Credit Shocks and Aggregate Fluctuations In An Economy With Production Heterogeneity

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Motivation

- Most recent recession - difficult to explain by TFP shocks in standard RBC model
- This Model - disruption of credit can generate deep and lengthy recession
- Interaction of financial frictions (borrowing constraints) and real frictions (capital irreversibility)
- Frictions generate misallocation in response to credit shock, delaying recovery
The Model - Setup

- Heterogenous Firms with \( y = z \epsilon F(k, n) \), Representative HH
- Aggregative productivity shocks \( z \), idiosyncratic productivity shocks \( \epsilon \)
- Firms idiosyncratic state \((b, k, \epsilon)\)
- Aggregate state \((z, \mu)\)
- Exogenous exit w.p. \( \pi_d \)
  - If survive: choose investment, dividends, borrowing
- Capital Irreversibility: \( k' = \begin{cases} (1 - \delta)k + i & \text{if } i \geq 0 \\ (1 - \delta)k + \frac{1}{\theta_k}i & \text{if } i < 0 \end{cases} \)
- Borrowing constraint \( b \leq \theta_b \theta_k k \)
The Model - Firms Problem

\[ v_0(k, b, \epsilon_i; z_l, \mu) = \pi_d \max_n[z_l \epsilon_i F(k, n) - w(z_l, \mu)n + \theta_k(1 - \delta)k - b] + (1 - \pi_d)v(k, b, \epsilon_i; z_l, \mu) \]

\[ v(k, b, \epsilon_i; z_l, \mu) = \max\{v^u(k, b, \epsilon_i; z_l, \mu), v^d(k, b, \epsilon_i; z_l, \mu)\} \]

\[ v^i(k, b, \epsilon_i; z_l, \mu) = \max_{n, k', b', D}[ D + \sum_{m=1}^{N_z} \pi^z_{lm} d_m(z_l, \mu) \sum_{j=1}^{N_\epsilon} \pi_{ij}v_0(k', b', \epsilon_j; z_m, \mu')] \]

s.t. \( 0 \leq D \leq z_l \epsilon_i F(k, n) - w(z_l, \mu)n + q(z_l, \mu)b' - b - J^c(i)[k' - (1 - \delta)k] \)

\( b' \leq \theta_b \theta_k k \) and \( \mu' = \Gamma(z, \mu) \)

\[ k' \begin{cases} 
(1 - \delta)k & \text{if } i = u \\
\leq (1 - \delta)k & \text{if } i = d 
\end{cases} , J^c(i) = \begin{cases} 
1 & \text{if } i = u \\
\theta_k & \text{if } i = d 
\end{cases} \]
The Model - Household Problem

\[ V^h(\lambda; z_l, \mu) = \max_{c, n^h, \lambda'} \left[ U(c, 1 - n^h) + \beta \sum_{m=1}^{N_z} \pi^z_m V^h(\lambda'; z_m, \mu') \right] \]

s.t. \[ c + \int_S \rho_1(k', b', \epsilon'; z_l, \mu) d\lambda(k', b', \epsilon') \leq w(z_l, \mu)n^h + \int_S \rho_0(k, b, \epsilon; z_l, \mu) d\lambda(k, b, \epsilon) \]
and \[ \mu' = \Gamma(z, \mu) \]

Where \( \rho_0(k, b, \epsilon; z_l, \mu) \) and \( \rho_1(k', b', \epsilon'; z_l, \mu) \) are current period and next-period prices for (1-period) shares in firms, and \( \lambda(k, b, \epsilon) \) is a measure over shares.
The Model - Equilibrium

- Equilibrium Requires that the Following Expressions Hold:
  1) \( w(z, \mu) = \frac{U_2(C, 1-N)}{U_1(C, 1-N)} \)
  2) \( q(z, \mu) = \beta \sum_{m=1}^{N_z} \pi_{l,m} \frac{U_1(C', 1-N')}{U_1(C, 1-N)} \)
  3) \( d_m(z, \mu) = \beta \frac{U_1(C_m', 1-N_m')}{U_1(C, 1-N)} \)

