# Competitive Equilibrium in Asset Markets with Adverse Selection

by Veronica Guerrieri and Robert Shimer

Dan Greenwald

October 5, 2011

# Introduction

- Motivation: characterize asset markets in which sellers know the quality of assets but not buyers.
- Goal is to obtain a separating equilibrium. Sellers choose the price at which to offer each asset, and rational buyers correctly infer the type of asset by its price. This contrasts with previous literature focused on pooling equilibrium, in which all assets are sold at a common price.
- Incentive compatibility is obtained through probabilistic selling (rationing). At equilibrium, an asset offered at price $p$ only sells with probability $\Theta(p)$, which is decreasing in $p$ so that $\Theta(p) < 1$ for all but the lowest price.
- Idea is that the cost of keeping a high quality asset is lower than the cost of keeping a low quality asset, so only sellers of high quality assets are willing to set a high price (and incur a high chance of no sale).

# Introduction (Continued)

- Under this model, an increase in dispersion in the quality of assets can deliver falling asset prices and illiquidity even if average asset quality is held constant.
- The government can increase liquidity and asset prices by offering to buy assets at some intermediate price (although it will take a loss on these purchases). In contrast, government intervention is ineffective under a pooling equilibrium.

# Assets and Agents

- There are $J$ types of infinitely-lived asset (or "tree"), indexed by $j$, each with measure $K_j$. A type $j$ asset produces $\delta_j$ units of the consumption good (or "fruit") each period, where $0 \leq \delta_1 < \cdots < \delta_J$.
- There is a continuum of infinitely-lived risk-neutral agents with measure 1, indexed by $i$.
- In each period, each individual receives a preference shock $s_t \in \{l, h\}$, which determines their discount factor $\beta_{s_t}$, with $0 < \beta_l < \beta_h < 1$. Preference shocks are i.i.d. across individuals and time.

# Timing

- At the start of each period, each individual owns a vector $(k^1_{i,t}, \ldots, k^J_{i,t})$ of trees, which produce fruit.
- Next, each individual's preference shock (i.e. his or her discount factor between $t$ and $t+1$) is realized.
- Individuals then trade trees for fruit on a competitive market. Those with $\beta_l$ will sell trees, and those with $\beta_h$ will buy trees (this is actually an assumption).
- At the end of the period, agents consume their fruit.

# Adverse Selection

- Only the owner of a tree knows its quality. Sellers choose at what price to bring each tree to market, and buyers must try to infer quality based on the price of the tree.
- Individuals have rational beliefs about the types of trees sold in each market. A buyer believes that a tree sold at price $p$ is type $j$ with probability $\gamma_j(p)$. Under a separating equilibrium, type of tree will be known with certainty.

# Rationing

- Let $\Theta(p)$ indicate the ratio of buyers to sellers in the market for trees at price $p$ (more correctly, ratio of buyer's fruit to the value of trees for sale).
- If a seller brings a tree to market at price $p$, it will sell with probability $\min\{\Theta(p), 1\}$.
- Similarly, a buyer that brings value $p$ of fruit to market will be able to purchase an asset with probability $\min\{\Theta^{-1}(p), 1\}$.
- Important difference: seller rationing is a necessary feature of equilibrium under adverse selection, occurs at all but lowest price. Buyer rationing is a more arbitrary phenomenon that occurs when there is a relative shortage of sellers.

# Notation

- Let $k^j_{i,t}(s^{t-1})$ denote individual $i$'s holdings of type $j$ trees at the start of period $t$, following state $s^{t-1}$.
- Let $q^j_{i,t}(p; s^t)$ denote individual $i$'s time $t$ purchases of type $j$ trees at price $p$. If we define

$$Q_{i,t}(p; s^t) = \sum_{j=1}^{J} q^j_{i,t}(p, s^t)$$

then $Q_{i,t}(p; s^t)$ is individual $i$'s total time $t$ purchases of trees at price $p$ under state $s^t$. By rational expectations and a "law of large numbers" argument, we have

$$q^j_{i,t}(p, s^t) = \gamma^j_t(p; s^t) Q_{i,t}(p; s^t).$$

- Let $c_{i,t}(s^t)$ denote individual $i$'s time $t$ consumption of fruit under state $s^t$.

