Credit Shocks and Aggregate Fluctuations In An Economy With Production Heterogeneity

by Aubhik Khan and Julia Thomas

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Motivation

- Most recent recession - difficult to explain by TFP shocks in standard RBC model
- This Model - disruption of credit can generate deep and lengthy recession
- Interaction of financial frictions (borrowing constraints) and real frictions (capital irreversibility)
- Frictions generate misallocation in response to credit shock, delaying recovery

The Model - Setup

- Heterogenous Firms with $y = z\epsilon F(k, n)$, Representative HH
- Aggregative productivity shocks z, idiosyncratic productivity shocks ϵ
- Firms idiosyncratic state (b, k, ϵ)
- Aggregate state (z, μ)
- Exogenous exit w.p. π_d
  - If survive: choose investment, dividends, borrowing
- Capital Irreversibility: $k' = \begin{cases} (1-\delta)k + i & \text{if } i \geq 0 \\ (1-\delta)k + \frac{1}{\theta_k} i & \text{if } i < 0 \end{cases}$
- Borrowing constraint $b \leq \theta_b \theta_k k$

The Model - Firms Problem

- $v_0(k, b, \epsilon_i; z_l, \mu) = \pi_d \max_n[z_l \epsilon_i F(k, n) - w(z_l, \mu)n + \theta_k(1-\delta)k - b] + (1-\pi_d)v(k, b, \epsilon_i; z_l, \mu)$
- $v(k, b, \epsilon_i; z_l, \mu) = max\{v^u(k, b, \epsilon_i; z_l, \mu), v^d(k, b, \epsilon_i; z_l, \mu)\}$
  $v^i(k, b, \epsilon_i; z_l, \mu) = \max_{n,k',b',D} \; [\, D + \sum_{m=1}^{N_z} \pi^z_{lm} d_m(z_l, \mu) \sum_{j=1}^{N_\epsilon} \pi_{ij} v_0(k', b', \epsilon_j; z_m, \mu')]$
  s.t. $0 \leq D \leq$
  $z_l \epsilon_i F(k, n) - w(z_l, \mu)n + q(z_l, \mu)b' - b - \mathcal{J}^c(i)[k' - (1-\delta)k]$
  $b' \leq \theta_b \theta_k k$ and $\mu' = \Gamma(z, \mu)$
  k' $\begin{cases} \geq (1-\delta)k & \text{if } i = u \\ \leq (1-\delta)k & \text{if } i = d \end{cases}$, $\mathcal{J}^c(i) = \begin{cases} 1 & \text{if } i = u \\ \theta_k & \text{if } i = d \end{cases}$

The Model - Household Problem

- $V^h(\lambda; z_l, \mu) =$
  $\max_{c,n^h,\lambda'}[U(c, 1 - n^h) + \beta \sum_{m=1}^{N_z} \pi^z_{lm} V^h(\lambda'; z_m, \mu')]$
- s.t. $c + \int_S \rho_1(k', b', \epsilon'; z_l, \mu) d\lambda(k', b', \epsilon') \leq$
  $w(z_l, \mu) n^h + \int_S \rho_0(k, b, \epsilon; z_l, \mu) d\lambda(k, b, \epsilon)$
  and $\mu' = \Gamma(z, \mu)$
- Where $\rho_0(k, b, \epsilon; z_l, \mu)$ and $\rho_1(k', b', \epsilon'; z_l, \mu)$ are current period and next-period prices for (1-period) shares in firms, and $\lambda(k, b, \epsilon)$ is a measure over shares

The Model - Equilibrium

- Equilibrium Requires that the Following Expressions Hold:
  - 1) $w(z,\mu) = \frac{U_2(C,1-N)}{U_1(C,1-N)}$
  - 2) $q(z,\mu) = \beta \sum_{m=1}^{N_z} \pi_{lm}^z \frac{U_1(C',1-N')}{U_1(C,1-N)}$
  - 3) $d_m(z,\mu) = \beta \frac{U_1(C_m{}',1-N_m{}')}{U_1(C,1-N)}$
- Accordingly, rescale firms problem so that dividends are valued at price $p(z,\mu) = U_1(C,1-N)$
- $V_0(k,b,\epsilon_i;z_l,\mu) = \pi_d \max_n p(z_l,\mu)[z_l\epsilon_i F(k,n) - w(z_l,\mu)n + \theta_k(1-\delta)k - b] + (1-\pi_d)V(k,b,\epsilon_i;z_l,\mu)$
- $V(k,b,\epsilon_i;z_l,\mu) = \max_{n,k',b',D}[p(z_l,\mu)D + \sum_{m=1}^{N_z}\sum_{j=1}^{N_\epsilon} \pi_{lm}^z \pi_{ij} V_0(k',b',\epsilon_j;z_m,\mu')]$
  - s.t. Budget Constraint, Borrowing Constraint

