Rational Pessimism, Rational Exuberance, and Asset Pricing Models

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Overview

This paper ESTIMATES and examines the empirical plausibility of two of the leading rational expectations asset pricing models.

- Long Run Risk model of Bansal and Yaron (2004) (LRR).
- Habit formation model of Campbell and Cochrane (1999) (HAB).

Both models are found to fit the data equally well at conventional significance levels.

Indirect Inference. Tony Smith(1993)

MODEL

- For a given parameterization simulate data from the model.
- Construct series for observables.

AUXILIARY MODEL

- Estimate it using real data and using simulated data

The goal is to minimize the distance between moments of the auxiliary model with real and simulated data.

LRR dynamics I

The log endownment c_t is generated as:

$$c_t = c_{t-1} + \mu_c + x_{t-1} + \epsilon_{ct}$$

Log consumption and log dividends are cointegrated

$$d_t - c_t = \mu_{dc} + s_t$$

where s_t is an I(0) process.

They also introduce a stochastic volatility factor v_t.

The four variables are collected into the vector:

$$q_t = \begin{pmatrix} \Delta c_t \\ x_t \\ s_t \\ v_t \end{pmatrix}$$

LRR dynamics II

$$q_t = a + Aq_{t-1} + \exp\left(\Lambda_t\right)\Psi z_t$$

$$a = \begin{pmatrix} \mu_c \\ 0 \\ -\mu_{dc} \\ \mu_\sigma \end{pmatrix}, \quad A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \rho_x & 0 & 0 \\ 0 & \lambda_{sx} & \rho_s & 0 \\ 0 & 0 & 0 & \rho_v \end{bmatrix}$$

with volatility structure

$$\Lambda = \begin{bmatrix} b_{cc}v_t & 0 & 0 & 0 \\ 0 & b_{xx}v_t & 0 & 0 \\ 0 & 0 & b_{ss}v_t & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Psi = \begin{bmatrix} \Psi_{cc} & \Psi_{cx} & \Psi_{sx} & 0 \\ 0 & \Psi_{xx} & 0 & 0 \\ 0 & 0 & \Psi_{ss} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

LRR dynamics III

Minimal state vector necessary to simulate the model is

$$u_t = \begin{pmatrix} x_t \\ s_t \\ v_t \end{pmatrix}$$

Asset Pricing LRR I

Epstein-Zin-Weil Utility Function

$$U_t = \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}$$

where γ is the coefficient of risk aversion and ψ is the elasticity of intertemporal substitution.
and

$$\theta = \frac{1-\gamma}{1-1/\psi}$$

Asset Pricing LRR II

Let P_{ct} denote the price of an asset that pays the consumption endownment and let

$$v_{ct} = \frac{P_{ct}}{C_t}$$

Then, the price dividend ratio, v_{ct} is the solution to the nonlinear expectational equation.

$$v_{ct} = E_t \left\{ \delta^{\theta} \exp\left[-\left(\theta/\psi\right) \Delta c_{t+1} + \left(\theta - 1\right) r_{c,t+1} \right] \left(1 + v_{c,t+1}\right) \exp\left(\Delta c_{t+1}\right) \right\}$$

where,

$$r_{c,t+1} = \log\left[\frac{1 + v_{c,t+1}}{v_{ct}} \exp\left(\Delta c_{t+1}\right) \right]$$

and the one-period marginal rate of substitutions is

$$M_{t,t+1} = \delta^{\theta} \exp\left[-\left(\theta/\psi\right) \Delta c_{t+1} + \left(\theta - 1\right) r_{c,t+1} \right]$$

Asset Pricing LRR III

The price dividend ratio $v_{dt} = P_{dt}/D_t$ is the solution to

$$v_{dt} = E_t \left\{ \delta^\theta \exp\left[-\left(\theta/\psi\right) \Delta c_{t+1} + \left(\theta - 1\right) r_{c,t+1} \right] \left(1 + v_{d,t+1}\right) \exp\left(\Delta d_{t+1}\right.\right.$$

and the risk free rate is given by

$$e^{-r_{ft}} = E_t \left\{ \delta^\theta \exp\left[-\left(\theta/\psi\right) \Delta c_{t+1} + \left(\theta - 1\right) r_{c,t+1} \right] \right\}$$

These are obtained by evaluating the pricing functions

$$\begin{aligned} v_{ct} &= v_c\left(u_t\right) \\ v_{dt} &= v_d\left(u_t\right) \\ r_{ft} &= r_f\left(u_t\right) \end{aligned}$$

