Measuring the effects of monetary policy a Factor-augmented vector autoregressive (FAVAR) approach

Bernanke, Boivin and Eliasz

QJE(2005)

Overview

Structural vector autoregressions (SVAR) are widely used to trace out the effect of monetary policy innovations on the economy.

- Policy innovations are likely to be contaminated.
- The choice of a specific data series to represent a general concept such as "real activity" is arbitrary to some degree.
- Impulse response functions can be observed only for the included variables.

This paper:
Combines VAR analysis with recent developments in factor analysis for large data sets.

Factor-augmented vector autoregression FAVAR

Y_t is a $M \times 1$ vector of observable economic variables.
F_t is a $K \times 1$ vector of unobservable factors.

Their dynamics are given by:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \nu_t \qquad (1)$$

X_t is a $N \times 1$ vector of "informational" time series.

where $K + M \ll N$

They assume that:

$$X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t \qquad (2)$$

Motivating the FAVAR Structure: An Example

Consider the simple backward-looking model:

$$\pi_t = \delta \pi_{t-1} + \kappa (y_{t-1} - y^n_{t-1}) + s_t \tag{3}$$
$$y_t = \phi y_{t-1} - \psi (R_{t-1} - \pi_{t-1}) + d_t \tag{4}$$

$$y^n_t = \rho y^n_{t-1} + \eta_t \tag{5}$$
$$s_t = \alpha s_{t-1} + \upsilon_t \tag{6}$$

R_t is set by the central bank according to a simple Taylor Rule:

$$R_t = \beta \pi_t + \gamma (y_t - y^n_t) + \varepsilon_t \tag{7}$$

The macroeconomic indicators are assumed to be related in the following way:

$$X_t = \Lambda[y_t^n \ s_t \ \pi_t \ y_t \ R_t]' + e_t \tag{8}$$

The model can then be written in as a VAR(1) where,

$$\Phi = \begin{bmatrix} \rho & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 \\ -\kappa & \alpha & \delta & \kappa & 0 \\ 0 & 0 & \psi & \phi & -\psi \\ -\gamma\psi & \beta\alpha & (\beta\delta + \lambda\psi) & (\beta\kappa + \lambda\phi) & -(\beta\kappa + \lambda\rho) \end{bmatrix}$$

$$\upsilon_t = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ \gamma & 1 & -\gamma & \beta \end{pmatrix} \begin{pmatrix} dt \\ \varepsilon t \\ \eta_t \\ \upsilon_t \end{pmatrix}$$

What is F_t and Y_t in this model?

If all the variables are observed then

$$\begin{aligned} Y_t &= [y_t^n \; s_t \; \pi_t \; y_t \; R_t]' \\ F_t &= \emptyset \end{aligned}$$

If potential output and the cost-push shock are not observed then:

$$\begin{aligned} Y_t &= [\pi_t \; y_t \; R_t]' \\ F_t &= [y_t^n \; s_t]' \end{aligned}$$

The most realistic setting is one in which:

$$\begin{aligned} Y_t &= [R_t]' \\ F_t &= [y_t^n \; s_t \; \pi_t \; y_t]' \end{aligned}$$

Estimation I

A two step principle components approach:

- A nonparametric way of uncovering the common space spanned by the factors of X_t, denoted $C(Y_t, F_t)$.
- First step involves using the first $K + M$ principal components of X_t to first estimate the space spanned by the factors $\hat{C}(Y_t, F_t)$. Then they can obtain $\hat{F}_t$.
- Second step involves estimating the FAVAR equation with F_t replaced by $\hat{F}_t$.
- To account for the uncertainty of the "generated regressor" in the second step they implement a bootstrap procedure when computing impulse response functions.

Estimation II

Likelihood based inference using the Gibbs sampler.

- Can handle the difficult nature of the likelihood function which makes MLE infeasible for models like this.
- Need to assume errors are normally distributed.
- Idea is to characterize the joint posterior $P(\Theta, F^T | X^T, Y^T)$ by sampling from the conditional densities $P(F^T | \Theta, X^T, Y^T)$ and $P(\Theta | F^T X^T, Y^T)$.

The state space form of the model is:

$$\begin{bmatrix} F_t \\ Y_t \end{bmatrix} = \Phi(L) \begin{bmatrix} F_{t-1} \\ Y_{t-1} \end{bmatrix} + \nu_t$$

$$\begin{bmatrix} X_t \\ Y_t \end{bmatrix} = \begin{bmatrix} \Lambda^f & \Lambda^y \\ 0 & 0 \end{bmatrix} \begin{bmatrix} F_t \\ Y_t \end{bmatrix} \begin{bmatrix} e_t \\ 0 \end{bmatrix}$$

Identification

Two identification issues:

