Model Specification and Risk Premia: Evidence from Futures Options

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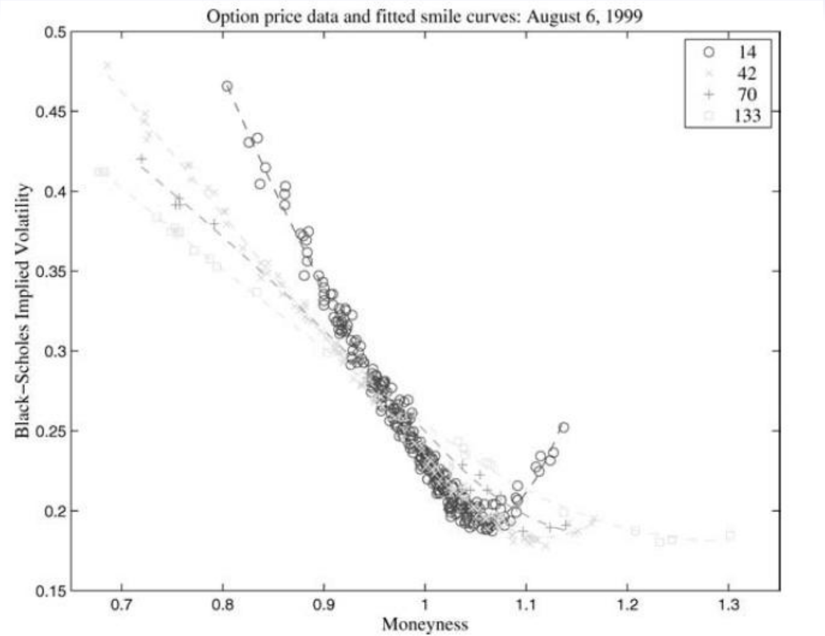
Black and Scholes

- Returns are log-normally distributed.
- The price of a European call option with exercise price K at time T on a stock currently trading at S is given in close form.

$$C = f(S, T, r, K; \sigma)$$

- This price is increasing in volatility (σ).
- The implied volatility is used as a quoting convention and is given by:

$$IV = f^{-1}(S, T, r, K)$$


Option price data and fitted smile curves: August 6, 1999
Black–Scholes Implied Volatility
Moneyness
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.7
0.8
0.9
1
1.1
1.2
1.3
14
42
70
133

Overview

Two central issues in empirical option pricing:

1. Model specification: identifying and modeling the factors that jointly determine returns and option prices.
2. Quantifying the risk premia associated with the factors.

The literature disagrees over the importance of jumps in prices or in volatility and on the magnitude and significance of volatility and jump risk premia.

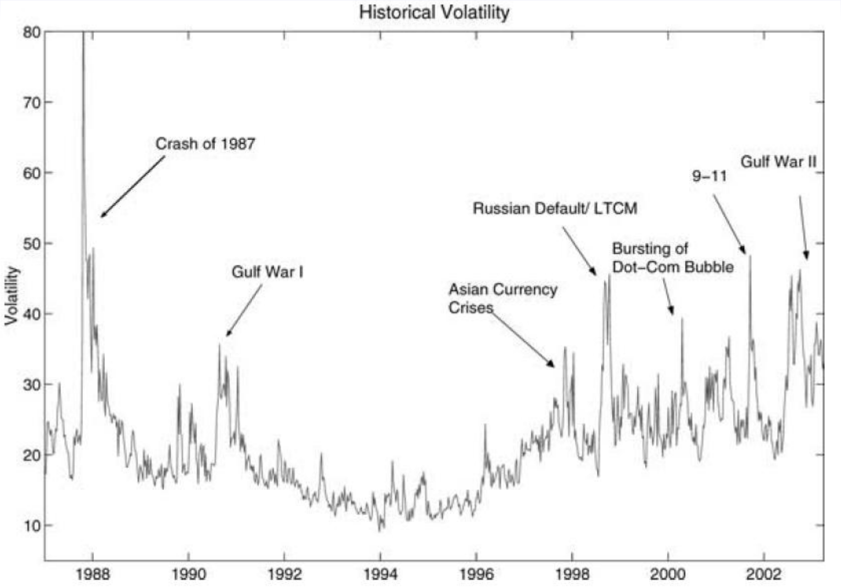


Figure 1. Time series of implied volatility. This figure displays the time series of implied volatility, as measured by the VIX index, from 1987 to March 2003.