The Solution - Method

- Since there are no labor frictions, optimal labor choice straightforward : $z\epsilon F_k(k, n) = w(z_l, \mu)$
- Split Firms into Unconstrained (borrowing constraint does not bind in current period or in any possible future period), and Constrained
  - Constrained Firms will set $D = 0$ (shadow value of retained earnings exceeds shadow value of current dividends)
  - Unconstrained Firms are indifferent between paying dividends and retained earnings

The Solution - Unconstrained Firms I

- For unconstrained firms, current debt b only impacts their value through current earnings
- Thus we can express its value as $W(k, b, \epsilon) = w(k, \epsilon) - pb$, with $w(k, \epsilon) \equiv W(k, 0, \epsilon)$
- Define $\pi \equiv z\epsilon F(k, N(k, \epsilon; z, \mu)) - wN(k, \epsilon; z, \mu) - b$
- $W^u(k, b, \epsilon_i; z_l, \mu) = p\pi(k, b, \epsilon_i) + p(1-\delta)k + \max_{k' \geq (1-\delta)k}[-pk' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi_{lm}^z \pi_{ij} w_0(k', \epsilon_j; z_m, \mu')]$
- $W^d(k, b, \epsilon_i; z_l, \mu) = p\pi(k, b, \epsilon_i) + p\theta_k(1-\delta)k + \max_{k' \leq (1-\delta)k}[-p\theta_k k' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi_{lm}^z \pi_{ij} w_0(k', \epsilon_j; z_m, \mu')]$

The Solution - Unconstrained Firms II

- Define Upward and Downward Capital Targets as Follows:
- $k_u^*(\epsilon_i) =$
  $\arg\max_{k'}[-pk' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi_{lm}^z \pi_{ij} w_0(k', \epsilon_j; z_m, \mu')]$
- $k_d^*(\epsilon_i) =$
  $\arg\max_{k'}[-p\theta_k k' + \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi_{lm}^z \pi_{ij} w_0(k', \epsilon_j; z_m, \mu')]$
  - Targets are independent of current capital and debt, only depend on aggregate state and ϵ
- Capital Adjustment Characterized by (S, s) rule:

$$K^w(k, \epsilon; z, \mu) = \begin{cases} k_u^*(\epsilon; z, \mu) & \text{if } k < \frac{k_u^*(\epsilon; z, \mu)}{1-\delta} \\ (1-\delta)k & \text{if } k \in [\frac{k_u^*(\epsilon; z, \mu)}{1-\delta}, \frac{k_d^*(\epsilon; z, \mu)}{1-\delta}] \\ k_d^*(\epsilon; z, \mu) & \text{if } k > \frac{k_d^*(\epsilon; z, \mu)}{1-\delta} \end{cases}$$

Unconstrained Firms Minimum Savings Policy

- Solve for minimum level of financial savings such that an unconstrained firm of type (ϵ, k) will never be affected by borrowing constraints
- Let $\tilde{B}(K^w(k, \epsilon; z, \mu), \epsilon_j; z_m, \mu'))$ be the maximum debt level at which a firm entering the next period with capital $K^w(k, \epsilon; z, \mu)$ and $(\epsilon_j; z_m)$ will remain unconstrained
- Minimum savings policy $B^w(k, \epsilon; z, \mu)$ defined recursively:
  - $B^w(k, \epsilon; z, \mu) = \min_{\{\epsilon_j | \pi_{ij} > 0 \text{ and } z_m | \pi^z_{lm} > 0\}} \tilde{B}(K^w(k, \epsilon; z, \mu), \epsilon_j; z_m, \mu'))$
  - $\tilde{B}(k, \epsilon; z, \mu)) = z\epsilon F(k, N(k, \epsilon)) - wN(k, \epsilon) + q\min\{B^w(k, \epsilon; z, \mu), \theta_b \theta_k k\} - \mathcal{J}(K^w(k, \epsilon) - (1 - \delta)k)[K^w(k, \epsilon) - (1 - \delta)k)]$