- Accordingly, rescale firms problem so that dividends are valued at price \( p(z, \mu) = U_1(C, 1 - N) \)

- \( V_0(k, b, \epsilon_i; z_l, \mu) = \pi_d \max_n p(z_l, \mu)[z_l \epsilon_i F(k, n) - w(z_l, \mu)n + \theta_k (1 - \delta) k - b] + (1 - \pi_d) V(k, b, \epsilon_i; z_l, \mu) \)

- \( V(k, b, \epsilon_i; z_l, \mu) = \max_{n, k', b', D}[p(z_l, \mu) D + \sum_{m=1}^{N_z} \sum_{j=1}^{N_{\epsilon}} \pi_{l,m} \pi_{ij} V_0(k', b', \epsilon_j; z_m, \mu')] \)

- s.t. Budget Constraint, Borrowing Constraint
The Solution - Method

- Since there are no labor frictions, optimal labor choice straightforward: \( z \in F_k(k, n) = w(z_l, \mu) \)

- Split Firms into Unconstrained (borrowing constraint does not bind in current period or in any possible future period), and Constrained
  - Constrained Firms will set \( D = 0 \) (shadow value of retained earnings exceeds shadow value of current dividends)
  - Unconstrained Firms are indifferent between paying dividends and retained earnings
The Solution - Unconstrained Firms I

- For unconstrained firms, current debt b only impacts their value through current earnings.
- Thus we can express its value as $W(k, b, \epsilon) = w(k, \epsilon) - pb$, with $w(k, \epsilon) \equiv W(k, 0, \epsilon)$.
- Define $\pi \equiv z \epsilon F(k, N(k, \epsilon; z, \mu)) - wN(k, \epsilon; z, \mu) - b$.
- $W^u(k, b, \epsilon_i; z_l, \mu) = p\pi(k, b, \epsilon_i) + p(1 - \delta)k + \max_{k' \geq (1 - \delta)k} [-pk' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_{\epsilon}} \pi_{lm}^{z} \pi_{ij} w_0(k', \epsilon_j; z_m, \mu')]$.
- $W^d(k, b, \epsilon_i; z_l, \mu) = p\pi(k, b, \epsilon_i) + p\theta_k(1 - \delta)k + \max_{k' \leq (1 - \delta)k} [-p\theta_k k' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_{\epsilon}} \pi_{lm}^{z} \pi_{ij} w_0(k', \epsilon_j; z_m, \mu')]$.
Define Upward and Downward Capital Targets as Follows:

\[ k_u^*(\epsilon_i) = \arg \max_{k'} [-pk' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_{\epsilon}} \pi_z \pi_{ij} w_0(k', \epsilon_j; z_m, \mu')] \]

\[ k_d^*(\epsilon_i) = \arg \max_{k'} [-p\theta_k k' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_{\epsilon}} \pi_z \pi_{ij} w_0(k', \epsilon_j; z_m, \mu')] \]

Targets are independent of current capital and debt, only depend on aggregate state and \( \epsilon \).

Capital Adjustment Characterized by \((S, s)\) rule:

\[ K^w(k, \epsilon; z, \mu) = \begin{cases} 
 k_u^*(\epsilon; z, \mu) & \text{if } k < \frac{k_u^*(\epsilon; z, \mu)}{1-\delta} \\
 (1 - \delta)k & \text{if } k \in \left[ \frac{k_u^*(\epsilon; z, \mu)}{1-\delta}, \frac{k_d^*(\epsilon; z, \mu)}{1-\delta} \right] \\
 k_d^*(\epsilon; z, \mu) & \text{if } k > \frac{k_d^*(\epsilon; z, \mu)}{1-\delta} 
\end{cases} \]
Unconstrained Firms Minimum Savings Policy

- Solve for minimum level of financial savings such that an unconstrained firm of type \((\epsilon, k)\) will never be affected by borrowing constraints

- Let \(\tilde{B}(K^w(k, \epsilon; z, \mu), \epsilon_j; z_m, \mu')\) be the maximum debt level at which a firm entering the next period with capital \(K^w(k, \epsilon; z, \mu)\) and \((\epsilon_j; z_m)\) will remain unconstrained