This paper:

Use an extensive data set of S&P 500 futures options form January 1987 to March 2003 to shed light on the following issues:

1. Is there option-implied time-series evidence for jumps in volatility?
2. Are jumps in prices and volatility important factors in determining the cross section of option prices?
3. What is the nature of the factor risk premia embedded in the cross section of option prices?

Model under the $\mathbb{P}$ measure

The equity index price, S_t, and its spot variance, V_t, solve:

$$
\begin{aligned}
dS_t &= S_t(r_t - \delta_t + \gamma_t)dt + S_t\sqrt{V_t}dW_t^s + d\left(\sum_{n=1}^{N_t} S_{\tau_{n-}}[e^{Z_n^s - 1}]\right) - S_t\bar{\mu_s}\lambda dt \\
dV_t &= \kappa_v(\theta_v - V_t)dt + \sigma_v\sqrt{V_t}dW_t^v + d\left(\sum_{n=1}^{N_t} Z_n^v\right)
\end{aligned}
$$

where,

W_t^s and W_t^v are two correlated Brownian motions.
δ_t is the dividend yield.
γ_t is the equity premium.
N_t is a Poisson process with intensity λ
$Z_n^s | Z_n^v \sim N(\mu_s + \rho_s Z_n^v, \sigma_s^2)$ are the jumps in prices.
$Z_n^v \sim exp(\mu_v)$ are the jumps in volatility.

Parameterizing the change of measure

The price of an asset is given by:

$$P = E^{\mathbb{P}}\left[M_t X_t\right] = E^{\mathbb{Q}}\left[X_t\right]$$

The change of measure or SDF:

$$\frac{d\mathbb{Q}}{d\mathbb{P}} = M_t = M_t^D M_t^J$$

Model under the $\mathbb{Q}$ measure

$$
\begin{aligned}
dS_t &= S_t(r_t - \delta_t)dt + S_t\sqrt{V_t}dW_t^s(\mathbb{Q}) + d\left(\sum_{n=1}^{N_t(\mathbb{Q})} S_{\tau_{n-}}[e^{Z_n^s(\mathbb{Q})-1}]\right) - S_t\bar{\mu}_s^{\mathbb{Q}}\lambda^{\mathbb{Q}}dt \\
dV_t &= [\kappa_v^{\mathbb{Q}}(\theta_v - V_t) - \eta_v V_t]dt + \sigma_v\sqrt{V_t}dW_t^v(\mathbb{Q}) + d\left(\sum_{n=1}^{N_t(\mathbb{Q})} Z_n^v(\mathbb{Q})\right)
\end{aligned}
$$

Risk premia:

- $\mu_s - \mu_s^{\mathbb{Q}}$ is the mean price jump risk premium.
- $\sigma_s^{\mathbb{Q}} - \sigma_s$ is the volatility of price jumps risk premium.
- $\eta_v = \kappa_v^{\mathbb{Q}} - \kappa_v$ is the diffusive volatility risk premium

The price of a European call option on the futures is:

$$C(F_t, V_t, \Theta, t, T, K, r) = e^{-r(T-t)}E_t^{\mathbb{Q}}[(F_T - K)^+]$$

Empirical strategy

- The model places joint restrictions on the return and volatility dynamics under $\mathbb{P}$ and $\mathbb{Q}$.
- Fix some parameters that they take from previous studies ($\mathbb{P}$) and choose the rest to match the panel of implied volatility curves.
- They use the following criterion function.

$$(\hat{\Theta^{\mathbb{Q}}}, \hat{V_t}) = argmin \sum_{t=1}^{T} \sum_{n=1}^{O_t} [IV_t(K_n, \tau_n, S_t, r) - IV_t(V_t, \Theta^{\mathbb{Q}} | \Theta^{\mathbb{P}}, K_n, \tau_n, S_t, r)]^2$$

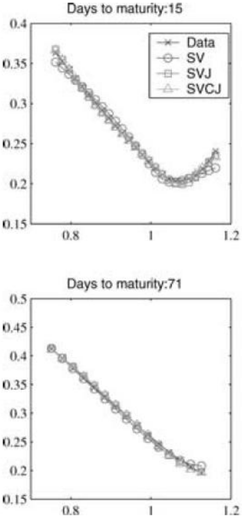


Figure 2. Calibrated implied volatility curves, August 5, 1999. Parameter estimates are obtained using all four curves for each of the models, with no restrictions on the parameters. The units on the X-axis are in terms of the options' moneyness, *K/F*, and the units on the Y-axis are the annualized Black–Scholes implied volatility.