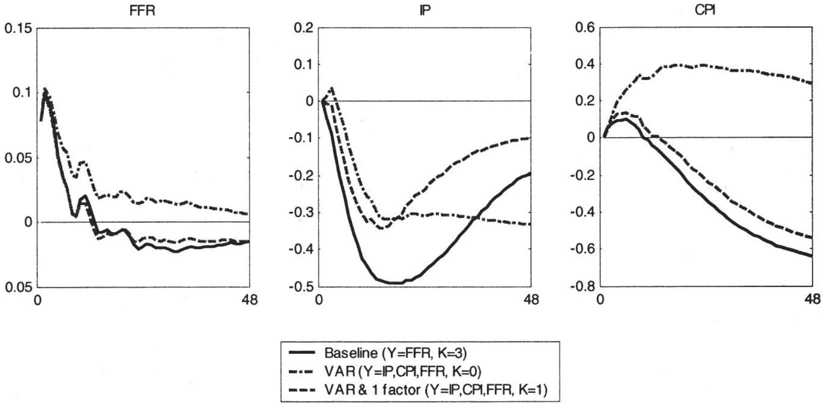
- **Normalization**
  - Recall that $X_t = \Lambda^f F_t + \Lambda^y Y_t + e_t$. So if $\hat{\Lambda^f}$ and $\hat{F}_t$ are a solution to the estimation problem then $\tilde{\Lambda^f} = \hat{\Lambda^f} H$ and $\tilde{F}_t = H^{-1} \hat{F}_t$ are also valid solutions.
- **Identification of the structural shocks**
  - Recursive structure in which all the factors respond with a lag to a change in the monetary policy instrument, which is ordered last in Y_t.
  - Notice that the FAVAR framework allows for other identification schemes. (e.g. long-run restrictions or arbitrary contemporaneous restrictions).

The Dynamic Effects of Monetary Policy

- In their application X_t consists of a balanced panel of 120 monthly macroeconomic time series.
- The recursive ordering imposes the identifying assumption that the unobserved factors do not respond to monetary policy innovations within the period.
- For the principle component estimation approach they need to remove the direct dependence of $\hat{C}(F_t, Y_t)$ on R_t. They estimate the coefficient through a multiple regression of the form

$$\hat{C}(F_t, Y_t) = b_{C^*} \hat{C}^*(F_t) + b_R R_t + e_t$$

where to obtain $\hat{C}^*(F_t)$ they extract principal components from a subset of slow-moving variables of X_t.


FFR
0.15
0.1
0.05
0
0.05
0
48
IP
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
48
CPI
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
48
Baseline (Y=FFR, K=3)
VAR (Y=IP,CPI,FFR, K=0)
VAR & 1 factor (Y=IP,CPI,FFR, K=1)

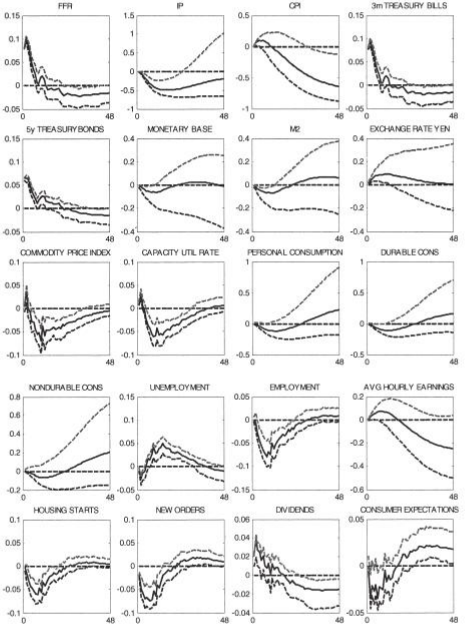


FIGURE II
Impulse Responses Generated from FAVAR with Three Factors and FFR
Estimated by Principal Components with Two-Step Bootstrap

CONTRIBUTION OF THE POLICY SHOCK TO VARIANCE OF THE COMMON COMPONENT

Variables	Variance decomposition	R^2
Federal funds rate	0.454	*1.000
Industrial production	0.054	0.707
Consumer price index	0.038	0.870
3-month treasury bill	0.433	0.975
5-year bond	0.403	0.925
Monetary base	0.005	0.104
M2	0.005	0.052
Exchange rate (Yen/$)	0.007	0.025
Commodity price index	0.049	0.652
Capacity utilization	0.100	0.753
Personal consumption	0.006	0.108
Durable consumption	0.005	0.062
Nondurable cons.	0.002	0.062
Unemployment	0.103	0.817
Employment	0.066	0.707
Aver. hourly earnings	0.007	0.072
Housing starts	0.032	0.387
New orders	0.081	0.624
S&P dividend yield	0.062	0.549
Consumer expectations	0.036	0.700

The column titled Variance decomposition reports the fraction of the variance of the forecast error, at the 60-month horizon, explained by the policy shock. R^2 refers to the fraction of the variance of the variable explained by the common factors, $(\hat{F}_t', Y_t')$. See text for details.

* This is by construction.