Rollover Risk and Market Freezes

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The Model

- $t \in [0, 1]$, state $s_t \in \{L, H\}$.
- Risk neutral market participants: banks, lenders.
- 1 asset with finite life
    - no intermediate payoff
    - terminal value at $t = 1$: $v(s_1) \in \{50, 100\}$.
- 0 risk free interest rate.

Debt

- Banks want to repo an asset
- Debt has fixed maturity $\tau \in (0,1)$.
  $\Longrightarrow$ Debt has to rolled over N times

  $$\tau = \frac{1}{N+1}$$

  at each $t_n = n\tau$, $n = 0, 1, ..., N+1$.

Default

In case of default:

- lenders seize collateral
- collateral is liquidated
- lenders recover λ of the sale price (buyers are finance constrained).

Information Structure

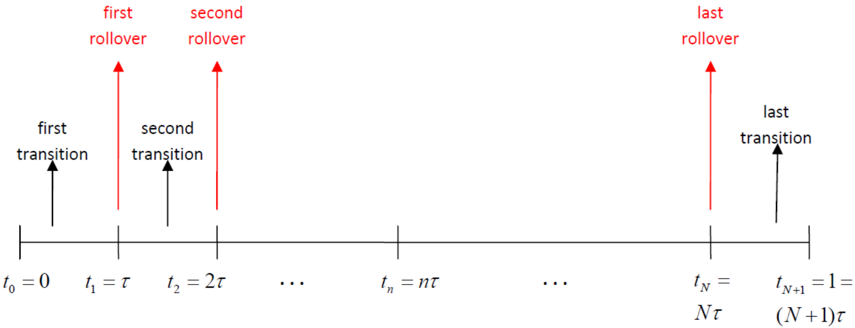
- s_t is publicly observable each t.
- Information arrives each t at a rate α.
- When information arrives, the state changes randomly according to

$$P = \begin{bmatrix} p_{LL} & p_{LH} \\ p_{HL} & p_{HH} \end{bmatrix}$$

- Number of information events in a given interval of time is random.
- Transition form t to $t + T$

$$P(T) = \begin{bmatrix} p_{LL}(T) & p_{LH}(T) \\ p_{HL}(T) & p_{HH}(T) \end{bmatrix}$$

Timing


first
rollover
second
rollover
last
rollover
first
transition
second
transition
last
transition
$t_0 = 0$
$t_1 = \tau$
$t_2 = 2\tau$
$\cdots$
$t_n = n\tau$
$\cdots$
$t_N = N\tau$
$t_{N+1} = 1 = (N+1)\tau$

Example

- $\alpha = 10$
- $$P = \begin{bmatrix} 0.2 & 0.8 \\ 0.01 & 0.99 \end{bmatrix}$$
- $\tau = 0.1,\ N = 100.$
- $\lambda = 0.9$

Fundamental Value

- Expected value at $t = 0$

$$
\begin{aligned}
V_0^H &= p_{HH}(1) \times 100 + p_{HL}(1) \times 50 = 99.383 \\
V_0^L &= p_{LH}(1) \times 100 + p_{LL}(1) \times 50 = 99.367
\end{aligned}
$$

Face Value of Debt

- $D > 100 \implies$ Default in all states
- $D \in (0.50) \cup (50, 100) \implies$ can increase D holding default prob. constant.
- 2 possible options:

$$D \in \{50, 100\}$$

Debt Capacity

$t = t_{99}$

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$$D_{99}^{H} = 100 \text{ and } D_{99}^{L} = 50.$$

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- Face value of debt

$$D_{99}^{H} = 100 \text{ and } D_{99}^{L} = 50.$$

- Debt capacity

$$B_{99}^{H} = 99.854 \text{ and } B_{99}^{L} = 50.$$

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Debt Capacity

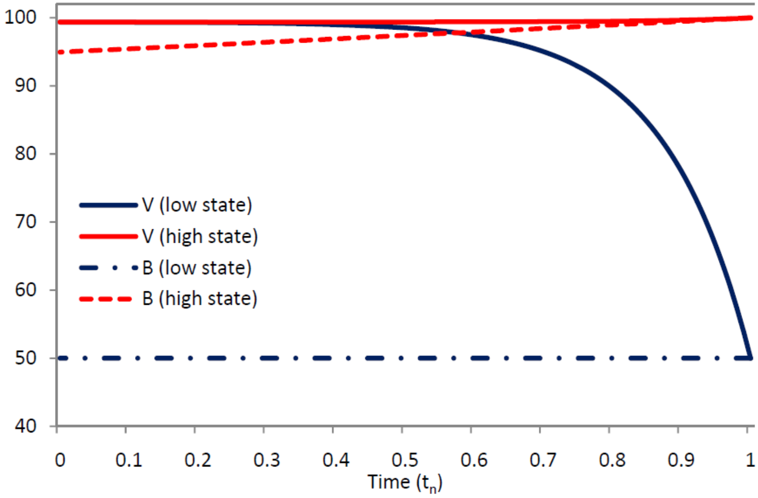
- ... $t = 0$

$$B_0^H = 94.9603 \text{ and } B_0^L = 50$$

but

$$V_0^H = 99.383 \text{ and } V_0^L = 99.367.$$

Market Freeze

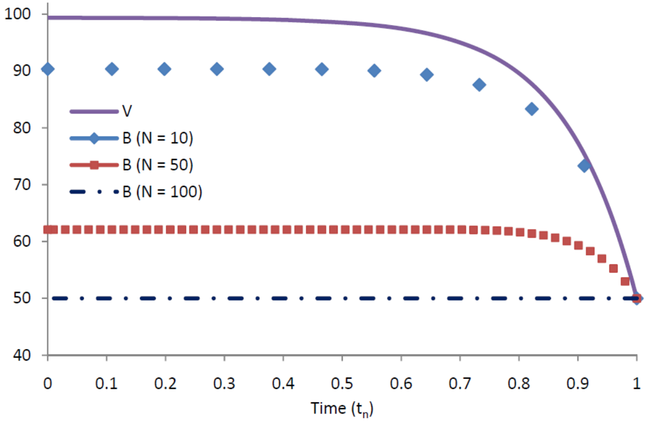


Market Freeze

In this example, small changes in V, huge change in B.
In general, market freezes occur when

- refinancing frequency is high,
- prob. of switching states in the low state is low enough, or
- liquidation cost is high enough.

Market Freeze



Market Freeze

Key assumptions:

- Credit risk, $v_L \neq v_H$.
- Need for rollover, $\tau < 1$.
- Liquidation cost, $\lambda < 1$.
- Constrained buyers, market value $=$ debt capacity.