A Parsimonious Macroeconomic Model for Asset Pricing
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Empirical Facts

For the period from 1890 to 1991

- equity premium puzzle
  - (i) risky asset premium is "high": 6.17%
  - (ii) riskless asset return is "low": 1.94%
- volatilities
  - (i) risky asset preimum volatility is "high": 19.4%
  - (ii) riskless asset real return volatility is "low": 5.44%

Key Mechanism

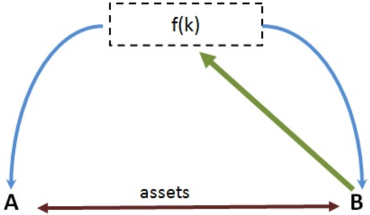
Two factors

1. limited market participation
2. heterogeneous EIS across agents

Key Mechanism

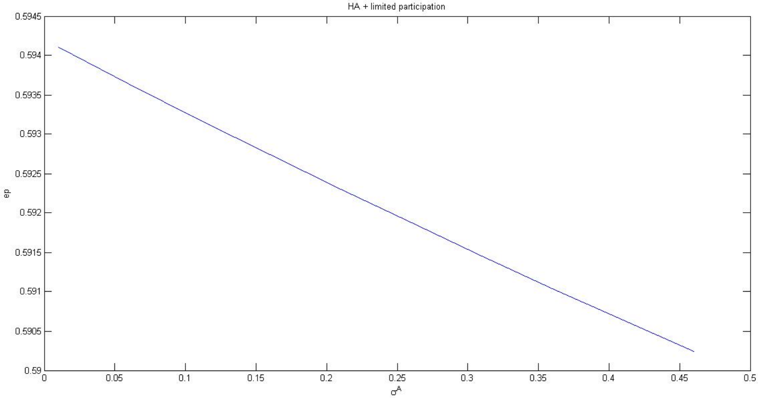
An Example

- two periods: $t = 0, 1$, known endowments $\{y_0, y_1\}$
- two agents: A and B
- prefences: $\frac{c^{1-1/\sigma^i}}{1-1/\sigma^i}$
- risky technology: $f(k)$
- only agent B can invest

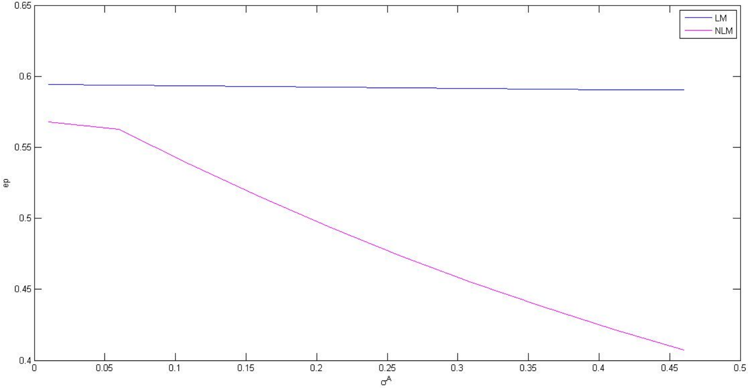


Key Mechanism

An Example



Key Mechanism
An Example



Model

Households

- Two type of households: μ stockholders and $1-\mu$ non-stockholders
- stockholders have access to equity and debt
- non-stockholders have access to debt only
- Epstein-Zin preferences:

$$U_t^i = \left[(1-\beta) u^i(c_t, 1-l_t) + \beta \left(E_t (U_{t+1}^i)^{1-\alpha^i} \right)^{\frac{1-\rho^i}{1-\alpha^i}} \right]^{1/(1-\rho^i)}$$

