Trade with Heterogeneous Prior Beliefs and Asymmetric Information

Stephen Morris 1994, presented by Shengxing Zhang

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Setup

- exchange economy with uncertainty
- H agents, $h \in \mathbb{H} = \{1, \ldots, H\}$
- L commodities
- s: payoff relevant state, $s \in S$
- payoff $u_h : \mathbb{R}^L \times S \to \mathbb{R}$,
  concave, increasing, twice differentiable
- signal: $t_h \in T_h$, private at the interim stage, public ex post
- state space: $\Omega = T \times S$, $T = T_1 \times \ldots \times T_H$
- prior beliefs: $\pi = (\pi_1, \ldots, \pi_H)$, $\pi_h \in \Delta(\Omega)$
- endowment: $e = (e_1, \ldots, e_H)$, $e_h : \Omega \to \mathbb{R}^L$

Setup II

four subperiods according to information structure

- *initial period*: not even know there will be information,
  - endowment priors on S assigned
- *ex ante period*: information structure known,
  - priors on $T \times S$ assigned
- *interim period*: private signals arrive,
  - belief updated by Bayes rule
- *ex post period*: ex post: signals revealed publicly,
  - belief updated by Bayes rule

Assumptions

- agents agree on the set of possible signals
  - Belief profile π has T^* **support**
  - $\sum_{s \in S} \pi_h(t, s) > 0 \Leftrightarrow t \in T^*$
- agents' belief have **full marginal support (FMS)** on
  - T_h: $\sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \pi_{h'}((t_h, t_{-h}), s) > 0, \forall t_h \in T_h, h \in \mathbb{H}$
  - S: $\sum_{t \in T} \pi_h(t, s) > 0, \forall s \in S$
- endowment is payoff relevant and is initially efficient

Objective

Starting from an initially efficient allocation,
find the necessary and sufficient conditions on beliefs so that no trade will take place as information unfolds in a competitve equilibrium.

Efficiency and no trade

- no trade is tightly related to efficiency
- An allocation is efficient if there does not exist trade that Pareto improves the allocation
- feasibility of trade: $\sum_{t\in T}\sum_{s\in S} x(t,s) \leq 0$
- notions of efficiency depends on :
  - available information
  - restrictions on trade and allocation from available information

Initial efficiency

- An allocation is initial efficient if
  - the allocation is payoff relevant
  - no feasible and payoff relevant trade Pareto improves the allocation
- payoff relevant: allocation only contingent on the payoff relevant state
  $e_h(t,s) = e_h(t',s)$, for all $t, t' \in T, s \in S, h \in \mathbb{H}$
- our problem starts from an initially efficient allocation

Ex ante efficiency

- before the realization of signals and payoff relevant state
- ex ante payoff: $E_h(u_h(e_h(t,s),s))$

definition of efficiency	restriction on allocation
ex ante efficient	state contingent
ex ante incentive efficient	incentive compatible
ex ante public efficient	public

Ex ante efficiency II

An allocation is

- *incentive compatible* if
  $E_h(u_h(e_h(t,s))|t_h) \geq E_h(u_h(e_h((t'_h, t_{-h})))|t_h)$
  $\forall t_h, t'_h \in T_h, h \in \mathbb{H}$
- *public* if trade is contingent on public events of common knowledge at the signal t
  - if it is measurable with respect to public events, and
  $x_h(t,s) = x_h(t',s)$ for all $s \in S, t' \in P(t)$, where
  $P(t) = \{E|t \in E$ and E is public$\}$.
  - E is public if all agents know when it is true, $E \subset K_h E, \forall h$
  - $E \subset T$ is common knowledge at t if and only if $\exists$ a public event F such that $t \in F \subset E$
  - $P(t)$ is common knowledge at t

Interim efficiency

- after the realization of signals and payoff relevant state, before signals become public
- interim payoff: $E_h(u_h(y_h(t,s),s)|t_h)$

definition of efficiency	restriction on allocation
interim efficient	no restriction
interim incentive efficient	incentive compatible
interim public efficient	public

Ex post efficiency

- signals become public
- ex post payoff: $E_h(u_h(t, s), s|t)$
- we can define in a similar fashion ex post efficiency and ex post public efficiency

Geometric interpretation of efficiency and trade

assume $u_h(x_h, s) = x_h$,
ex ante payoff: $Eu_h = \pi_h \cdot x_h$,

- belief: a vector in the state space $T \times S$
- trade x_h: direction of a class of hyperplanes in the state space
- Eu_h: the hyperplane π_h lies on in the class
- $\exists$ Pareto improving trade $\Rightarrow$ $\exists$ a hyperplane in the class that separates belief from zero
- restriction from feasibility of trade $\Rightarrow$ a hyperplane in the class that separates "feasibility vectors" from zero

By Fakas Lemma

- there either exists a feasible and Pareto improving trade
- or the zero point lies in the convex cone of the belief and the "feasibility vectors"

allocation is efficient $\Rightarrow$
the feasibility of trade imposes a (only) restriction across beliefs of different agents.