Habit Persistence(HAB) I

The agent is assumed to maximize the utility function

$$E\sum_{i=0}^{\infty}\delta^i\frac{\left(C_{t+i}-X_{t+i}\right)^{1-\gamma}-1}{1-\gamma}$$

where X_t denotes the habit stock.
Let the surplus ratio be

$$H_t=\frac{C_t-X_t}{C_t}$$

and assume that

$$h_t=\log\left(H_t\right)$$

evolves as

$$h_{t+1}=\left(1-\rho_h\right)\bar{h}+\rho_h h_t+\lambda\left(h_t\right)\epsilon_{c,t+1}$$

HAB II

Under the assumption of external habit, the intertemporal marginal rate of substitution is:

$$M_{t+1} = \delta \left(\frac{H_{t+1}}{H_t} \frac{C_{t+1}}{C_t} \right)^{-\gamma}$$

so here the expectational equation for the price dividend ratio for the asset that pays the consumption endownment C_t is:

$$v_{ct} = E_t \left\{ \delta \exp \left[-\gamma \left(\Delta h_{t+1} + \Delta c_{t+1} \right) \right] \left(1 + v_{c,t+1} \right) \exp \left(\Delta c_{t+1} \right) \right\}$$

and for the asset that pays the dividend D_t is:

$$v_{dt} = E_t \left\{ \delta \exp \left[-\gamma \left(\Delta h_{t+1} + \Delta c_{t+1} \right) \right] \left(1 + v_{d,t+1} \right) \exp \left(\Delta d_{t+1} \right) \right\}$$

HAB III

Unlike Campbell and Cochrane (1999) they incorporate cointegration between consumption and dividends.

$$d_t - c_t = \mu_{dc} + s_t$$

They set $x_t = 0$ for all t. and $v_t = 0$ so that the endowment and the dividends evolve as follows:

$$\begin{aligned} \Delta c_t &= \mu_c + \epsilon_{ct} \\ \Delta d_t &= \mu_c + (\rho_s - 1)\, s_{t-1} + \epsilon_{ct} + \epsilon_{st} \end{aligned}$$

Minimal state vector for HAB model is s_t and h_t

Aggregation I

The models are written at a monthly frequency, but their observables are annual.

From the simulations annual aggregates can be computed as twelve month moving sums sampled annually.

$$
\begin{aligned}
c_t^a &= \log\left(C_t^a\right) \quad t = 12, 24, 36, \ldots \\
d_t^a &= \log\left(D_t^a\right) \quad t = 12, 24, 36, \ldots
\end{aligned}
$$

Aggregation II

The annual geometric returns are computed as:

$$
\begin{aligned}
r_{dt}^{a} &= \sum_{k=0}^{11} r_{d,t-k} \\
r_{ft}^{a} &= \sum_{k=0}^{11} r_{f,t-k}
\end{aligned}
$$

and the log of the within-year realized variance.

$$q_t^a = \log\left(\sum_{k=0}^{11} r_{d,t-k}^2\right)$$

The Auxiliary Model

The auxiliary model is a four-variable VAR(1) at the annual frequency.

$$y_t = b_0 + By_{t-12} + e_t$$

$$y_t = \begin{pmatrix} d_t^a - c_t^a \\ c_t^a - c_{t-12}^a \\ p_{dt}^a - d_t^a \\ r_{dt}^a \end{pmatrix}$$

Sample period 1931-2001

Model Solution I

They use a simulation estimator, which meas that a long simulation of the state vector $\{u_t\}_{t=1}^{N}$ will be available for each value of the parameters considered during estimation. One might as well use the simulations in order to solve the model.

Recall that an unconditional expectation of a random variable of the form $g(u_{t+1}, u_t)$ can be computed from a simulation as

$$E(g) \simeq \frac{1}{N} \sum_{t=1}^{N} g(u_{t+1}, u_t)$$

This approximation can be made arbitrarily accurate by making N large.

Model Solution II

Given a value for the parameters of the state dynamics and values for the parameters of the utility function collected in a vector ρ. We need to find the pricing function $v_c(u)$ that solves:

$$E\left\{v_c(u_t) - M(u_{t+1}, u_t)\left[1 + v_c(u_{t+1})\right]\exp(\Delta c_{t+1})\,|u_t = u\right\} = 0$$

for all $u \in R^3$, where

$$M(u_{t+1}, u_t) = \delta^{\theta}\exp\left[-(\theta/\psi)\,\Delta c_{t+1} + (\theta - 1)\,r_c(u_{t+1}, u_t)\right]$$

$$r_c(u_{t+1}, u_t) = \log\left[\frac{1 + v_c(u_{t+1})}{v_c(u_t)}\exp(\Delta c_{t+1})\right]$$

Model Solution III

Assume $v_c(u)$ is adequately approximated by:

$$v_c(u) \simeq a_0 + a_1' u + u' A_2 u$$

put

$$g(u_{t+1}, u_t) = \{v_c(u_t) - M(u_{t+1}, u_t)[1 + v_c(u_{t+1})] \exp(\Delta c_{t+1})\} Z_t$$

where $Z_t = [1, u_t', vech'(u_t, u_t)]'$ and solve the unconditional moment conditions

$$Eg \simeq \frac{1}{N} \sum_{t=1}^{N} g(u_{t+1}, u_t, \alpha) = 0$$

for $\alpha = [a_0, a_1', vech'(A_2)]$

One then does the same for $v_d(u)$ and $r_f(u)$.