- EIS is proportional to $1/\rho^i$
- Assume: $1/\rho^h > 1/\rho^n$

Model

Firms

- $Y_t = Z_t K_t^{\theta} L_t^{1-\theta}$
- $\log(Z_{t+1}) = \phi \log(Z_t) + \varepsilon_{t+1}$, where $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$
- Firm can issue on-period bonds, at price P_t^f
- Let P_t^s be the price of 1 unit of equity
- Total debt is fixed at $\chi \bar{K}$ and equity normalized to 1
- Firms problem then is

$$
\begin{aligned}
P_t^s &= \max_{\{I_{t+j}, L_{t+j}\}} \sum_{j=1}^{\infty} \beta^j \frac{\Lambda_{t+j}}{\Lambda_t} D_{t+j} \\
\text{s. to} & \\
K_{t+1} &= \Phi\left(\frac{I_t}{K_t}\right) K_t + (1-\delta) K_t \\
D_t &= Z_t K_t^{\theta} L_t^{1-\theta} - W_t L_t - I_t - (1 - P_t^f)\chi \bar{K}
\end{aligned}
$$

Model

Dynamic Programming Problem

- Let B be bonds held by non-stockholders
- Let $\mathcal{Y} = (K, B, Z)$
- Dynamic programming for stockholders is

$$
\begin{aligned}
V^h(\omega, \mathcal{Y}) &= \max_{c,l,b',s'} [(1-\beta)u^i(c, 1-l) \\
&+ \beta(E[V^h(\omega', \mathcal{Y}')|Z]^{1-\alpha^h})^{\frac{1-\rho^h}{1-\alpha^h}}]^{1/(1-\rho^h)} \\
\text{s. to} & \\
c + P^f(\mathcal{Y})b' &+ P^s(\mathcal{Y})s' \leq \omega + W(Z,K)l \\
\omega' &= b' + s'(P^s(\mathcal{Y}') + D(\mathcal{Y}')) \\
K' &= \Gamma_K(\mathcal{Y}),\ B' = \Gamma_B(\mathcal{Y}) \\
b' &\geq \underline{B}
\end{aligned}
$$

A recursive competitive equilibrium for this economy is given by:

- $\{V^i(\omega,\mathcal{Y}), c^i(\omega,\mathcal{Y}), b^{i\prime}(\omega,\mathcal{Y}), l^i(\omega,\mathcal{Y}), s^{i\prime}(\omega,\mathcal{Y})\}$, $i = h, n$
- price functions $\{P^f(\mathcal{Y}), P^s(\mathcal{Y}), W(Z,K)\}$
- policies for the firm $L(\mathcal{Y}), I(\mathcal{Y})$ and $D(\mathcal{Y})$
- law of motions $\Gamma_K(\mathcal{Y})$ and $\Gamma_B(\mathcal{Y})$

such that, given prices

(i) $\{c^i(\omega,\mathcal{Y}), b^{i\prime}(\omega,\mathcal{Y}), l^i(\omega,\mathcal{Y}), s^{i\prime}(\omega,\mathcal{Y})\}$ are optimal for households

(ii) $\{L(\mathcal{Y}), I(\mathcal{Y}), D(\mathcal{Y})\}$ are optimal for firms

(iii) markets clear:
  - (bond market) $\mu b^{h\prime}(\omega^h,\mathcal{Y}) + (1-\mu) b^{n\prime}(\omega^n,\mathcal{Y}) = \chi \bar{K}/P^f(\mathcal{Y})$,
  - (stock market) $\mu s^{h\prime}(\omega,\mathcal{Y}) = 1$
  - (labor market) $\mu l^h(\omega^h,\mathcal{Y}) + (1-\mu) l^n(\omega^n,\mathcal{Y}) = L(\mathcal{Y})$

(iv) law of motions
  - $K' = \Gamma_K(\mathcal{Y}) = (1-\delta)K + \Phi(\frac{I(\mathcal{Y})}{K})K$
  - $B' = \Gamma_B(\mathcal{Y}) = (1-\mu) b^{n\prime}(\omega^n,\mathcal{Y})$