# Individual's Problem

- Each individual chooses $(c_{i,t}(s^t), k^j_{i,t}(s^t), q^j_{i,t}(s^t))$ to maximize

$$\sum_{t=0}^{\infty} \sum_{s^t} \left( \prod_{\tau=0}^{t-1} \pi_{s_\tau} \beta_{s_\tau} \right) \pi_{s_t} c_{i,t}(s^t)$$

subject to the budget constraint

$$\sum_{j=1}^{J} \delta_j k^j_{i,t}(s^{t-1}) = c_{i,t}(s^t) + \int_0^\infty p \cdot Q_{i,t}(p; s^t)\, dp$$

for all $t$ and $s^t$ and the law of motion for tree holdings

$$k^j_{i,t+1}(s^t) = k^j_{i,t}(s^{t-1}) + \int_P q^j_{i,t}(p; s^t)\, dp.$$

# Individual's Problem (Continued)

- Individuals also face constraints depending on whether they are buyers or sellers in that period.
- Buyer's constraints: must have enough fruit on hand for tree purchases

$$\sum_{j=1}^{J} \delta_j k_{i,t}^j(s^{t-1}) \geq \int_0^\infty \max\{\Theta_t(p;s^t),1\} p \cdot Q_{i,t}(p;s^t)\, dp$$

and cannot buy more trees than are for sale

$$\int_P q_{i,t}^j(p;s^t) = \int_P \gamma_t^j(p;s^t) Q_{i,t}(p;s^t)\, dp.$$

- Seller's constraint: cannot sell more trees than he or she owns.

$$k_{i,t}^j(s^{t-1}) \geq -\int_0^\infty \max\{\Theta_t(p;s^t)^{-1},1\} q_{i,t}^j(p;s^t)\, dp.$$

# Value Functions

- Let $\bar{V}(k)$ be the value function for an individual holding tree vector $k$ at the start of the period (evaluated before you know your type). The value function has the intuitive form

$$\bar{V}(k) = \pi_h V_h(k) + \pi_l V_l(k)$$

where $V_s(k)$ is the value of $k$ after the realization $s_t = s$.

- Since everything is linear, the state-contingent value functions have the convenient form

$$V_h(k) = \sum_{j=1}^{J} v_{h,j} k_j$$

$$V_l(k) = \sum_{j=1}^{J} v_{l,j} k_j$$

where $v_{s,j}$ is expected value of holding a type $j$ tree in state $s$.

# Value Functions (Continued)

- If we define $\bar{v}^j = \pi_h v_{h,j} + \pi_l v_{l,j}$, then we have

$$\bar{V}(k) = \sum_{j=1}^{J} \bar{v}_j k_j.$$

- The $v_{s,j}$ functions solve simple recursive problems. Let $1 + \lambda$ be the value of a unit of fruit to a buyer. Then

$$v_{h,j} = \delta_j(1 + \lambda) + \beta_h \bar{v}_j \quad (1)$$

$$v_{l,j} = \delta_j + \max_p \left( \min\{\Theta(p)\} p + (1 - \min\{\Theta(p)\}) \beta_l \bar{v}_j \right). \quad (2)$$

- $\lambda$ can be calculated as

$$\lambda = \max_{p \geq 0} \left\{ \min\{\Theta(p)^{-1}, 1\} \left( \frac{\beta_h \sum_{i=1}^{J} \gamma_j(p) \bar{v}_j}{p} - 1 \right) \right\}.$$

