 (0.042)
7	0.115 (0.067)
8	0.005 (0.048)
9	0.010 (0.040)
10	0.093 (0.081)
11	0.081 (0.068)
12	0.017 (0.057)

A model with time-nonseparable preferences(1)

Preferences of the consumer are assumed to be time separable over a stochastic process, $s \equiv \{s(t) : 0 \leq t < \infty\}$ called services.

$$U(s) \equiv -\frac{1}{2}E\left\{\int_0^\infty exp(-\rho t)(s(t) - b(t))^2 dt\right\}, \rho > 0$$

where $\{b(t) : 0 \leq t < \infty\}$ is a deterministic process describing the bliss point movement.

A model with time-nonseparable preferences(2)

Time non separability of preferences is introduced by making $s(t)$ a linear function of current and past consumption.

$$s(t) = g * c(t)$$

For many examples this is given by the integral.

$$s(t) = \int_0^t g(\tau)c(t-\tau)d\tau$$

A model with time-nonseparable preferences(3)

- There is also a capital accumulation technology that allows the transfer of resources over time at the constant instantaneous rate ρ:

$$Dk(t) = \rho k(t) + e(t) - c(t)$$

  where $k(0)$ is given.
- Heaton assumes that the rate of return on capital is equal to the pure rate of time preference.

A model with time-nonseparable preferences(4)

The Lagrangian for this problem is then:

$$\begin{aligned}\mathcal{L} &= -E\Big\{ \int_0^\infty exp(-\rho t)\{\frac{1}{2}[g * c(t) - b(t)]^2 \\ &\quad -\lambda(t)[k(t) - k(0) - \rho\int_0^t k(\tau)d\tau - \int_0^t e(\tau)d\tau + \int_0^t c(\tau)d\tau]\}dt\Big\}\end{aligned}$$

First order conditions for the choice of $c(t)$ and $k(t)$ are then given by:

$$E\left\{-g^f * [g * c(t) - b(t)] \mid \mathcal{F}(t)\right\} + E\left\{\int_0^\infty exp(-\rho t)\lambda(t+\tau)d\tau \mid \mathcal{F}(t)\right\} = 0$$

and

$$\lambda(t) - \rho E\left\{\int_0^\infty exp(-\rho t)\lambda(t+\tau)d\tau \mid \mathcal{F}(t)\right\} = 0$$

The marginal utility of consumption is a martingale

$$g^f * [s(t+\tau) - b(t+\tau)] = g^f * [s(t) - b(t)] + u(t+\tau)$$

and applying the one-sided forward looking inverse of g^f we get that

$$s(t+\tau) - b(t+\tau) = s(t) - b(t) + (g^f)^{-1} u(t+\tau)$$

Let's invert the convolution using the fact that $(g)^{-1} * s = c$

$$c(t) - c(t-1) = (g)^{-1} * \int_{t-1}^{t} D\xi(\tau) + (g)^{-1} * [b(t) - b(t-1)]$$

Implications for Time-Averaged Consumption

But since consumption consists of averages of consumption expenditures over a unit time, we obtain the following general representation for observed consumption:

$$\bar{c}(t) - \bar{c}(t-1) = (g)^{-1} * w(t) + (g)^{-1} * \left\{ \int_{t-1}^{t} [b(\tau) - b(\tau - 1)] d\tau \right\}$$

where

$$\begin{aligned} w(t) &= \int_{t-1}^{t} \int_{\tau-1}^{\tau} D\xi(r) d\tau \\ &= \int_{t-1}^{t} (t-\tau) D\xi(\tau) + \int_{t-2}^{t-1} (\tau - t + 2) D\xi(\tau) \end{aligned}$$

Identification of g with Sampled Data

- One needs to assume that the bliss point moves in a deterministic fashion. Otherwise the dynamics of consumption could be completely explained by movements in the bliss point.
- Then we have a continuous time MA representation for changes in consumption of the form

$$\overline{c}(t) - \overline{c}(t-1) = (g)^c * D\xi(t)$$

- But we only have discrete-time data and therefore the best we can do is to identify the discrete-time MA representation

$$\overline{c}(t) - \overline{c}(t-1) = \sum_{j=0}^{\infty} (G)^c(j)\varepsilon(t-j)$$

Exponential Depreciation

The first time-nonseparability considered assumes that consumption can be easily substituted over short periods of time.
To do this let,

$$g(t) = \begin{cases} 0 & if\ t < 0 \\ exp(\delta t) & if\ t \geq 0 \end{cases}, \quad \delta < 0$$

The exponential depreciation model implies that

$$\bar{c}(t) - \bar{c}(t-1) = \int_0^1 (1 - \delta\tau) D\xi(t-\tau) - \int_0^1 [1 + \delta(1-\tau)] D\xi(t-1-\tau)$$

As in the time-separable case, first differences in time-average consumption follow an MA(1) process. But now the first-order autocorrelation value need not be 0.25 nor even positive and is given by:

$$R(1) \equiv \frac{\delta^2/6 - 1}{2 + (2/3)\delta^2}$$

First autocorrelation implied by exponential depreciation model

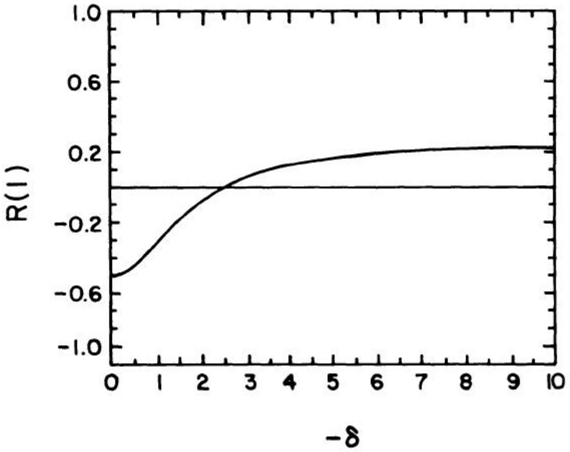

FIGURE 1.—$R(1)$ implied by exponential depreciation model.