(v) a stationary distribution $\mathbf{P}$ over $\mathcal{Y}$

Results

- Define returns as

$$
\begin{aligned}
1 + R^s &= \frac{D' + P^{s\prime}}{P^s} \\
1 + R^f &= 1/P^f \\
R^{ep} &= R^s - R^f
\end{aligned}
$$

- Utility function is

$$u(c, 1 - l) = c^{1-\rho^i}$$

Results

	US Data	Consumption Model			
α^h, α^n		6/6	6/6	6/6	6/12
$1/\rho^h$, $1/\rho^n$		0.3/0.1	0.3/0.3	0.1/0.1	0.3/0.1
$E(R^{ep})$	6.17	5.46	2.44	7.65	5.52
$E(R^f)$	1.94	1.31	3.20	0.24	1.35
$\sigma(R^{ep})$	19.4	21.9	15.3	27.1	22.0
$\sigma(R^f)$	5.44	6.65	4.55	8.52	6.71
$\frac{\sigma(\Delta \log c^h)}{\sigma(\Delta \log c^n)}$	$>$ 1.5-2	2.42	0.78	1.12	2.44

Results

Why is $E(R^{ep})$ high?

- consumption can be expressed as

$$
\begin{aligned}
c^h &= \underbrace{\left(W + \frac{\theta Z K^\theta L^{1-\theta} - I}{\mu}\right)}_{A^h} - \underbrace{\frac{B - P^f B'}{\mu}}_{a^h} \\
c^n &= \underbrace{W}_{A^n} + \underbrace{\frac{B - P^f B'}{1-\mu}}_{-a^n}
\end{aligned}
$$

Results

Why is $E(R^{ep})$ high?

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$$c^h = \underbrace{\left(W + \frac{\theta Z K^{\theta} L^{1-\theta} - I}{\mu}\right)}_{A^h} - \underbrace{\frac{B - P^f B'}{\mu}}_{a^h}$$

$$c^n = \underbrace{W}_{A^n} + \underbrace{\frac{B - P^f B'}{1-\mu}}_{-a^n}$$

- then

$$var(\Delta \log c^i) \approx var(\Delta \log A^i) + var\left(\Delta \frac{a^i}{A^i}\right) + 2cov\left(\Delta \log A^i, \Delta \frac{a^i}{A^i}\right)$$

Results

Why is $E(R^{ep})$ high?

	$E(A^i)/E(c^i)$	$E(a^i)/E(c^i)$	$\sigma^2(\Delta \log c^i)$
Stochholders	1.011	0.011	$(3.61\%)^2$
Non-stochholders	0.995	0.005	$(1.48\%)^2$

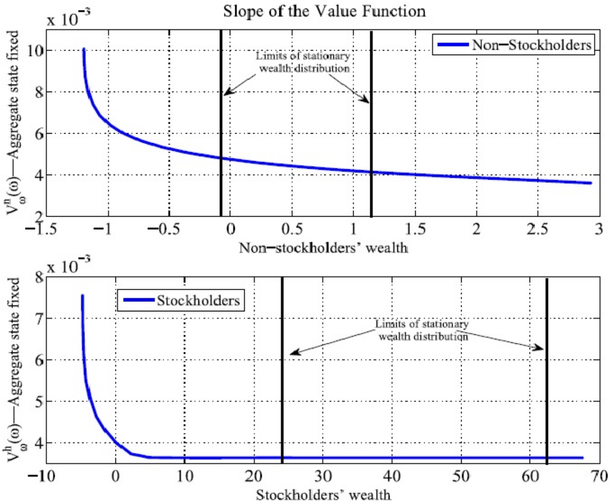
	Fraction of $\sigma^2(\Delta \log c^i)$		
	$\sigma^2(\Delta \log A^i)$	$\sigma^2(\Delta \frac{a^i}{A^i})$	$2\sigma(\Delta \log A^i, \Delta \frac{a^i}{A^i})$
Stochholders	0.185	0.340	0.475
Non-stochholders	3.13	0.61	-2.74

	$\text{corr}(\Delta \log A^i,\ \Delta \frac{a^i}{A^i})$
Stochholders	0.947
Non-stochholders	-0.99

Results

Why is $E(R^{ep})$ high?

look at the value functions



Results

Why is $\sigma(R^f)$ so low?

