On the Welfare Costs of Consumption Uncertainty

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What's this paper about?

- Lucas (1987) argued that the welfare gain from eliminating fluctuations in aggregate consumption is trivial.
- Satisfactory calculations of the welfare cost of aggregate consumption uncertainty require a framework that replicates major features of asset prices and returns.
- Barro(2006) "Rare disasters and asset markets in the twentieth century".
- Welfare cost of consumption uncertainty imply that society would be willing to give up 20% of real GDP each year to eliminate all disaster risk.

The Model

- Lucas's (1978) representative-agent, fruit-tree economy with exogenous stochastic production.
- Output of fruit in period t equals real GDP, Y_t.
- Number of trees is fixed. There is neither investment nor depreciation.
- In equilibrium, $C_t = Y_t$.

Endowment Process

$$\log(Y_{t+1}) = \log(Y_t) + g - \frac{1}{2}\sigma^2 + u_{t+1} + v_{t+1}$$

- $g > 0$ is the growth rate of the economy
- $u_{t+1} \sim N(0, \sigma^2)$
- v_{t+1} picks up low-probability disasters, as in Rietz(1988) and Barro(2006).
  - probability $1 - p$ $\quad v_{t+1} = 0$
  - probability p $\quad v_{t+1} = \log(1 - b)$.

Calibration of p and b

- Uses time series on per capita GDP for 35 countries for the full 20th century.
- p is calibrated to match the empirical distribution of contractions of 15% or more of GDP over consecutive years.
  $p = 1.7\%$ per year.(60 events).
- The contraction proportion b for the observed 20th century disasters ranged from 15% to 64% with a mean of 29%.
  $E[b] = 0.29$.
- The expected growth rate of GDP and consumption is given by:
  $$g^* = g - pE[b]$$

- The representative consumer maximizes a time-additive utility function

$$U_t = E_t \sum_{i=0}^{\infty} \frac{1}{(1+\rho)^i} \frac{C_{t+i}^{1-\theta} - 1}{1-\theta}$$

where $\rho \geq 0$ is the rate of time preference, and θ is the coefficient of relative risk aversion and the reciprocal of the elasticity of intertemporal substituion for consumption.

- Let V be the price of a tree that initially produces one unit of fruit.

$$\frac{1}{V} = \rho + (\theta - 1) g^* - \frac{1}{2}\theta(\theta - 1)\sigma^2 - p\left(E\left[(1-b)^{1-\theta}\right] - 1 - (\theta - 1) Eb\right)$$

- Also,

$$U_t = \frac{1}{1-\theta} V Y_t^{1-\theta}$$

Marginal calculations of welfare effects

- The marginal effect on utility from a proportionate change in Y_t is given by

$$\frac{\partial U_t}{\partial Y_t} Y_t = Y_t^{1-\theta} V$$

- The marginal effect from a change in g^* is then

$$\frac{\partial U_t}{\partial g^*} = Y_t^{1-\theta} V^2$$

- Therefore, the utility rate of transformation between proportionate changes in Y_t and changes in g^* is:

$$-\frac{\frac{\partial U_t}{\partial g^*}}{\frac{\partial U_t}{\partial Y_t} Y_t} = -V$$

  This gives us the proportionate decrease in Y_t that compensates, at the margin, for an increase in g^*

- $\rho = 0.03$ $\theta = 4$ $g = 0.025$ $\sigma = 0.02$ $g^* = 0.02$
- This implies that $V = 19.7$.
- So a small rise in the expected growth rate g^* say of 0.1% per year has to be compensated by a fall in the initial level of Y_t of about 2%.
- Confirms the intuition that an economy should be willing to give up a lot in its initial level of GDP to obtain a small increase in its long-term growth rate.

- What about changes in σ?

$$-\frac{\frac{\partial U_t}{\partial \sigma}}{\frac{\partial U_t}{\partial Y_t} Y_t} = \theta \sigma V = 1.58$$

Increasing σ by 10% from 0.02 to 0.022 requires a rise in the initial level of GDP of about 0.32%

- Changes in the probability of disasters

$$-\frac{\frac{\partial U_t}{\partial p}}{\frac{\partial U_t}{\partial Y_t} Y_t} = \frac{V\left[E\left(1-b\right)^{1-\theta}-1\right]}{\left(\theta-1\right)} = 20$$

A rise in p by 10% matches up with a proportionate initial increase in GDP of 3.4%.

Previous formulas apply for small changes only

Recall that

$$U_t = \frac{1}{1-\theta} V Y_t^{1-\theta}$$

we can see that:

$$\frac{Y_t^*}{Y_t} = \left(\frac{V^*}{V}\right)^{\frac{1}{\theta-1}}$$

where Y^* and V^* are values that apply in any alternative situation that delivers the same expected utility U_t.