Estimating the exponential depreciation model monthly SA data

ESTIMATES OF EXPONENTIAL DEPRECIATION MODEL USING PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES, 59, 1 TO 86, 12

Parameter	Estimated Value[a]
μ $(\times 10^2)$	0.164 (0.017)
θ_0	0.215 (0.010)
δ	−1.358 (0.161)
Log-Likelihood	−54.22

Estimates of the restricted model with quarterly data

- Consumption goods have a half life of 0.51 months.
- This implies a corresponding estimate of δ for quarterly data of $3 \times -1.358 = -4.07$
- A likelihood ratio test cannot reject this restriction at the ten per cent significance level, which implies that quarterly data are consistent with the model.

Habit Persistence (1)

In this case:

$$s(t) = c(t) - \alpha(-\gamma)\int_0^{\infty} exp(\gamma\tau)c(t-\tau)d\tau,\ \gamma < 0,\ 0 < \alpha < 1$$

In this example, g is given by

$$g = \Delta - \alpha(-\gamma)\eta$$

where Δ is the Dirac delta function and η is given by

$$\eta(t) = \begin{cases} 0 & if\ t \leq 0 \\ exp(\gamma t) & if\ t > 0 \end{cases}$$

Habit Persistence (2)

The habit-persistence model implies that:

$$D[\bar{c}(t) - \bar{c}(t-1)] = (1-\alpha)\gamma[\bar{c}(t) - \bar{c}(t-1)]$$
$$+ \int_0^1 (1-\gamma\tau) D\xi(t-\tau) - \int_0^1 [1+\gamma(1-\tau)] D\xi(t-1-\tau)$$

- Now the discrete-time observations satisfy an ARMA(1,2) model where the autoregressive parameter is given by $exp[(1-\alpha)\gamma]$
- Independent of the effects of time-averaging the data, habit persistence induces smoothness into consumption in the sense that the first difference of consumption is positively autocorrelated. The fact that consumption is time-averaged reinforces this effect.
- This model is clearly rejected by the monthly data.

A model with habits and durability

A potentially better way to specify the habit persistence model is to first define an intermediate service process. This will capture the fact that consumption can be substituted over short periods of time.

The intermediate service process is generated according to

$$\widetilde{s}(t) = \int_0^\infty exp(\delta\tau)c(t-\tau)d\tau, \ \delta < 0$$

But now the habit develops over the intermediate service instead of consumption as

$$s(t) = \widetilde{s}(t) - \alpha(-\gamma)\int_0^\infty exp(\gamma\tau)\widetilde{s}(t-\tau)d\tau, \ \gamma < 0, \ 0 < \alpha < 1$$

Habit persistence with exponential depreciation model

MAXIMUM LIKELIHOOD ESTIMATES OF PARAMETERS OF HABIT PERSISTENCE WITH EXPONENTIAL DEPRECIATION MODEL, USING PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION EXPENDITURES ON NONDURABLES AND SERVICES, 59, 1 TO 86, 12

Parameter	Parameter Estimates[a]
$\mu\ (\times 10^2)$	0.156 (0.024)
σ	0.132 (0.010)
δ	−1.558 (0.743)
γ	−0.211 (0.414)
α	0.438 (0.308)
Log-Likelihood	−52.54

[a] Standard errors are in parentheses.

TABLE IX

ESTIMATES OF HALF-LIVES IN MONTHS FOR DURABILITY AND HABIT PERSISTENCE EFFECTS WITH MONTHLY DATA

Depreciation Parameter	Estimates[a]
δ	0.656 (0.104)
γ	3.293 (6.473)

Seasonally Adjusted Expenditure on Durables

ESTIMATES OF EXPONENTIAL DEPRECIATION MODEL USING PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION EXPENDITURES ON DURABLES, 59,1 TO 86,12

Parameter	Estimated Value[a]
μ $(\times 10^2)$	0.389 (0.036)
θ_0	0.137 (0.009)
δ	−1.647 (0.280)
Log-Likelihood	−30.10

- The half life of the durable good implied by this estimate of δ is 0.42 moths. Puzzle first noted by Mankiw(1982).

Habit persistence with exponential depreciation model

MAXIMUM LIKELIHOOD ESTIMATES OF PARAMETERS FOR HABIT PERSISTENCE WITH EXPONENTIAL DEPRECIATION MODEL, USING PER CAPITA, SEASONALLY-ADJUSTED MONTHLY CONSUMPTION EXPENDITURES ON DURABLES, 59,1 TO 86,12

Parameter	Parameter Estimates[a]
μ ($\times 10^2$)	0.380 (0.035)
σ	0.047 (0.005)
δ	−0.461 (0.137)
γ	−4.176 (0.059)
α	0.754 (0.069)
Log-Likelihood	−26.89

- Considerable better fit.
- Now the half-life for the durable good is 1.5 months.
- The model also fits quarterly expenditures on durables.
- However, higher order ARMA models provide significant evidence against this model which is an ARMA(1,2)

Two remarks

- Heaton(1995) extends some of the analysis of this paper to an endownment economy where asset returns are endogenous and the representative agent has CRRA preferences.
- Annual data?