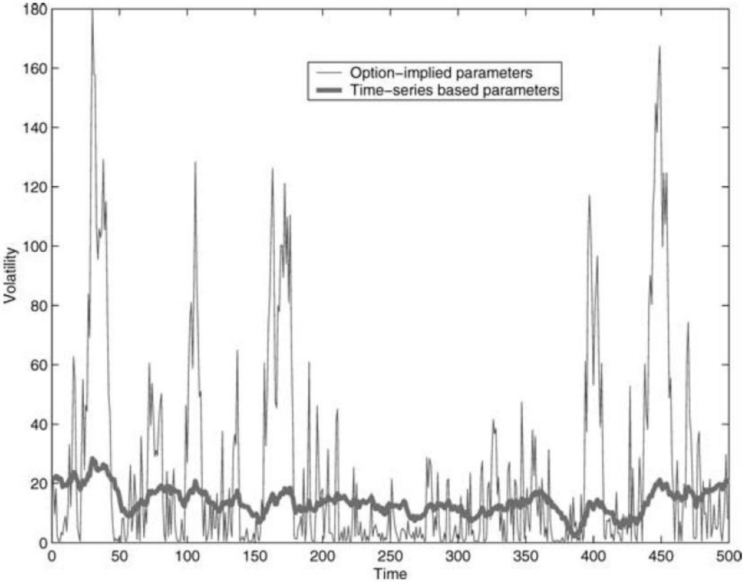


Figure 3. Simulated volatility paths. This graph provides volatility paths simulated based on options ($\theta_v = 3.63$, $\kappa_v = 0.06$, $\sigma_v = 2.8$, $\rho = -0.66$), and index returns ($\theta_v = 0.90$, $\kappa_v = 0.025$, $\sigma_v = 0.15$, $\rho = -0.40$). The time corresponds to 2 years (500 trading days) and the same Brownian increments are used for both paths to allow for a direct comparison.

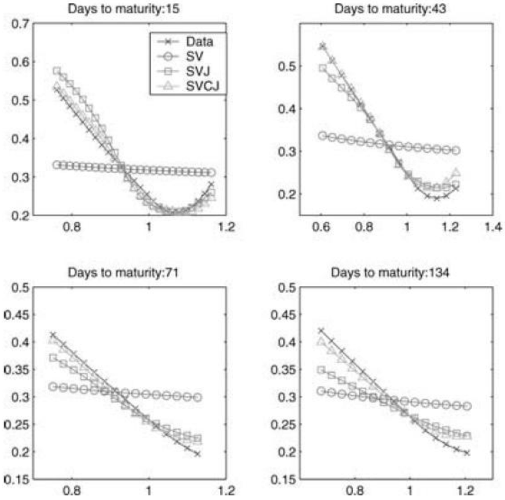


Figure 4. Calibrated Black–Scholes implied volatility curves, August 5, 1999. Parameter estimates are obtained using all four curves for each of the models, constraining the $\mathbb{P}$-measure parameters to be equal to their time-series counterparts.

Table II
Volatility and Return Summary Statistics

The first three rows provide summary statistics for variance increments and standardized returns using the VIX index, a time series of call option implied volatility (see Appendix B), and the ATM interpolated implied volatility (see Appendix B). In these three cases, the variance used is that from the Black–Scholes model. The second, third, and fourth panels contain model implied variances for the SV, SVJ, and SVCJ models assuming options are priced based on the objective measure. We also include risk premiums (RP) and document the effect of increasing σ_v in the SV model.

