Equilibrium Yield Curves

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Yields are then defined as:

$$y_{n,t} = -\frac{1}{n}\ln P_{n,t}$$

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- Recursive Utility (Kreps-Porteus-Epstein-Zin-Weil)
  - Disentangle risk aversion from the intertemporal elasticity of substitution.
  - Agents are not indifferent to the temporal distribution of risk
- Assume IES=1 and homoskedastic lognormal shocks.

The Pricing Kernel

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-1} \left(\frac{V_t}{E_t \left(V_{t+1}^{1-\gamma} \right)^{\frac{1}{1-\gamma}}} \right)^{1-\gamma}$$

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Taking logs, and assuming normal and homoskedastic shocks

$$m_{t+1} = constant - \Delta c_{t+1} - (\gamma - 1) \sum_{i=0}^{T-t-1} \alpha_{t+1,i} \left(E_{t+1} - E_t \right) \Delta c_{t+1+i}$$

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What is a bad state here?

- low realized consumption growth.
- bad news about future consumption growth.

The Nominal Pricing Kernel

$$m_{t+1}^{\$} = m_{t+1} - \pi_{t+1}$$

Unconditional mean of the one-period real rate

$$\mu^{(1)} = -\ln\beta + \mu_c - \frac{1}{2} var_t\left(\Delta c_{t+1}\right) \underbrace{- (\gamma - 1)\, cov_t\left(\Delta c_{t+1}, \sum_{i=0}^{T-t-1} \alpha_{t+1,i}\left(E_{t+1} - E_t\right)\Delta c_{t+1+i}\right)}_{New\ \ term\ \ with\ \ recursive\ \ utility}$$

Unconditional mean of the one-period nominal rate

$$\mu^{(1)\$} = \underbrace{\mu^{(1)} + \mu_\pi - \frac{1}{2} var_t\left(\pi_{t+1}\right)}_{Fisher\ \ Equation}$$

$$\underbrace{-cov_t\left(\pi_{t+1}, \Delta c_{t+1}\right)}_{Traditional\ \ Inflation\ \ risk\ \ premium}$$

$$\underbrace{-\left(\gamma - 1\right) cov_t\left(\pi_{t+1}, \sum_{i=0}^{T-t-1} \alpha_{t+1,i}\left(E_{t+1} - E_t\right)\Delta c_{t+1+i}\right)}_{Additional\ \ premium\ \ because\ \ Inflation\ \ is\ \ bad\ \ news}$$

What about yield spreads?

$$\textit{average real yield spread} \approx cov_t\left(m_{t+1}, E_{t+1}\left(\sum_{i=1}^{n-1}\Delta c_{t+1+i}\right)\right)$$

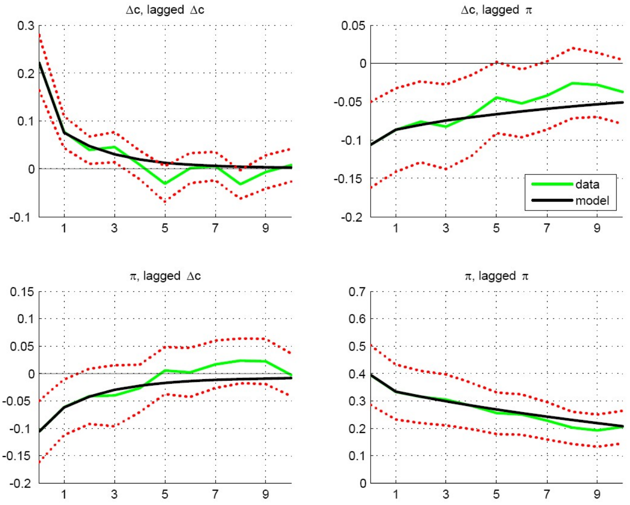
$$\textit{average nom. yield spread} \approx cov_t\left(m_{t+1}-\pi_{t+1}, E_{t+1}\left(\sum_{i=1}^{n-1}\Delta c_{t+1+i}+\pi_{t+1+i}\right)\right)$$

Where do investors beliefs come from?

The vector of consumption growth and inflation $z_{t+1} = (\Delta c_{t+1}, \pi_{t+1})^T$ has the state-space representation:

$$
\begin{aligned}
z_{t+1} &= \mu_z + x_t + e_{t+1} \\
x_{t+1} &= \phi_x x_t + \phi_x K e_{t+1}
\end{aligned}
$$

Rational Expectations Benchmark



Rational Expectations Benchmark

Values for preference parameters β and γ to match average nominal yields.

No discounting $\beta = 1.005$ and high risk aversion $\gamma = 69$

Average Nominal Yield Curve

	1 quarter	1 year	2 year	3 year	4 year	5 year
Data	5.15	5.56	5.76	5.93	6.06	6.14
SE	(0.43)	(0.43)	(0.43)	(0.42)	(0.41)	(0.41)
Benchmark Model	5.15	5.33	5.56	5.78	5.97	6.14
Expected (Log) Utility	4.92	4.92	4.91	4.90	4.89	4.88
SE Spreads	5-year minus 1 quarter yield (0.13)			5-year minus 2-year yield (0.07)		

Average Real Yield Curve

	1 quarter	1 year	2 year	3 year	4 year	5 year
Benchmark Model	0.84	0.64	0.49	0.38	0.30	0.23
Expected (Log) Utility	1.22	1.21	1.21	1.21	1.21	1.21

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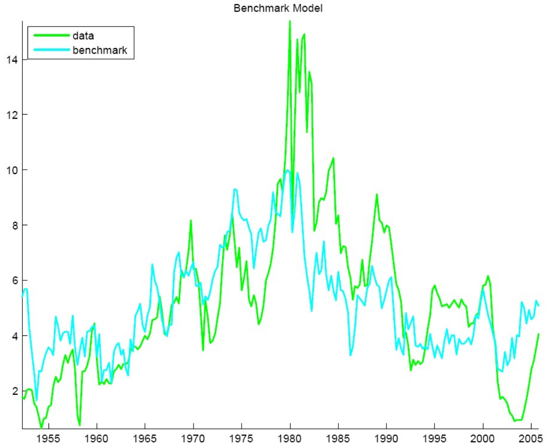

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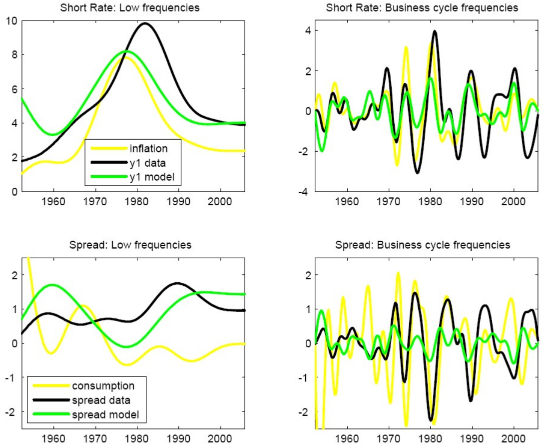

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- Parameter Uncertainty about the mean.

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- What about the equity premium implied by their model.