Constrained Firms Solution I

- First, check whether previously constrained firm can adopt unconstrained decision rules for capital and savings while paying non-negative dividends
  - For those still constrained - since $D = 0$, can reduce their optimization problem to 1 dimension - debt determined residually
- Identify choice sets for upward/downward capital adjustment
  - $\Lambda^u = [(1-\delta)k, (1-\delta)k + q\theta_b\theta_k k + \pi(k, b, \epsilon)]$
  - $\Lambda^d = [0, \min((1-\delta)k, (1-\delta)k + \frac{1}{\theta_k}[q\theta_b\theta_k k + \pi(k, b, \epsilon)])]$

Constrained Firms Solution II

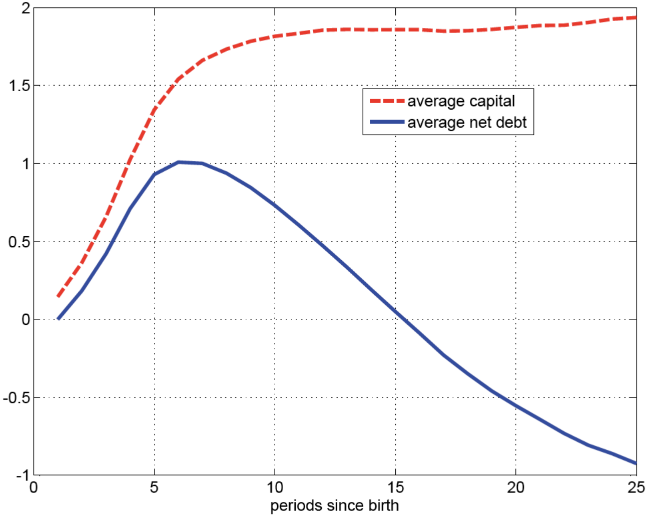
- $V^c(k, b, \epsilon; z, \mu) = \max\{V^u(k, b, \epsilon; z, \mu), V^d(k, b, \epsilon; z, \mu)\}$
- With $V^i(k, b, \epsilon; z, \mu) = \max_{k' \in \Lambda^i(k,b,\epsilon)} \beta \sum_{m=1}^{N_z} \sum_{j=1}^{N_\epsilon} \pi^z_{lm} \pi_{ij} V_0(k', {b_u}'(k'), \epsilon_j; z_m, \mu')$
  - s.t. ${b_u}'(k') = \frac{1}{q}(-\pi(k, b, \epsilon) + \mathcal{J}(i)[k' - (1 - \delta)k]),$ $\mu' = \Gamma(z, \mu)$
  - For $i = \{u, d\}$, where $\mathcal{J}(i) = \begin{cases} 1 & \text{if } i \geq 0 \\ \theta_k & \text{if } i < 0 \end{cases}$

Calibration

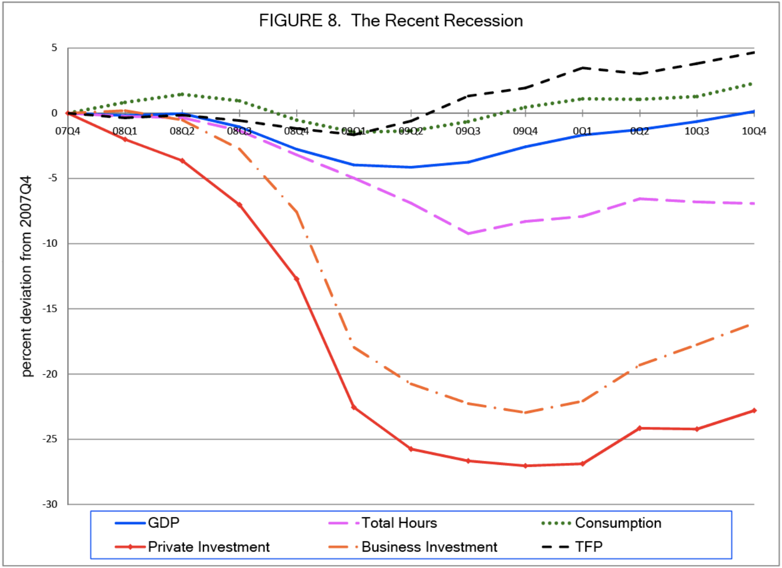
- Utility: $u(c, L) = log(c) + 2.15L$
- Production: $F(k, n) = k^{.27} L^{.60}$
- $\theta_b = 1.35$ set to match debt/assets ratio of .366
- $\theta_k = 0.95, \rho_\epsilon = .653, \sigma_\epsilon = .135$ (Idiosyncratic AR(1) persistence, S.D.) chosen jointly to match Average, Standard Deviation, and Serial Correlation of investment rates across establishments