Estimation method I

- Efficient Method of Moments (EMM)(Gallant and Tauchen (1996)) with a one lag VAR as auxiliary model.
- Let $f\left(y_t | y_{t-12}, \theta\right)$ denote the transition density of the VAR(1) and let $\tilde{\theta}$ denote the ML estimates of θ computed from the data $\{\tilde{y}_t\}_{t=12,24}^{n}$.
- Then collect the parameters of the structural model into a vector ρ. Simulations from the structural model will follow a stationary density $p\left(y_t, y_{t-12} | \rho\right)$.

Estimation method II

Consider the unconditional moment function

$$m\left(\rho, \tilde{\theta}\right) = \int \int \frac{\partial}{\partial \theta} \log f\left(y_t | y_{t-12}, \tilde{\theta}\right) p\left(y_t, y_{t-12} | \rho\right) dy_t dy_{t-12}$$

GMM criterion function

$$s\left(\rho\right) = m'\left(\rho, \tilde{\theta}\right) \left(\tilde{I}\right)^{-1} m\left(\rho, \tilde{\theta}\right)$$

where

$$\tilde{I} = \sum_{t=12,24,\ldots.}^{n/12} \left\{\log f\left(\tilde{y}_t | \tilde{y}_{t-12}, \tilde{\theta}\right)\right\} \left\{\log f\left(\tilde{y}_t | \tilde{y}_{t-12}, \tilde{\theta}\right)\right\}'$$

The EMM estimator is $\hat{\rho}$ that minimizes the criterion function.

Ex Ante Risk Free Rate

The ex ante real risk free rate of interest is not directly observable but any reasonable asset pricing model must accommodate the evidence that the risk free rate r_{ft} is about 0.896 percent per year with low volatility (Campbell 2002)

They impose this restriction on $E\left[r_{ft}\right]$ by only using parameters that imply that

$$\frac{1}{N}\sum_{t=1}^{n} r_{ft} = 0.000743618 \pm 0.0004167$$

on the monthly simulation.

Note: This restriction is very important for their results. Otherwise the estimation method chooses a risk free rate on the same order of magnitude as the real stock return.

Table 1. Parameter Estimates: Four Variable Score

	SRR Model		LRR Model	
Parameter	Estimate	Std Err	Estimate	Std Err
μ_c	0.002178	0.000290	0.001921	0.000332
ρ_s	0.9892	0.0080	0.9583	0.1032
λ_{sx}			2.5004	16.9346
ρ_x			0.9871	0.0088
b_{cc}			0.1432	c
b_{ss}			0.8188	0.2788
b_{xx}			0.1100	c
ψ_{cc}	0.00646	0.00458	0.0034	c
ψ_{cs}	0.00146	0.01746		
ψ_{ss}	0.02487	0.00809	.4109e-06	.2203e-05
ψ_{xx}			0.000120	c
ρ_σ			0.9866	0.0011
δ	0.997488	0.004369	0.999566	0.000343
θ	-195.79	233.59	-12.2843	7.6243
ψ	2.00	c	2.00	c
γ	98.8969	116.7968	7.1421	3.8122
μ_{dc}	-3.3965	0.0428	-3.3857	0.0540
	$\chi^2(5) = 41.051$ (.9e-7)		$\chi^2(3) = 10.501$ (0.0148)	

Table 2. Parameter Estimates for the Habit Persistence (**HAB**) Model

Parameter	Estimate	Std Err
μ_c	0.002116	0.000250
ρ_s	0.9719	0.0154
ψ_{cc}	0.006151	0.000896
ψ_{ss}	0.036503	0.007716
ρ_h	0.9853	0.002597
δ	0.9939	0.000526
γ	0.8386	0.2463
μ_{dc}	-3.3587	0.0380
	$\chi^2(5) = 7.109$	(0.213)