- Minimum savings policy \(B^w(k, \epsilon; z, \mu)\) defined recursively:
  - \(B^w(k, \epsilon; z, \mu) = \min\{\epsilon_j|\pi_{ij}>0 \text{ and } z_m|\pi_{im}^z>0\} \tilde{B}(K^w(k, \epsilon; z, \mu), \epsilon_j; z_m, \mu')\)
  - \(\tilde{B}(k, \epsilon; z, \mu) = z\epsilon F(k, N(k, \epsilon)) - wN(k, \epsilon) + q\min\{B^w(k, \epsilon; z, \mu), \theta_b \theta_k k\} - \mathcal{J}(K^w(k, \epsilon) - (1 - \delta)k)[K^w(k, \epsilon) - (1 - \delta)k]\)
First, check whether previously constrained firm can adopt unconstrained decision rules for capital and savings while paying non-negative dividends

- For those still constrained - since $D = 0$, can reduce their optimization problem to 1 dimension - debt determined residually

- Identify choice sets for upward/downward capital adjustment

  - $\Lambda^u = [(1 - \delta)k, (1 - \delta)k + q\theta_b\theta_kk + \pi(k, b, \epsilon)]$
  - $\Lambda^d = [0, \min((1 - \delta)k, (1 - \delta)k + \frac{1}{\theta_k}[q\theta_b\theta_kk + \pi(k, b, \epsilon)])]$
Constrained Firms Solution II

- $V^c(k, b, \epsilon; z, \mu) = \max\{V^u(k, b, \epsilon; z, \mu), V^d(k, b, \epsilon; z, \mu)\}$
- With $V^i(k, b, \epsilon; z, \mu) = \max_{k' \in \Lambda^i(k, b, \epsilon)} \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi^z_{lm} \pi_{ij} V_0(k', b_u'(k'), \epsilon_j; z_m, \mu')$
  - s.t. $b_u'(k') = \frac{1}{q}(-\pi(k, b, \epsilon) + J(i)[k' - (1 - \delta)k])$
  - $\mu' = \Gamma(z, \mu)$
- For $i = \{u, d\}$, where $J(i) = \begin{cases} 1 & \text{if } i \geq 0 \\ \theta_k & \text{if } i < 0 \end{cases}$
Calibration

- Utility: \( u(c, L) = \log(c) + 2.15L \)
- Production: \( F(k, n) = k^{0.27} L^{0.60} \)
- \( \theta_b = 1.35 \) set to match debt/assets ratio of \( .366 \)
- \( \theta_k = 0.95, \rho_\epsilon = .653, \sigma_\epsilon = .135 \) (Idiosyncratic AR(1) persistence, S.D.) chosen jointly to match Average, Standard Deviation, and Serial Correlation of investment rates across establishments
Results - Firm Life-Cycle

FIGURE 2. Cohort in steady state

- average capital
- average net debt
FIGURE 8. The Recent Recession
Unexpected 55% drop in value of firms collateral (drop in $\theta_b$)

Households and Firms expect financial conditions will return to normal with 40% probability

- Experiment I - Financial conditions never actually return to normal
- Experiment II - After 4 periods, complete recovery of financial conditions, agents know change is permanent
Results - Persistent Financial Crisis

**FIGURE 9.** Persistent financial crisis

- **Output and Capital:**
  - Red dash: Output
  - Blue line: Capital

- **Employment and Consumption:**
  - Red dash: Employment
  - Blue line: Consumption

- **Investment:**
  - Red dash: Investment

- **Measured TFP and Exogenous TFP:**
  - Red dash: Measured TFP
  - Dotted line: Exogenous TFP

The graphs illustrate the percentage change over time for various economic indicators during a persistent financial crisis.
Results - Financial Crisis and Recovery

FIGURE 13. Financial crisis and recovery

- Output and Capital
- Employment and Consumption
- Investment
- Measured TFP and Exogenous TFP
Results - Financial Crisis and Recovery

- Peak-to-trough comparison of 2007 US Recession:

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