# Partial Equilibrium

- Before proceeding to general equilibrium, calculate partial equilibrium taking $\lambda$ as fixed. Later, we will choose $\lambda$ to make markets clear.
- Partial equilibrium is $(v_h, v_l, \Theta, \gamma, P, \mu)$, where where $P$ is a set of prices $P$, $\mu$ is the measure corresponding to the distribution of assets for sale on $P$, such that
  1. The Bellman equations (1) and (2) hold.
  2. Buyers optimize: for all $p \in \mathbb{R}_+$

$$\lambda \geq \min\{\Theta(p)^{-1}, 1\} \left( \frac{\beta_h \sum_{j=1}^{J} \gamma_j(p) \bar{v}_j}{p} - 1 \right)$$

  with equality for $p \in P$.

  3. Sellers optimize:

$$v_{l,j} \geq \delta_j + \min\{\Theta(p), 1\}(p - \beta_l \bar{v}_j) + \beta_l \bar{v}_j$$

  with equality if $\Theta(p) < \infty$ and $\gamma_j(p)$

  4. All trees owned by sellers are offered for sale at some price $p \in P$, i.e. for each $j$ we have

$$\pi_l K_j = \int_P \gamma_j(p)\, d\mu(p).$$

# Partial Equilibrium (Continued)

- Method of computation:
  1. Solve for $p_1$, $\bar{v}_1$ such that the worst good sells with probablity 1.
  2. Recursively solve for $p_j$, $\bar{v}^j$ such that incentive compatibility constraint

$$\Theta(p_j)(p_j - \beta_l \bar{v}_{j-1}) = \min\{\Theta(p_{j-1}), 1\}(p_{j-1} - \beta_l \bar{v}_{j-1})$$

  holds given $(p_{j'}, \bar{v}_{j'})_{j'<j}$.

**Proposition**

A partial equilibrium $(v_h, v_l, \Theta, \gamma, P, \mu)$ exists for each $\lambda \in [0, \beta_h/\beta_l - 1]$

.

# General Equilibrium

- A competitive equilibrium consists of the objects of a partial equilibrium $(v_h, v_l, \Theta, \gamma, P, \mu)$, and $\lambda \in [0, \beta_h/\beta_l - 1]$ such that
  1. $(v_h, v_l, \Theta, \gamma, P, \mu)$ is a partial equilibrium given $\lambda$.
  2. Markets clear:

$$\pi_h \sum_{j=1}^{J} \delta_j K_j = \int_P \Theta(p) \cdot p \, d\mu(p).$$

**Proposition**

A competitive equilibrium $(\lambda, v_h, v_l, \Theta, \gamma, P, \mu)$ exists and is unique.

- Note that uniqueness implies there are no non-separating equilibria.

# Continuous Spectrum of Trees

- Tree distribution is dense on some interval $[\underline{\delta}, \bar{\delta}]$ with density $\kappa(\delta)$.
- New incentive compatibility constraint

$$\Theta'(p_j)(p_j - \beta_l \bar{v}_j) + \Theta(p_j) = 0 \tag{3}$$

- Solve for $\underline{\delta}$, then use (3) to obtain partial equilibrium:

$$\Theta(p) = \left(\frac{p(\underline{\delta})}{p}\right)^{\frac{\beta h}{\beta_h - \beta_l(1+\lambda)}}$$

Finally, choose $\lambda$ so that markets clear.

# Persistent Shocks

- Need to introduce persistence (i.i.d. every period makes state meaningless in continuous time), so allow state to follow a first-order Markov chain, where $\pi_{ss'}$ is the probability of moving from $\beta_s$ to $\beta_{s'}$.
- To move to continuous time, fix

$$\rho_s = \frac{1 - \beta_s}{\Delta}$$

$$z_{ss'} = \frac{\pi_{ss'}}{\Delta}$$

and take the limit as $\Delta \to 0$.