Example: ex ante efficient

Let $L = 1$,

- Pareto improving trade$\pi_h \cdot x_h \geq 0$
- feasibility: $\sum_h x_h \geq 0$

$$\begin{bmatrix} \pi_1 & \pi_2 & \dots & \pi_h \\ -I & -I & \dots & -I \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_h \end{bmatrix} = \begin{bmatrix} \pi \\ -\mathbf{1} \end{bmatrix} x$$

I: the identity matrix of dimension $(T \times S) \times (T \times S)$

By Fakas lemma,

- either $\begin{bmatrix} \pi \\ -\mathbf{1} \end{bmatrix} x \geq 0$
- or $\exists \lambda, k(t,s) > 0$ such that $\lambda \pi_h(t,s) - k(t,s) = 0, \forall t, s$

$$\therefore \pi_h(t,s) = \pi_{h'}(t,s), \forall t, s$$

Takeaway from the example

- restrictions over possible trade $\Rightarrow$ restrictions on beliefs if efficiency holds
- the restrictions vary over information structure $\Rightarrow$ restrictions on beliefs vary
- restrictions over trade signal contingent $\Rightarrow$ restrictions on beliefs signal contingent

Ex ante efficiency and beliefs

definition of efficiency	restriction on beliefs
ex ante efficient	concordant beliefs
ex ante incentive efficient	noisy version of concordant beliefs
ex ante public efficient	public version of concordant beliefs

- concordant: FMS and $\pi_h(t|s) = \pi'_h(t|s), \forall t \in T, s \in S, h', h$
- noisy version of beliefs θ:
  $\pi_h(t, s) = \alpha_h(t_h)\theta_h(t, s) + \sum_{t'_h \in T_h} \beta_h(t'_h, t_h)\pi_h((t'_h, t_{-h}), s)$,
  $\exists \alpha_h : T_h \to (0, 1]$, $\beta_h : T_h^2 \to \mathbb{R}_+$, $\forall t \in T, s \in S, h \in \mathbb{H}$
- public version of beliefs θ: $\sum_{t \in E} \pi_h(t, s) = \sum_{t \in E} \theta_h(t, s)$, $\forall s \in S$ and public events E

Interim efficiency and beliefs

definition of efficiency	restriction on beliefs
interim efficient	$\Leftarrow$ consistent version of concordant beliefs $\Rightarrow$ weakly consistent version of concordant beli

- an allocation is interim noisy/public efficient if a noisy/public version of consistent concordant beliefs
- if an allocation is interim noisy/public efficient then beliefs must be a noisy/public version of weakly consistent concordant

Interim efficiency and beliefs II

- π is a weakly consistent version of beliefs θ if
  - π and θ have public support
  - if $\sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \pi_h((t_h, t_{-h}), s) > 0$ and $\sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \theta_h((t_h, t_{-h}), s) > 0$
  - then $\pi_h(t_{-h}, s | t_h) = \theta_h(t_{-h}, s | t_h), \forall t_{-h} \in T_{-h}, s \in S$
- π are a consistent version of beliefs θ if
  - they have FMS
  - are a weakly consistent version of θ

IC condition cannot guarantee FMS property of the belief.

Ex post efficiency and beliefs

- an initially efficient allocation is ex post (public) efficient if and only if
  - beliefs are (public) revelation consistent concordant
- π is a revelation consistent version of θ if
  - π and θ satisfy FMS and
  - $\pi_h(s|t) = \theta_h(s|t)$, $\forall s \in S, t \in T$

Definition

- information revelation is endogenous to prices
- hard to categorize

Definition

An allocation e is a *rational expectations equilibrium* if there exists a price vector: $q : T \times S \to \mathbb{R}^L_+$ such that

- $q(t', s) = q(t, s)$, $\forall s \in S$, and $t'_h = t_h \Rightarrow e_h(t', s) = e_h(t, s)$, $\forall s \in S$
- there does not exist allocation x such that
  - $q(t', s) = q(t, s)$, $\forall s \in S$, and $t'_h = t_h \Rightarrow x_h(t', s) = x_h(t, s)$, $\forall s \in S$
  - $\sum_{s \in S} q(t, s) x_h(t, s) \leq 0$, $\forall t \in T, h \in \mathbb{H}$
  - $e + x$ ex ante dominates e.

Characterization

- initially efficient allocation is rational expectations equilibrium
  $\Rightarrow$ beliefs are public revelation consistent concordant
  $\Leftrightarrow$ ex post efficient
- initially efficient allocation is rational expectations equilibrium
  $\Leftarrow$ beliefs are consistent concordant (*interim*)
  $\Rightarrow$ interim efficient

But

- not shown that private information swamped by prices

Discussion

- focuses on beliefs in competitive equilibrium
- silent about price formation under the arrival of information