Results - Firm Life-Cycle

FIGURE 2. Cohort in steady state



Data - Recent Recession

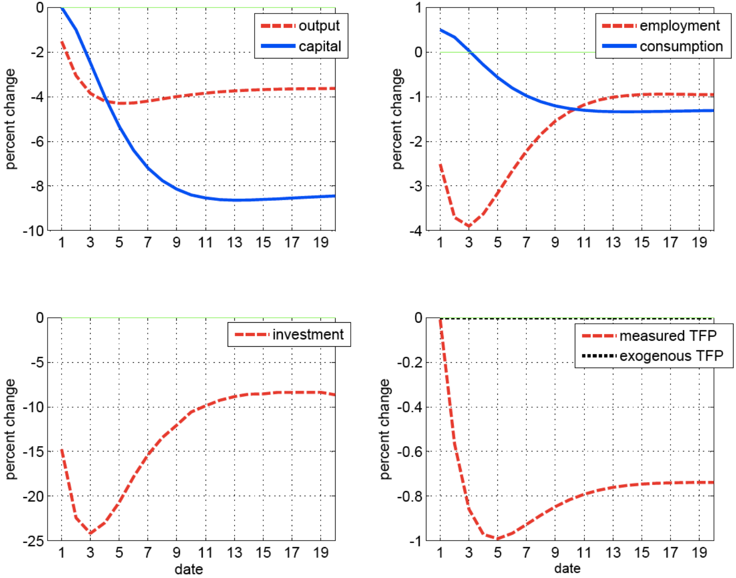


Experiment- Financial Crisis

- Unexpected 55% drop in value of firms collateral (drop in θ_b)
- Households and Firms expect financial conditions will return to normal with 40% probability
  - Experiment I - Financial conditions never actually return to normal
  - Experiment II - After 4 periods, complete recovery of financial conditions, agents know change is permanent

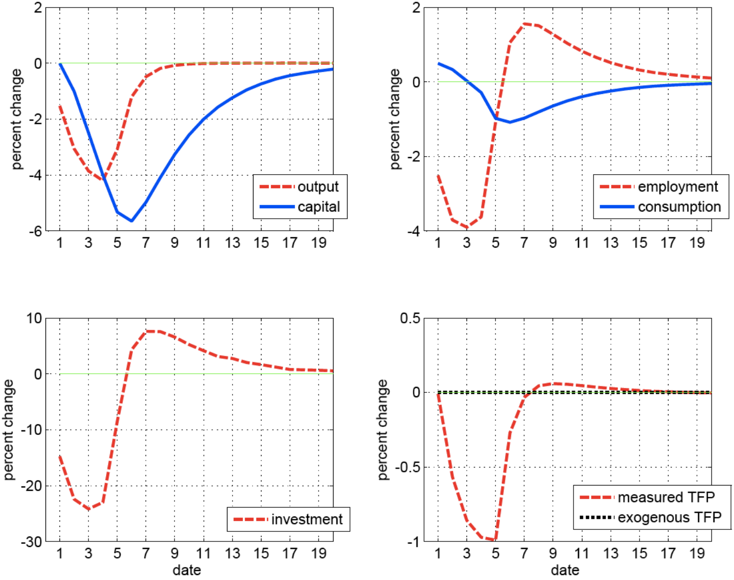
Results - Persistent Financial Crisis

FIGURE 9. Persistent financial crisis



Results - Financial Crisis and Recovery

FIGURE 13. Financial crisis and recovery



Results - Financial Crisis and Recovery

- Peak-to-trough comparison of 2007 US Recession:

	GDP	I	N	C	TFP
Data	-4.14	-25.75	-6.89	-1.36	-0.60
Model	-4.20	-22.98	-3.62	-0.58	-0.97