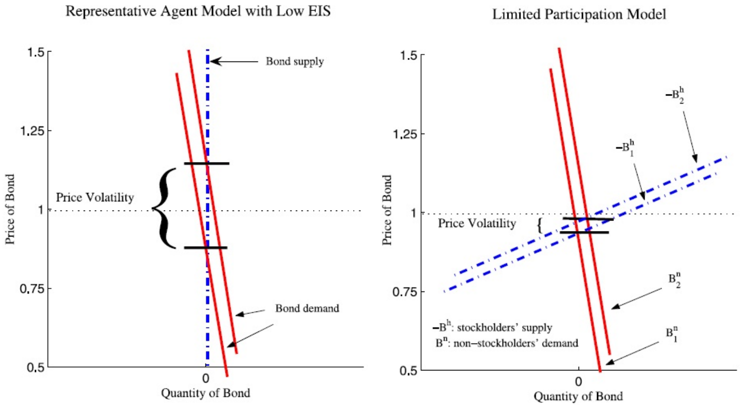


Figure: bond price volatility

Results

Why is $E(R^f)$ so low?

A good approximation is

$$R_t^f \approx -\ln\beta + \rho^h E_t(\Delta \log c_{t+1}^h) + \kappa$$

Two things are important

- Epstein-Zin preferences : $\rho^h \neq 1/\alpha^h$
- no growth : $E[E_t(\Delta \log c_{t+1}^h)] = 0$

Algorithm

Dynamic Programming Problem

$$
\begin{aligned}
V^h(\omega, \mathcal{Y}) &= \max_{c,l,b',s'} [(1-\beta)u^i(c, 1-l) \\
&+ \beta(E[V^h(\omega', \mathcal{Y}')|Z]^{1-\alpha^h})^{\frac{1-\rho^h}{1-\alpha^h}}]^{1/(1-\rho^h)} \\
\text{s. to} & \\
c + P^f(\mathcal{Y})b' &+ P^s(\mathcal{Y})s' \leq \omega + W(Z, K)l \qquad (1) \\
\omega' &= b' + s'(P^s(\mathcal{Y}') + D(\mathcal{Y}')) \qquad (2) \\
K' &= \Gamma_K(\mathcal{Y}), \;\; B' = \Gamma_B(\mathcal{Y}) \qquad (3) \\
b' &\geq \underline{B} \qquad (4)
\end{aligned}
$$

Algorithm

step 0

(a) Choose grids for ω^h, ω^n and $\mathcal{Y}$

(b) Guess initial functions $P^{f,1}(\mathcal{Y}), P^{s,1}(\mathcal{Y}), \Gamma_K^1(\mathcal{Y}), \Gamma_B^1(\mathcal{Y})$ and $D^j(\mathcal{Y})$

(c) denote j to the iteration number

step 1

Given $P^{f,j-1}, P^{s,j-1}, \Gamma_K^{j-1}, \Gamma_B^{j-1}$ and $D^{j-1}(\mathcal{Y})$ solve each agent problem by iterating over the value function

Algorithm

step 2

(a) use $c^{h,j}(\omega, \mathcal{Y})$ and $V^{h,j}(\omega, \mathcal{Y})$ to compute $\frac{\Lambda(\mathcal{Y}')}{\Lambda(\mathcal{Y})}$

(b) use the stochastic discount factor to solve for firm policy $I^j(\mathcal{Y})$