	Period	V_{kurt}	V_{skew}	R_{kurt}	R_{skew}
VIX (BSIV)	1987 to 2003	2996.58	50.41	13.72	−1.02
	1988 to 2003	20.93	1.74	5.69	−0.43
Calls (BSIV)	1987 to 2003	1677.16	32.78	22.99	−1.46
	1988 to 2003	15.17	1.25	5.64	−0.40
Interpolated (BSIV)	1987 to 2003	2076.58	38.21	21.04	−1.38
	1988 to 2003	25.17	1.79	5.82	−0.43
SV Model	1987 to 2003	1035.71	23.85	17.39	−1.18
	1988 to 2003	14.33	1.29	6.04	−0.41
SV Model (RP)	1987 to 2003	907.57	21.85	17.87	−1.20
	1988 to 2003	13.44	1.25	5.74	−0.40
SV Model ($\sigma_v = 0.2$)	1987 to 2003	1039.98	23.91	17.97	−1.22
	1988 to 2003	14.41	1.29	6.10	−0.43
SVJ Model	1987 to 2003	850.41	21.16	17.75	−1.20
	1988 to 2003	15.66	1.37	6.10	−0.42
SVJ Model (RP)	1987 to 2003	1048.51	24.21	15.91	−1.06
	1988 to 2003	16.01	1.40	7.09	−0.42
SVCJ Model	1987 to 2003	1015.13	23.62	16.62	−1.12
	1988 to 2003	15.16	1.34	6.31	−0.40
SVCJ Model (RP)	1987 to 2003	546.52	16.08	15.96	−1.02
	1988 to 2003	13.44	1.38	6.77	−0.35

Table III

Statistics' Finite Sample Distribution

For each model and set of parameters, we report the appropriate quantiles from the statistics' finite sample distribution. The base parameters are taken from Eraker, Johannes, and Polson (2003) as reported in Table I.

	Quantile	V_{kurt}	V_{skew}	R_{kurt}	R_{skew}
SV model	0.50	3.27	0.34	3.02	−0.05
	0.95	3.51	0.41	3.14	−0.10
	0.99	3.67	0.43	3.19	−0.12
SV model	0.50	3.55	0.48	3.05	−0.06
$\sigma_v = 0.2$	0.95	3.96	0.55	3.16	−0.12
	0.99	4.26	0.60	3.23	−0.14
	0.50	3.05	0.15	22.05	−1.48
SVJ model	0.95	3.23	0.22	106.05	−5.07
	0.99	3.34	0.26	226.77	−8.66
	0.01	261.02	9.94	7.73	−0.63
	0.05	320.24	13.18	10.72	−0.92
SVCJ model	0.50	615.40	21.04	24.77	−1.91
	0.95	1649.03	34.87	78.90	−4.15
	0.99	2500.51	43.73	175.67	−6.66
	0.01	3.28	0.20	3.02	0.06
SVCJ model	0.05	13.21	1.01	3.21	−0.02
$\mu_v = 0.85$	0.50	217.70	8.98	7.13	−0.46
$\lambda = 0.0026$	0.95	1150.76	27.53	37.92	−2.20
	0.99	2012.62	39.34	94.16	−4.11

Table IV
Risk-Neutral Parameter Estimates

For each parameter and model, the table gives the point estimate, computed as the average parameter value across 50 bootstrapped samples, and the bootstrapped standard error. For the SVJ and SVCJ models, an entry of σ_s in the $\sigma_s^{\mathbb{Q}}$ column indicates that we impose the constraint $\sigma_s = \sigma_s^{\mathbb{Q}}$.

	η_v	$\mu_s^{\mathbb{Q}}$ (%)	$\sigma_s^{\mathbb{Q}}$ (%)	$\mu_v^{\mathbb{Q}}$	RMSE (%)
SV	0.005 (0.07)	—	—	—	7.18
SVJ	0.010 (0.03)	−9.97 (0.51)	σ_s	—	4.08
SVJ	0.006 (0.02)	−4.91 (0.36)	9.94 (0.41)	—	3.48
SVJ	0	−9.69 (0.58)	σ_s	—	4.09
SVJ	0	−4.82 (0.33)	9.81 (0.58)	—	3.50
SVCJ	0.030 (0.21)	−6.58 (0.53)	σ_s	10.81 (0.45)	3.36
SVCJ	0.031 (0.18)	−5.39 (0.40)	5.78 (0.70)	8.78 (0.42)	3.31
SVCJ	0	−7.25 (0.50)	σ_s	5.29 (0.18)	3.58
SVCJ	0	−5.01 (0.38)	7.51 (0.83)	3.71 (0.22)	3.39

Jump risk premia is important

- Jump risk premia is important and accounts for one third of the equity premium.
- Where does this process come from?