Table 3. Comparision of Model Predicitons with Observed Unconditional Moments

		Observed		Predicted-**SRR**		Predicted-**LRR**		Predicted-**HAB**	
		Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
log Dividend consumption ratio	$d_t^a - c_t^a$	-3.399	0.162	-3.414	0.163	-3.384	0.154	-3.37	0.148
Consumption growth (% Per Year)	$100(\times c_t^a - c_{t-12}^a)$	1.95	2.24	2.58	1.91	2.34	2.36	2.52	1.76
Price dividend ratio	$\exp(v_{dt}^a)$	28.24	12.08	28.86	4.18	27.47	8.01	27.75	7.037
Return (% Per Year), dividend asset	$100 \times r_{dt}^a$	6.02	19.29	6.15	2.82	6.29	16.00	6.54	16.89
$100 \times \sqrt{\text{Quadratic variation}}$	$100 \times \text{std}_t^a$	16.69	09.32	3.22	0.64	14.95	8.21	14.41	9.69
Risk free rate (% Per Year)	$100 \times r_{ft}^a$			0.39	0.00	0.78	0.41	1.07	0.99
Return (% Per Year), consumption asset	$100 \times r_{ct}^a$			5.60	2.33	2.34	3.95	6.60	16.99
Equity premium (% Per Year)	$100 \times r_{dt}^a - r_{ft}^a$			5.76	2.82	5.51	16.09	5.46	17.14

Notes. Observed values are sample statistics computed from annual data, 1930–2001; predicted values are computed from a long simulation from the indicated model.

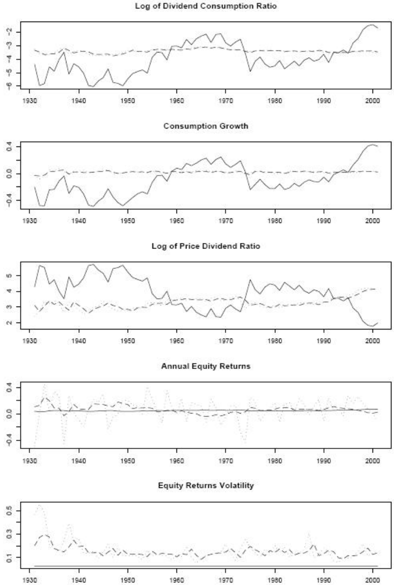

Figure 3. One Step Ahead Forecasts of the Data Confronted by the SRR Model The variables shown are $d^a_t - c^a_t$, $c^a_t - c^a_{t-12}$ $p^a_{dt} - d^a_t$, r^a_{dt}, and q^a_t of the text, in order. The dotted line is the data. The dashed line is a one-step-ahead forecast of a VAR fitted to the data. The solid line is the one-step-ahead forecast of a VAR computed from a simulation from the **SRR Model**.

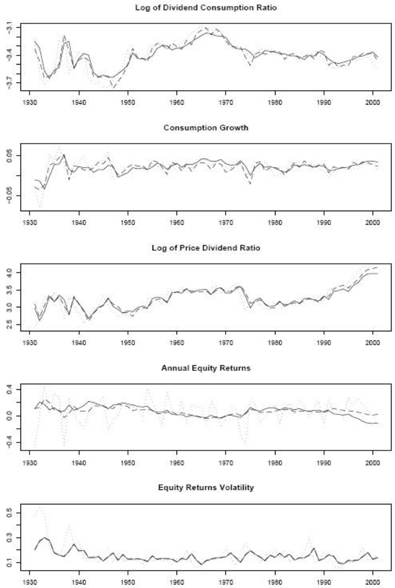

Figure 4. One Step Ahead Forecasts of the Data Confronted by the LRR Model The variables shown are $d_t^a - c_t^a$, $c_t^a - c_{t-12}^a$ $p_{dt}^a - d_t^a$, r_{dt}^a, and q_t^a of the text, in order. The dotted line is the data. The dashed line is a one-step-ahead forecast of a VAR fitted to the data. The solid line is the one-step-ahead forecast of a VAR computed from a simulation from the **LRR Model**.

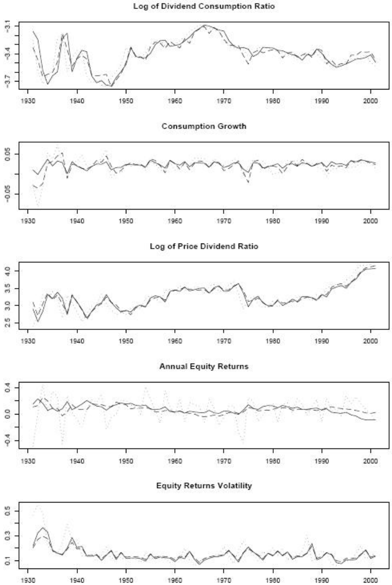

Figure 5. One Step Ahead Forecasts of the Data Confronted by the Habit Persistence (HAB) Model The variables shown are $d_t^a - c_t^a$, $c_t^a - c_{t-12}^a$ $p_{dt}^a - d_t^a$, r_{dt}^a, and q_t^a of the text, in order. The dotted line is the data. The dashed line is a one-step-ahead forecast of a VAR fitted to the data. The solid line is the one-step-ahead forecast of a VAR computed from a simulation from the **HAB model**.