(c) obtain $D^j(\mathcal{Y}) = ZK^\theta L^{1-\theta} - WL - I^j(\mathcal{Y}) - (1 - P^{f,j-1}(\mathcal{Y}))\chi\bar{K}$ and update $\Gamma^j_K(\mathcal{Y}) = (1-\delta)K + \Phi(\frac{I^j(\mathcal{Y})}{K})K$

(d) set $\tilde{P}^0 = P^{s,j-1}(\mathcal{Y})$ and iterate over

$$\tilde{P}^m(\mathcal{Y}) = E\left[\beta \frac{\Lambda^j(\mathcal{Y}')}{\Lambda^j(\mathcal{Y})}(D^j(\mathcal{Y}') + \tilde{P}^{m-1}(\mathcal{Y}'))|\mathcal{Y}\right]$$

set $P^{s,j} = \tilde{P}^M(\mathcal{Y})$

Algorithm

step 3

(a) Solve

$$\begin{aligned} \tilde{V}^h(\omega, \mathcal{Y}, \hat{q}) &= \max_{c,l,b',s'} [(1-\beta)u^i(c, 1-l) \\ &+ \beta(E[V^{h,j}(\omega', \mathcal{Y}')|Z]^{1-\alpha^h})^{\frac{1-\rho^h}{1-\alpha^h}}]^{1/(1-\rho^h)} \\ \text{s. to} & \\ c + \hat{q}b' &+ P^{s,j}(\mathcal{Y})s' \leq \omega + W(K,Z)l \end{aligned}$$

and (2) – (4)

(b) last step gives policies $\tilde{b}^h(\omega, \mathcal{Y}, \hat{q})$ and $\tilde{b}^n(\omega, \mathcal{Y}, \hat{q})$

(c) For each $\mathcal{Y}$, search for $\hat{q}$ such that bond markets clear. Call it q^*

(d) Set $P^{f,j}(\mathcal{Y}) = q^*(\mathcal{Y})$

Algorithm

step 4

Obtain $\Gamma^{j}_{B}(\mathcal{Y}) = (1-\mu)\tilde{b}^{n}(\omega^{n}, \mathcal{Y}, q^{*}(\mathcal{Y}))$

step 5

Iterate over steps 1 to 4 until convergence

step 6 check!

We haven't used equity market clearing. Check if $|\mu s'(\omega, \mathcal{Y}) - 1| < 10^{-5}$

Algorithm

Question: why not $P^f(\mathcal{Y}) = E\left[\beta \frac{\Lambda^j(\mathcal{Y}')}{\Lambda^j(\mathcal{Y})} | \mathcal{Y}\right]$ instead of *step 3*?

Results

Preferences

Three utility functions

1. Consumption : $u(c, 1-l) = c^{1-\rho^i}$
2. Cobb-Douglas: $u(c, 1-l) = (c^{\gamma}(1-l)^{(1-\gamma)})^{1-\rho^i}$
3. GHH : $u(c, 1-l) = (c - \psi \frac{l^{1+\gamma}}{1+\gamma})^{1-\rho^i}$

Results

	US Data	Cons	CD	GHH
$E(R^{ep})$	6.17	5.46	2.65	4.21
$\sigma(R^{ep})$	19.4	21.9	15.4	17.4
$\sigma(R^{s})$	19.3	20.6	14.8	16.5
$E(R^{ep})/\sigma(R^{ep})$	0.32	0.25	0.17	0.24
$E(R^{f})$	1.94	1.31	2.87	1.42
$\sigma(R^{f})$	5.44	6.65	4.91	4.10

Results

Why is $E(R^{ep})$ countercyclical?

For the sharpe ratio

$$\frac{E(R^{ep})}{\sigma(R^{ep})} \approx \alpha^h \sigma_t(\Delta \log c^h_{t+1}) \underbrace{corr_t(\Delta \log c^h_{t+1}, R^{ep}_{t+1})}_{\approx 1}$$

but $\sigma_t(\Delta \log c^h_{t+1})$ is counterciclycal
why?...