Lucas vs. Rare Disasters

- Lucas focused on the consequences of eliminating all consumption uncertainty associated with usual business fluctuations.
  So set $\sigma = 0$

  $$\frac{1}{V^*} = \frac{1}{V} + \frac{1}{2}\theta\left(\theta - 1\right)\sigma^2$$

  and therefore,

  $$1 - \frac{Y_t^*}{Y_t} = \left(1 + \frac{1}{2}\theta\left(\theta - 1\right)\sigma^2 V\right)^{\frac{1}{1-\theta}} = 1.5\%$$

- However, eliminating the possibility of disasters has much greater consequences for welfare.

  $$1 - \frac{Y_t^*}{Y_t} = \left(1 + pV\left[E\left(1 - b\right)^{1-\theta} - 1\right]\right)^{\frac{1}{1-\theta}} = 20.9\%$$

Sensitivity of welfare gains to changes in θ

θ	proportionate cut in initial GDP
4	20.9%
3	15.7%
2	14.8%
1	18.6%

A Model with Endogenous Savings

$$Y_t = AK_t$$

The capital stock (trees) evolves as follows:

$$K_{t+1} = K_t + I_t - \delta_{t+1} K_t$$

with stochastic depreciation rate

$$\delta_{t+1} = \delta + u_{t+1} + v_{t+1}$$

where similarly to before

- $u_{t+1} \sim N(0, \sigma^2)$
- v_{t+1} represents rare disasters.
    - probability $1 - p$ $\quad v_{t+1} = 0$
    - probability p $\quad v_{t+1} = -b$

The expected return on equity shares is given by

$$r^e = A - \delta - pEb$$

Given that the shocks are iid, the ratio of gross investment to the capital stock will be optimally chosen as a constant

$$v = \delta + \frac{1}{\theta}\left[A - \delta - \rho + \frac{1}{2}\theta\left(\theta - 1\right)\sigma^2 - p + pE\left[\left(1 - b\right)^{1-\theta}\right]\right]$$

The expected growth rate of the economy is

$$E_t\left(\frac{K_{t+1}}{K_t} - 1\right) = v - \delta - pEb$$

The model is calibrated so that the expression above is equal to $g^* = 0.020$. This pins down the parameter combination $A - \delta$ (0.076 per year).
Given these values we obtain that $v = 0.025 + \delta$

- The welfare compensations in the endogenous-saving model coincide with those for the endowment economy if the gross saving ratio, v, is constrained to remain fixed at its initial value.

- If the ratio of savings v, is free to adjust to changes in σ or p
  - setting $\sigma = 0$, saving ratio v falls from 0.075 to 0.0744, welfare effect $=$1.55%.
  - setting $p = 0$, saving ratio v falls to 0.0620, welfare effect $=$26.1%.

Epstein-Zin-Weil Preferences

$$U_t = \frac{\left[(1-\beta)\, C_t^{1-\theta} + \beta\left[(1-\beta)(1-\gamma)\, E_t U_{t+1}\right]^{\frac{1-\theta}{1-\gamma}}\right]^{\frac{1-\gamma}{1-\theta}}}{(1-\beta)(1-\gamma)}$$

Because the underlying shocks are i.i.d, attained utility, U_t is a simple function of contemporaneous consumption C_t

$$U_t = \Phi C_t^{1-\gamma}$$

where Φ is a constant that depends on the parameters of the model.

The first order condition for utility maximization is:

$$C_t^{-\gamma} = \frac{1}{1+\rho^*} E_t \left[R_t C_{t+1}^{-\gamma} \right]$$

where the effective discount factor is:

$$\rho^* = \rho - (\gamma - \theta) \left\{ g - \frac{1}{2}\gamma\sigma^2 - \frac{p}{\gamma - 1} \left[E (1-b)^{1-\gamma} - 1 \right] \right\}$$

Now the formula for the price-dividend ratio is:

$$\frac{1}{V} = \rho + (\theta - 1)\, g - \frac{1}{2}\gamma\,(\theta - 1)\,\sigma^2 - p\frac{\theta - 1}{\gamma - 1}\left[E\left[(1 - b)^{1-\gamma}\right] - 1\right]$$

And the value function U_t

$$U_t = \frac{\rho^{\frac{\theta-\gamma}{1-\theta}}}{1-\gamma} V^{\frac{1-\gamma}{1-\theta}} Y_t^{1-\gamma}$$

The result under standard preferences still applies here:

$$\frac{Y_t^*}{Y_t} = \left(\frac{V^*}{V}\right)^{\frac{1}{\theta-1}}$$

Effect of Preference Parameters on Welfare Costs

Table 1			
Effects of Preference Parameters on Welfare Costs			
		welfare effect from eliminating uncertainty $[1-(Y_t)^*/Y_t)]$	
γ	θ	p=0	σ=0
4	4	20.9%	1.5%
3	3	15.7%	1.1%
2	2	14.8%	0.9%
1	1	18.5%	0.7%
3	4	12.6%	0.9%
2	4	8.5%	0.5%
1	4	6.3%	0.2%
4	3	25.2%	1.8%
4	2	31.9%	2.1%
4	1	43.8%	2.6%
4	3*	22.9%	1.5%
4	2*	25.4%	1.6%
4	1*	28.9%	1.6%

Relationship to Robustness

Under Barro's framework:

$$g^* = g - pE[b]$$

An agent who is concern about model misspecification and whose approximating model for consumption growth is a random walk with drift g, values consumption streams following his worst case model:

$$\log(C_{t+1}) = \log(C_t) + g - \frac{1}{2}\sigma_\varepsilon^2 + \sigma_\varepsilon(w_{t+1} + \varepsilon_{t+1})$$

where w_{t+1} is the worst case distortion.
For an agent with log utility the distortion is a constant

$$w = -\frac{\sigma_\varepsilon}{(1-\beta)\theta}$$

Thus under his worst case model the expected growth rate of the economy is

$$g^* = g - \frac{\sigma_\varepsilon^2}{\theta(1-\beta)}$$