# Results

- Keep assumption of continuous spectrum of trees, and also assume $\lambda = 0$ (to focus on adverse selection).
- Solution using usual method (solve for $\underline{\delta}$ and apply IC constraint).
- $\Theta(p) \to 0$ as $\Delta \to 0$, but the sale rate per unit of time converges

$$\alpha(p) = \lim_{\Delta \to 0} \frac{\Theta(p)}{\Delta} = \frac{z_{hl} + z_{lh} + \rho_l}{\left(\frac{p}{p(\underline{\delta})}\right)^{\frac{z_{hl}+z_{lh}+\rho_l}{\rho_l - \rho_h}} - 1} \geq 0.$$

- From the perspective of a seller, $\alpha(p)$ is the arrival rate of a Poisson process that allows him or her to sell the asset listed at price $p$.
- Contrast to search frictions, which can disappear as the interval goes to zero.

# Modeling the Financial Crisis

- Can consider financial crisis as experiment in which assets which were formerly considered to be relatively uniform are now recognized to be of different types.
- Assume that trees were formerly believed to be of type $\delta_0$, but are now distributed on the interval $[\underline{\delta}, \bar{\delta}]$. To focus on adverse selection effects, can keep the mean of the distribution at $\delta_0$.
- Price of trees with $\delta < \delta_0$ must fall.
- Price of trees with $\delta > \delta_0$ can also fall, because of liquidity effects. Because of rationing, buyers value trees less (especially high $\delta$ trees) because they may not be able to sell them when they have $\beta_l$ later.
- Delivers "fire sale" outcome without issues of debt/leverage, or decline in average quality of assets.

# Government Intervention

- Assume that the government announces it is willing to purchase any asset at price $\hat{p}$, and destroys all trees that it buys (similar to original TARP proposal). All sellers with trees that would have sold for less than $\hat{p}$ sell them to the government (what about $p > \hat{p}$?).
- After intervention, it is common knowledge that the worst type of tree is now $\hat{\delta} > \underline{\delta}$, where $p(\hat{\delta}) = \hat{p}$. This tree sells with certainty at price $\hat{p}$, and the liquidity and price of all trees with $\delta > \hat{\delta}$ increase.

# Pooling Equilibrium

- Extensive literature on adverse selection in financial markets, but typically under pooling equilibria, where all assets sold at a common price $p$ (see Eisfeldt, 2004; Kurlat, 2009; Daley and Green, 2010; Chari, Shourideh and Zetlin-Jones, 2010).
- Guerrieri and Shimer consider pooling equilibria under i.i.d. shocks and a continuum of trees by imposing common price requirement.

## Proposition

A pooling equilibrium exists. Under any pooling equilibrium, only trees with $\delta \leq \delta^*$ are sold, for

$$\delta^* = \frac{\beta_h(1-\bar{\beta})}{\beta_l(1-\beta_h+\lambda(1-\pi_h\beta_h))} \cdot \frac{\int_{\underline{\delta}}^{\delta^*} \delta\kappa(\delta)}{\int_{\underline{\delta}}^{\delta^*} \kappa(\delta)}.$$

# Differences Between Separating and Pooling Equilibrium

- Under separating equilibrium, model predicts that higher quality trees will sell at a higher price but take longer to sell. In pooling equilibrium these are uncorrelated.
- This could be tested empirically: within a class of securities that look outwardly similar, those that sell for a higher price will take longer to sell but will generate higher dividends on average.
- Different responses to increase in dispersion ("fire-sale" scenario): Under pooling, both prices and quality of assets sold must fall. Under separating equilibrium, assets of all qualities are sold and prices of high quality assets can either go up or down.

# Differences (Continued)

- "TARP"-style government intervention is much less effective under pooling.
- Reason is that pooling equilibrium depends only on distribution (specifically, expectation) of assets sold at pooling price. If government cannot differentiate between assets then intervention does not change market price or induce more agents to sell.
- Key difference is that separating equilibrium allows government to selectively purchase assets by type, because of information contained in prices.